Topic : *Random variables (PART-1)*

Q.1 *A box has 5 Blue and 3 Red balls. If 2 balls are to be drawn at random without replacement and X denotes the number of Blue balls, find the probability distribution for X.*

Q.2 *A random variable X has the following probability distribution:*

Value of X, x:	0	1	2	3	4	5	6	7	8
$p(x)$:	a	$3a$	$5a$	$7a$	$9a$	$11a$	$13a$	$15a$	$17a$

(*i*) *Determine the value of* a.

(*ii*) *Find* $P(X < 3), P(X \geq 3), P(2 \leq X < 5)$

(*iii*) *What is the smallest value of* x *for which* $P(X \leq x) > 0.5$?

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DPP-1 Maths-4 Unit - IV

Topic - Random Variables

Q-1- A box has 5 Blue, 3 Red Balls. If 2 balls are to be drawn at random without replacement and X denotes the number of Blue Balls, find probability distribution for X.

Sol- Number of Blue Balls = 5

Number of Red Balls = 3

Total number of Balls = 8

X = Number of Blue Balls

X = 0, 1, 2

Probabilities ⇒

$$P(X=0) = \frac{{}^3C_2}{{}^8C_2} = \frac{3!}{2!} \times \frac{6! \times 2!}{8!}$$

$$= \frac{3 \times 2}{48 \times 7}$$

$$P(X=0) = \frac{3}{28}$$

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$$P(X=1) = \frac{{}^5C_1 \times {}^3C_1}{{}^8C_2}$$

$$= \frac{5 \times 3 \times 2}{4 \times 7}$$

$$P(X=1) = \frac{15}{28}$$

$$P(X=2) = \frac{{}^5C_2}{{}^8C_2}$$

$$= \frac{5!}{3!2!} \times \frac{6! \times 2!}{8!}$$

$$= \frac{5 \times 4}{2} \times \frac{2}{8 \times 7}$$

$$P(X=2) = \frac{5}{14}$$

X	0	1	2
P(X)	$\frac{3}{28}$	$\frac{15}{28}$	$\frac{5}{14}$

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Q-2- A random variable X has the following probability distribution

x:	0	1	2	3	4	5	6	7	8
P(x):	a	3a	5a	7a	9a	11a	13a	15a	17a

Sol- i) We know, $\Sigma P(x_i) = 1$

$$a + 3a + 5a + 7a + 9a + 11a + 13a + 15a + 17a = 1$$

$$81a = 1$$

$$\boxed{a = \frac{1}{81}}$$

ii) $P(X < 3) = P(x=0) + P(x=1) + P(x=2)$

$= a + 3a + 5a$

$= 9a$

$= 9 \times \frac{1}{81}$

$$\boxed{P(X<3) = 1/9}$$

$P(X \geq 3) = P(X=3) + P(X=4) + P(X=5) + P(X=6) + P(X=7) + P(X=8)$

$= 7a + 9a + 11a + 13a + 15a + 17a$

$= 72a$

$= 72 \times 1/81$

$$\boxed{P(x \geq 3) = \frac{8}{9}}$$

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$P(2 \leq X < 5) = P(X=2) + P(X=3) + P(X=4)$

$= 5a + 7a + 9a$

$= 21a$

$= 21 \times \frac{1}{81}$

$\boxed{P(2 \leq X < 5) = \frac{7}{27}}$

v) $P(X \leq 1) = a + 3a$

$= 4a$

$= \frac{4}{81} < \frac{1}{2}$

$P(X \leq 3) = a + 3a + 5a + 7a$

$= 16a$

$= \frac{16}{81} < \frac{1}{2}$

$P(X \leq 5) = a + 3a + 5a + 7a + 9a + 11a$

$= \frac{36}{81} < \frac{1}{2}$

$P(X \leq 6) = a + 3a + 5a + 7a + 9a + 11a + 13a$

$= \frac{49}{81} > \frac{1}{2}$

Hence minimum value of X is 6.

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DPP- 02

Topic : *Random variables (PART-2)*

$Q.1$ *Let X denote the number of heads when a pair of coin is tossed. What is the expected value?*

$Q.2$ *Two dice are thrown simultaneously. if X denote the number of sixes, find the expectation of X*

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DPP-2 Maths-4 Unit-4

Topic- Random Variables

Q-1- Let X denote the number of heads when a pair of coin is tossed? What is expected value?

