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Maths - IV

DPP - 10

Topic - Particular Integral
Case - 1 (Type-2)

Q-1: Solve the linear partial differential equation

$$\frac{\delta^3 z}{\delta x^3} - \frac{4\delta^3 z}{\delta x^2 \delta y} + 4\frac{\delta^3 z}{\delta x \delta y^2} = 4\sin(2x+y)$$ {GBTU-2011}

$$(D^3 - 4D^2D' + 4DD'^2)z = 4\sin(2x+y)$$

$$D(D^2 - 4DD' + 4D'^2)z = 4\sin(2x+y)$$

Auxiliary Equation

$D = 0$ $\qquad$ $D^2 - 4DD' + 4D'^2 = 0$

Replace $D = m$ and $D' = 1$

C.F. $= f_1(y)$

$$m^2 - 4m + 4 = 0$$
$$(m-2)^2 = 0$$
$$m = 2, 2$$
$$C.F = f_2(y+2x) + x f_3(y+2x)$$

$$\boxed{C.F. = f_1(y) + f_2(y+2x) + x f_3(y+2x)}$$

$$PI = \frac{1}{D^3 - 4D^2D' + 4DD'^2} 4\sin(2x+y)$$

Put $D = 2$, $D' = 1$

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$$PI = \frac{1}{2^3 - 4\times4\times1 + 4\times2\times1} 4\sin(2x+y)$$

$$PI = \frac{1}{8-16+8} 4\sin(2x+y)$$

{ Case of failure {

Diffrentiate w.r.t 'D' and multiply with x.

$$PI = \frac{x}{3D^2 - 8DD' + 4D'^2} 4\sin(2x+y)$$

Put $D = 2$, $D' = 1$

$$PI = \frac{x}{3\times4 - 8\times2 + 4} 4\sin(2x+y)$$

$$PI = \frac{x}{12-16+4} 4\sin(2x+y)$$

{Again Case of failure {

Diffrentiate w.r.t 'D' and multiply with x.

$$PI = \frac{x^2}{6D - 8D'} 4\sin(2x+y)$$

Put $D = 2$, $D' = 1$, $2x+y = u$

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$$PI = 4x^2 \times \frac{1}{12-8} \int \sin u \, du$$

$$PI = 4x^2 \times \frac{1}{4} \times -\cos u$$

$$PI = -x^2 \cos u$$

$$PI = -x^2 \cos(2x+y)$$

Complete solution $\Rightarrow$ $Z = CF + PI$.

$$\boxed{Z = f_1(y) + f_2(y+2x) + x f_3(y+2x) - x^2 \cos(2x+y)}$$

Q-2 : Solve : $2r - s - 3t = \frac{5e^x}{e^y}$

$$2r - s - 3t = 5e^{x-y}$$

$$\frac{2\delta^2 z}{\delta x^2} - \frac{\delta^2 z}{\delta x \delta y} - \frac{3\delta^2 z}{\delta y^2} = 5e^{x-y}$$

$$(2D^2 - DD' - 3D'^2)z = 5e^{x-y}$$

Auxiliary Equation $\Rightarrow$ $2D^2 - DD' - 3D'^2 = 0$

Put $D = m$, $D' = 1$

$$2m^2 - m - 3 = 0$$
$$2m^2 - 3m + 2m - 3 = 0$$
$$m(2m-3) + 1(2m-3) = 0$$
$$(m+1)(2m-3) = 0$$
$$m = -1, 3/2$$

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$$C.F. = f_1(y-x) + f_2\left(y+\frac{3}{2}x\right)$$

$$PI = \frac{1}{2D^2-DD'-3D'^2}\, 5e^{x-y}$$

Put $D=1$ and $D'=-1$

$$PI = \frac{1}{2+1-3}\, 5e^{x-y}$$

{ Case of failure }

Differentiate w.r.t D and multiply with x

$$PI = x\times\frac{1}{4D-D'}\, 5e^{x-y}$$

Put $D=1$, $D'=-1$, $u=x-y$

$$PI = 5x\times\frac{1}{4+1}\int e^u du$$

$$PI = 5x\times\frac{1}{5}\times e^u$$

$$PI = xe^u$$

$$PI = xe^{x-y}$$

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Complete solution :- $z = CF + PI$

$$\boxed{z = f_1(y-x) + f_2\left(y + \frac{3}{2}x\right) + xe^{x-y}}$$

Q-3 :- Solve : $\frac{\delta^3 z}{\delta x^3} - 7\frac{\delta^3 z}{\delta x \delta y^2} - 6\frac{\delta^3 z}{\delta y^3} = \sin(x+2y) + e^{3x+y}$

Sol :- $(D^3 - 7DD'^2 - 6D'^3)z = \sin(x+2y) + e^{3x+y}$

Auxiliary Equation :- $D^3 - 7DD'^2 - 6D'^3 = 0$

Put $D = m$, $D' = 1$

$$m^3 - 7m - 6 = 0$$
$$(m+1)(m+2)(m-3) = 0$$
$$m = -1, -2, 3$$

$$\boxed{C.F. = f_1(y-x) + f_2(y-2x) + f_3(y+3x)}$$

$$PI = \frac{1}{D^3 - 7DD'^2 - 6D'^3}\left[\sin(x+2y) + e^{3x+y}\right]$$

$$PI = \frac{1}{D^3 - 7DD'^2 - 6D'^3}\sin(x+2y) + \frac{1}{D^3 - 7DD'^2 - 6D'^3}e^{3x+y}$$

$$PI = P_1 + P_2$$

$$P_1 = \frac{1}{D^3 - 7DD'^2 - 6D'^3}\sin(x+2y)$$

Put $D = 1$, $D' = 2$, $\sin(x+2y) = u$

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$$P_1 = \frac{1}{(1)^3 - 7(1)(2)^2 - 6(2)^3} \iiint \sin u \, du \, du \, du$$

$$P_1 = \frac{-1}{75} \cos u$$

$$\boxed{P_1 = \frac{-1}{75} \cos(x+2y)}$$

$$P_2 = \frac{1}{D^3 - 7DD'^2 - 6D'^3} e^{3x+y}$$

Put $D = 3$, $D' = 1$

$$P_2 = \frac{1}{27 - 21 - 6} e^{3x+y}$$

{ Case of failure }

Differentiate w.r.t 'D' and multiply with x

$$P_2 = x \times \frac{1}{3D^2 - 7D'^2} e^{3x+y}$$

Put $D = 3$, $D' = 1$, $3x+y = u$

$$P_2 = x \times \frac{1}{3 \times 9 - 7 \times 1} e^{3x+y}$$

$$P_2 = x \times \frac{1}{20} \iint e^u \, du \, du$$

$$P_2 = \frac{x}{20} e^u$$

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$$P_2 = \frac{x}{20} e^{3x+y}$$

$$PI = P_1 + P_2$$

$$PI = \frac{-1}{75} \cos(x+2y) + \frac{x}{20} e^{3x+y}$$

Complete solution is

$$z = CF + PI$$

$$z = f_1(y-x) + f_2(y-2x) + f_3(y+3x) - \frac{1}{75}\cos(x+2y) + \frac{x}{20} e^{3x+y}$$

Ans

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