Engineering Mathematics

UNIT : Partial Differential Equation

Today's Target

- Homogeneous Linear Partial Differential Equation with constant coefficient

  Particular Integral(PI) : Case-II and Case-III

- PYQs
- DPP

Course Details(Paid) : All Subjects

1	Recorded Video Lectures (100 % Syllabus Coverage)
2	Pdf Notes
3	Lecture wise DPP with Answers
4	DPP Solutions
5	Unit wise set of PYQs with Answers
6	Course will be completed by 'Last week of Nov 2023'
	Tentative AKTU Semester Exam- Jan/Feb

Paid Courses are available in Gateway Classes Application
Link in Description

Homogeneous Linear PDE with constant coefficient

(All partial Derivatives are of same order)

GW GATEWAY CLASSES

Particular Integral

Case-II

$$F(x, y) = x^m y^n$$

$$PI = \frac{1}{f(D, D')} x^m y^n$$

$$PI = [1 + \phi(D, D')]^{-1} x^m y^n$$

Note :-

(i) If $n = m$ (Taking either D or D' as common factor)

(ii) If $n > m$ (Taking D' as a common factor)

(iii) If $m > n$ (Taking D as a common factor)

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After taking common, Apply the formulae given below

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(1) $(1+x)^{-1} = 1 - x + x^2 - x^3 + x^4 + \ldots\ldots$

(2) $(1-x)^{-1} = 1 + x + x^2 + x^3 + x^4 + \ldots\ldots$

(3) $(1+x)^{-2} = 1 - 2x + 3x^2 - 4x^3 + \cdots$

(4) $(1-x)^{-2} = 1 + 2x + 3x^2 + 4x^3 + \cdots$

Q.1 Solve : $\frac{\partial^2 z}{\partial x^2} + 3\frac{\partial^2 z}{\partial x \partial y} + 2\frac{\partial^2 z}{\partial y^2} = 12xy.$ **(UPTU-2015)**

$$(D^2 + 3DD' + 2D'^2)z = 12xy$$

Put $D = m, D' = 1$

$$(m^2 + 3m + 2)z = 12xy$$

Aux. Eqⁿ

$$m^2 + 3m + 2 = 0$$

$$(m+1)(m+2) = 0$$

$$m = -1, -2$$

$$CF = f_1(y - x) + f_2(y - 2x)$$

$$PI = \frac{1}{(D^2 + 3DD' + 2D'^2)} 12xy$$

$$PI = \frac{1}{D^2\left[1 + 3\frac{D'}{D} + 2\frac{D'^2}{D^2}\right]} 12xy$$

$$PI = \frac{1}{D^2}\left[1 + \left(3\frac{D'}{D} + 2\frac{D'}{D^2}\right)\right]^{-1} \times 12xy$$

$$PI = \frac{1}{D^2}\left[1 - \left(3\frac{D'}{D} + 2\frac{D'^2}{D^2}\right) + \left(3\frac{D'}{D} + 2\frac{D'^2}{D^2}\right)^2 - \cdots\right] 12xy$$

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$$PI = \frac{12}{D^2}\left[1 - 3\frac{D'}{D}\right] xy$$

$$PI = \frac{12}{D^2}\left[xy - \frac{3D'}{D}(xy)\right]$$

$$PI = \frac{12}{D^2}\left[xy - 3\times\frac{1}{D}x\right]$$

$$PI = \frac{12}{D^2}\left[xy - \frac{3x^2}{2}\right]$$

$$PI = \frac{12}{D}\left[\int\left(xy - \frac{3x^2}{2}\right)dx\right]$$

$$PI = \frac{12}{D}\left[\frac{x^2}{2}y - \frac{3x^3}{3\times 2}\right]$$

$$PI = 12\int\left(\frac{x^2}{2}y - \frac{x^3}{2}\right)dx$$

$$PI = 12\left(\frac{x^3}{6}y - \frac{x^4}{8}\right)$$

$$\boxed{PI = 2x^3y - \frac{3x^4}{2}}$$

complete solution

$$\boxed{z = CF + PI}$$

Q.2 Solve : $\frac{\partial^3 u}{\partial x^3} - \frac{\partial^3 u}{\partial y^3} = x^3 y^3$.