Sol- $S = \{HH, HT, TH, TT\}$

X = Number of Heads

$X(HH) = 2$

$X(HT, TH) = 1$

$X(TT) = 0$

Probabilities $\Rightarrow P(X=0) = \frac{1}{4}$

$P(X=1) = \frac{2}{4}$

$P(X=2) = \frac{1}{4}$

Probability Distribution

X	0	1	2
P(x)	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{1}{4}$

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$$E(X) = \bar{x} = 0 \times \frac{1}{4} + 1 \times \frac{2}{4} + 2 \times \frac{1}{4}$$

$$\boxed{E(X) = 1}$$

Q-2: Two dice are thrown simultaneously. If X denote the number of sixes. Find the expectation of X

S = {(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)}.

Total cases = 36

X = No. of sixes

Probabilities ⇒ $P(X=0) = \frac{25}{36}$

$P(X=1) = \frac{10}{36}$

$P(X=2) = \frac{1}{36}$

Probability Distribution

X	0	1	2
P(X)	$\frac{25}{36}$	$\frac{10}{36}$	$\frac{1}{36}$

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Expectation $(E(x)) = 0 \times \frac{25}{36} + 1 \times \frac{10}{36} + 2 \times \frac{1}{36}$

$= \frac{10}{36} + \frac{2}{36}$

$= \frac{12}{36}$

$$E(x) = \frac{1}{3}$$

Ans

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Topic : *Random variables (PART-3)*

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Q. 1 Find the constant k so function $f(x)$ defined as follows be a density function:

$$\begin{cases} 1/k, & a \le x \le b \\ 0, & elsewhere \end{cases}$$

Q. 2 A continous random variable X has

$$\mathrm{f}(x) = \begin{cases} \frac{1}{2}(x+1), & for\ -1 \le x \le 1 \\ 0, & elsewhere \end{cases}$$

represent the density . Find mean, variance and standard deviation.

DPP-3 Maths-4 Unit-4

Topic- Random Variable

Q-1- Find the constant k so function f(x) defined as follows be a density function

$$\begin{cases} 1/k, & a \le x \le b \\ 0, & \text{elsewhere} \end{cases}$$

Sol- Since f(x) is a pdf i) $f(x) \ge 0 \ \forall x$

ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

From ii)

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_{-\infty}^{a} f(x)\,dx + \int_{a}^{b} f(x)\,dx + \int_{b}^{\infty} f(x)\,dx = 1$$

$$\int_{a}^{b} \frac{1}{k}\,dx = 1$$

$$\left[\frac{x}{k}\right]_a^b = 1$$

$$\frac{1}{k}(b-a) = 1$$

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$$\boxed{k = b - a}$$

Ans

Q 2- A continuous random variable X has,

$$f(x) = \begin{cases} \frac{1}{2}(x+1) & , \quad -1 \le x \le 1 \\ 0 & , \quad \text{elsewhere} \end{cases}$$

represent the density. Find mean, variance and S.D.

Sol- Mean :- $E(x) = \int_{-\infty}^{\infty} x f(x)\, dx$

$$E(x) = \int_{-\infty}^{-1} x f(x) dx + \int_{-1}^{1} x f(x) dx + \int_{1}^{\infty} x f(x)\, dx$$

$$= \int_{-1}^{1} \frac{x}{2}(x+1)\, dx$$

$$= \frac{1}{2}\int_{-1}^{1} (x^2 + x)\, dx$$

$$= \frac{1}{2}\left[\frac{x^3}{3} + \frac{x^2}{2}\right]_{-1}^{1}$$

$$= \frac{1}{2}\left[\frac{1}{3} + \frac{1}{2} + \frac{1}{3} - \frac{1}{2}\right]$$

$$\boxed{E(x) = \frac{1}{3}}$$

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ii) $E(x^2) = \int_{-\infty}^{\infty} x^2 f(x)\,dx$

$$E(x^2) = \int_{-\infty}^{-1} x^2 f(x)\,dx + \int_{-1}^{1} x^2 f(x)\,dx + \int_{1}^{\infty} x^2 f(x)\,dx$$

$$E(x^2) = \frac{1}{2}\int_{-1}^{1} (x^3 + x^2)\,dx$$

$$= \frac{1}{2}\left[\frac{x^4}{4} + \frac{x^3}{3}\right]_{-1}^{1}$$

$$= \frac{1}{2}\left[\frac{1}{4} + \frac{1}{3} - \frac{1}{4} + \frac{1}{3}\right]$$

$$= \frac{1}{3}$$

$$\text{Variance} = E(x^2) - [E(x)]^2$$

$$= \frac{1}{3} - \frac{1}{9}$$

$$= \frac{3-1}{9}$$

$$\boxed{\text{Variance} = \frac{2}{9}}$$

Standard deviation = $\sqrt{\text{Variance}}$

$$\boxed{S.D. = \frac{\sqrt{2}}{3}}$$

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Topic : *Binomial probability distribution*

Q.1 If the mean of a binomial distribution is 3 and the variabce is $\frac{3}{2}$, find the probability of obtaining at least 4 successes.

