ENGINEERING MATHEMATICS

UNIT-3 : *Statistical Techniques – I*

Lec-7

Today's Target

- Curve Fitting by the Method of Least Square : (i) Fitting of Parabola
  (ii) Fitting of Exponential Curve
- PYQ
- DPP

Curve Fitting by the Method of Least Square

1. Fitting of Straight line
2. Fitting of second degree parabola
3. Fitting of an Exponential curve

Fitting of second degree Parabola

Equation of parabola

$$y = a + bx + cx^2$$

Normal equations

$$\Sigma y = na + b\Sigma x + c\Sigma x^2 \quad -(1)$$

$$\Sigma xy = a\Sigma x + b\Sigma x^2 + c\Sigma x^3 \quad -(2)$$

$$\Sigma x^2 y = a\Sigma x^2 + b\Sigma x^3 + c\Sigma x^4 \quad -(3)$$

Q.1 *Fit a second degree parabola in the following data ?*

x	0	1	2	3	4
y	1	4	10	17	30

Equation of parabola

$$y = a + bx + cx^2 \quad \text{—— (1)}$$

Normal Equations

$$\Sigma y = na + b\Sigma x + c\Sigma x^2 \quad \text{— (2)}$$

$$\Sigma xy = a\Sigma x + b\Sigma x^2 + c\Sigma x^3 \quad \text{— (3)}$$

$$\Sigma x^2 y = a\Sigma x^2 + b\Sigma x^3 + c\Sigma x^4 \quad \text{— (4)}$$

$$n = 5$$

x	y	xy	x^2	x^3	x^4	x^2y
0	1	0	0	0	0	0
1	4	4	1	1	1	4
2	10	20	4	8	16	40
3	17	51	9	27	81	153
4	30	120	16	64	256	480
$\Sigma x =$ 10	$\Sigma y =$ 62	Σxy =195	$\Sigma x^2 =$ 30	$\Sigma x^3 =$ 100	$\Sigma x^4 =$ 354	$\Sigma x^2 y =$ 677

Put these values in eqn ②, ③ and ④

$5a + 10b + 30c = 62$ — ⑤

$10a + 30b + 100c = 195$ — ⑥

$30a + 100b + 354c = 677$ — ⑦

solve eqn ⑤, ⑥ and ⑦

$a = 1.2$

$b = 1.1$

$c = 1.5$

Put a, b, c in ①

$y = a + bn + cn^2$

$$y = 1.2 + 1.1n + 1.5n^2$$

Fitting of an Exponential Curve

Equation of Exponential curve

$$y = ae^{bx}$$

Taking log both side

$$\log_{10} y = \log_{10}(ae^{bx})$$

$$\log_{10} y = \log_{10} a + \log_{10} e^{bx}$$

$$\log_{10} y = \log_{10} a + bx \log_{10} e$$

$$\log mn = \log m + \log n$$

$$\log m^n = n \log m$$

$$\log_{10} y = \log_{10} a + (b \log_{10} e)x$$

Put

$$\log_{10} y = Y$$

$$\log_{10} a = A$$

$$b \log_{10} e = B$$

$$Y = A + Bx$$

Q. 2:- ***Find the curve of best fit of the type $y = ae^{bx}$ to the following data by the method of Least square :***

x:	1	5	7	9	12
y:	10	15	12	15	21

Equation of curve

$$y = a e^{bx} \quad \text{—— (1)}$$

Taking $\log_{10}$ both side

$$\log_{10} y = \log_{10} a + \log_{10} e^{bx}$$

$$\log_{10} y = \log_{10} a + (b \log_{10} e)x$$

Put

$$\log_{10} y = Y$$

$$\log_{10} a = A$$

$$b \log_{10} e = B$$

$$\boxed{Y = A + Bx} \quad \text{—— (2)}$$

Normal Equations

$$\Sigma Y = nA + B\Sigma x \quad \text{—— (3)}$$

$$\Sigma xY = A\Sigma x + B\Sigma x^2 \quad \text{—— (4)}$$

$$n = 5$$

x	y	$Y = \log_{10} y$	x^2	xY
1	10	1	1	1
5	15	1.1761	25	5.8805
7	12	1.0792	49	7.5544
9	15	1.1761	81	10.5849
12	21	1.3222	144	15.8664
$\Sigma x = 34$		$\Sigma Y =$ 5.7536	$\Sigma x^2 =$ 300	$\Sigma xY =$ 40.8862

Put these values in (3) and (4)

$5A + 34B = 5.7536$ — (5)

$34A + 300B = 40.8862$ — (6)

Solve (5) and (6)

$A = 0.9766$

$B = 0.0256$

Now

$\log_{10} a = A$

$\log_{10} a = 0.9766$

$a = \text{Antilog}_{10}(0.9766)$

$\boxed{a = 9.4754}$

$b \log_{10} e = B$

$b = \frac{B}{\log_{10} e} = \frac{0.0256}{0.4342}$

$\boxed{b = 0.059}$

Put a and b in (1)

$\boxed{y = 9.4754\, e^{0.059x}}$

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DPP- 7

Topic : :
1. Fitting of Parabola
2. Fitting of Exponential Curve

Q.1 *Using the method of least squares, fit second degree parabola $y = a + bx + cx^2$ for the following data:*

x:	2	3	4	5	6
y:	14	17	20	24	29

Q.2 *Using the method of least square, fit a parabolic curve of second degree to the following data:*

x	0	1	2	3	4
y	1	1.8	1.3	2.5	6.3

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Thank You