$$\frac{\partial^3 u}{\partial x^3} - \frac{\partial^3 u}{\partial y^3} = x^3 y^3$$

$$(D^3 - D'^3)u = x^3 y^3$$

Put $D = m$, $D' = 1$

$$(m^3 - 1)u = x^3 y^3$$

Aux. Equation

$$m^3 - 1 = 0$$

$$(m-1)(m^2 + m + 1) = 0$$

$m = 1, \omega, \omega^2$

$$CF = f_1(y+x) + f_2(y+\omega x) + f_3(y+\omega^2 x)$$

$$PI = \frac{1}{(D^3 - D'^3)} x^3 y^3$$

$$PI = \frac{1}{D^3\left[1 - \frac{D'^3}{D^3}\right]} x^3 y^3$$

$$PI = \frac{1}{D^3}\left[1 - \frac{D'^3}{D^3}\right]^{-1} x^3 y^3$$

$$PI = \frac{1}{D^3}\left[1 + \frac{D'^3}{D^3}\right] x^3 y^3$$

$$PI = \frac{1}{D^3}\left[x^3 y^3 + \frac{D'^3}{D^3} x^3 y^3\right]$$

$$PI = \frac{1}{D^3}\left[x^3 y^3 + 6x\frac{1}{D^3} x^3\right]$$

$$PI = \frac{1}{D^3}\left[x^3y^3 + \frac{6 \times x^6}{4 \times 5 \times 6}\right]$$

$$PI = \frac{1}{D^3}\left[x^3y^3 + \frac{x^6}{20}\right]$$

$$PI = \frac{x^6y^3}{4 \times 5 \times 6} + \frac{x^9}{20 \times 7 \times 8 \times 9}$$

$$\boxed{PI = \frac{x^6y^3}{120} + \frac{x^9}{10080}}$$

Complete solution is

$$u = CF + PI$$

$$\boxed{u = f_1(y+x) + f_2(y+\omega x) + f_3(y+\omega^2 x) + \frac{x^6y^3}{120} + \frac{x^9}{10080}}$$

CASE- III :- General case

Any Function of $(x,\ y)$

$$PI = \frac{1}{F(D, D')}\ \phi(x,\ y)$$

$$PI = \frac{1}{(D-m_1D')(D-m_2D')\ \ldots.(D-m_nD)}\ \phi(x,\ y)$$

$$\frac{1}{(D-m_1D')}\ \phi(x,\ y) = \int \phi(x,\ c-m_1x)\,dx$$

Put $y = c - m_1x$ $\quad y = (c + m_1x)$

Q.3 Solve : $(D^2 - DD' - 2D'^2)z = (y-1)e^x$. **(UPTU-2014)**

$$(D^2 - DD' - 2D'^2)z = (y-1)e^x$$

Put $D = m, D' = 1$

$$(m^2 - m - 2)z = (y-1)e^x$$

Aux. Eqn

$$m^2 - m - 2 = 0$$

$$(m+1)(m-2) = 0$$

$$m = -1, 2$$

$$CF = f_1(y-x) + f_2(y+2x)$$

$$PI = \frac{1}{D^2 - DD' - 2D'^2}(y-1)e^x$$

$$PI = \frac{1}{(D+D')(D-2D')}(y-1)e^x$$

$$PI = \frac{1}{(D-2D')}\left[\frac{1}{D+D'}(y-1)e^x\right]$$

$$PI = \frac{1}{(D-2D')} \int (c+x-1)\, e^x\, dx$$
(1) (2)

$$PI = \frac{1}{(D-2D')} \left[(c+x-1)\, e^x - 1 \times e^x \right]$$

$$PI = \frac{1}{D-2D'} \left[(c+x-1-1)\, e^x \right]$$

$$PI = \frac{1}{D-2D'} \left[(c+x-2)\, e^x \right]$$

$$PI = \frac{1}{D-2D'} (y-2)\, e^x$$

$$PI = \int (c-2x-2)\, e^x\, dx$$

$$PI = (c-2x-2)\, e^x - (-2)\, e^x$$

$$PI = (c-2x-2+2)\, e^x$$

$$PI = (c-2x)\, e^x$$

$$PI = y e^x$$

complete solution

$$z = CF + PI$$

Q.4 Solve the linear partial deferential equation $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 6\frac{\partial^2 z}{\partial y^2} = y\cos x$.

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 6\frac{\partial^2 z}{\partial y^2} = y\cos x$$

$$(D^2 + DD' - 6D'^2)z = y\cos x$$

Put $D = m$, $D' = 1$

$$(m^2 + m - 6)z = y\cos x$$

Aux. Eqn

$$m^2 + m - 6 = 0$$

$$(m-2)(m+3) = 0$$

$$m = 2, -3$$

$$CF = f_1(y+2x) + f_2(y-3x)$$

$$PI = \frac{1}{(D^2 + DD' - 6D'^2)} y\cos x$$

$$PI = \frac{1}{(D-2D')(D+3D')} y\cos x$$

$$PI = \frac{1}{(D-2D')}\left[\frac{1}{D+3D'} y\cos x\right]$$

$$PI = \frac{1}{(D-2D')} \int (c+3x)\cos x \, dx$$

$$PI = \frac{1}{(D-2D')} \int (c\times\cos x + 3x\cos x)\, dx$$

$$PI = \frac{1}{D-2D'} \left[c\int \cos x\, dx + 3\int \underset{①}{x}\,\underset{②}{\cos x}\, dx \right]$$

$$PI = \frac{1}{D-2D'} \left[c\sin x + 3\{x(\sin x) - 1\times(-\cos x)\} \right]$$

$$PI = \frac{1}{D-2D'} \left[c\sin x + 3x\sin x + 3\cos x \right]$$

$$PI = \frac{1}{D-2D'} \left[\underline{(c+3x)}\sin x + 3\cos x \right]$$

$$PI = \frac{1}{D-2D'} \left[y\sin x + 3\cos x \right]$$

$$PI = \int \left[(c-2x)\sin x + 3\cos x \right] dx$$

$$PI = \int (c\sin x - 2x\sin x + 3\cos x)\, dx$$

$$PI = c\int \sin x\, dx - 2\int x\sin x\, dx +$$

$$3\int \cos x\, dx$$

$$PI = -C\times\cos x - 2\{x(-\cos x) - 1\times(-\sin x)\} + 3\sin x$$

$$= -C\times\cos x + 2x\cos x - 2\sin x + 3\sin x$$

$$= -\cos x(C - 2x) + \sin x$$

$$PI = -y\cos x + \sin x$$

complete solution

$$z = CF + PI$$

$$\boxed{z = f_1(y+2x) + f_2(y-3x) - y\cos x + \sin x}$$

Engineering Mathematics by Gulshan Sir

PDE : DPP-11

Topic : Homogeneous Linear Partial Differential Equation with constant coefficient

Particular Integral(PI) : Case-II and Case-III

Q .1 *Solve* $\left[D^2 + (a+b)DD' + ab\,D'^2\right]z = xy$ *(GBTU-2011 & 2013)*

$$z = f_1(y-ax) + f_2(y-bx) + \frac{1}{6}x^3y - \frac{(a+b)}{24}x^4$$

Q .2 *Solve :* $\left(D^3 + 2D^2D' - DD'^2 - 2D'^3\right)z = (y+2)e^x.$

$$z = f_1(y+x) + f_2(y-x) + f_3(y-2x) + ye^x$$

Q . *Solve :* $\left(D^2 + DD' - 6\;D'^2\right)z = y\sin x.$ *(MTU-2011 & 2012)*

$$z = f_1(y+2x) + f_2(y-3x) - y\sin x - \cos x$$

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Thank You

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