Q.2 The probability that a bomb dropped from a plane will strike the target is $\frac{1}{5}$.

If six bombs are dropped, find the probability that (i)exactly two will strike the target, (ii)at least two will trike the target.

Q.3 Fit a binimial distribution to the following frequency data:

x:	0	1	2	3	4
f:	30	62	46	10	2

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DPP-4 Maths- IV Unit-4

Topic- Binomial Distribution

Q-1- If the mean of a binomial distribution is 3 and the variance is $\frac{3}{2}$. find the probability of obtaining at least 4 successes.

Sol- Mean

$$np = 3 \quad -(1)$$

Variance

$$npq = \frac{3}{2} \quad -(2)$$

Divide (2) by (1)

$$\frac{npq}{np} = \frac{3}{2 \times 3}$$

$$\boxed{q = \frac{1}{2}}$$

We know, $p + q = 1$

$$\therefore \boxed{p = \frac{1}{2}}$$

From eq (1)

$$\boxed{n = 6}$$

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At least 4 successes -

$$P(r \geq 4) = P(r=4) + P(r=5) + P(r=6)$$

$$= {}^6C_4 \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^2 + {}^6C_5 \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right) + {}^6C_6 \left(\frac{1}{2}\right)^6$$

$$= \left(\frac{1}{2}\right)^6 \left[{}^6C_4 + {}^6C_5 + {}^6C_6 \right]$$

$$= \left(\frac{1}{2}\right)^6 \left[\frac{6 \times 5 \times 4!}{4! \times 2!} + \frac{6 \times 5!}{5! \times 1!} + 1 \right]$$

$$= \left(\frac{1}{2}\right)^6 \left[15 + 6 + 1 \right]$$

$$= \left(\frac{1}{2}\right)^6 \times 22$$

$$= \frac{1}{2^5} \times 11$$

$$P(r \geq 4) = \frac{11}{32}$$

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Q-2- The probability that a bomb dropped from a plane will strike the target is $\frac{1}{5}$. If six bombs are dropped: find the probability that
i) exactly two will strike the target
ii) at least two will strike the target

Sol- $n = 6$

$p = \frac{1}{5}$

$q = 1 - \frac{1}{5}$

$q = \frac{4}{5}$

i) exactly two will strike the target

$$P(r=2) = {}^{6}C_2 \left(\frac{1}{5}\right)^2 \left(\frac{4}{5}\right)^4$$

$$= \frac{6!}{4!\,2!} \left(\frac{1}{5}\right)^2 \left(\frac{4}{5}\right)^4$$

$$= 15 \times \frac{1}{25} \times \frac{256}{625}$$

$$= \frac{3840}{15625}$$

$$= 0.246$$

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$$P(r \geq 2) = P(r=2) + P(r=3) + P(r=4) + P(r=5) + P(r=6)$$

$$= {}^6C_2 \left(\frac{1}{5}\right)^2 \left(\frac{4}{5}\right)^4 + {}^6C_3 \left(\frac{1}{5}\right)^3 \left(\frac{4}{5}\right)^3 + {}^6C_4 \left(\frac{1}{5}\right)^4 \left(\frac{4}{5}\right)^2$$
$$+ {}^6C_5 \left(\frac{1}{5}\right)^5 \left(\frac{4}{5}\right)^1 + {}^6C_6 \left(\frac{1}{5}\right)^6 \left(\frac{4}{5}\right)^0$$

$$= 0.345$$

Q-3- Fit a binomial distribution to the following frequency data

x:	0	1	2	3	4
f:	30	62	46	10	2

x	f	fx
0	30	0
1	62	62
2	46	92
3	10	30
4	2	8
	$\Sigma f = 150$	$\Sigma fx = 192$

Mean $\Rightarrow$ $\mu = \frac{\Sigma fx}{N} = \frac{192}{150}$

$$\boxed{\mu = 1.27}$$

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For Binomial Distribution

$$\mu = np$$
$$1.27 = 4 \times p$$
$$p = 0.31$$
$$q = 0.69$$

Binomial distribution

$$N(q+p)^n = 150(0.69+0.31)^4$$

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Topic : Binomial probability distribution(Part – II)

Q.1 Out of 800 families with 4 children each, how many families would be expected to have (i)2 boys and 2 girls (ii)at least one boy (iii)no girl (iv)almost two girls? Assume equal probabilities for boys and girls. UPTU 2014

Q.2 A bag contains 5 white , 7 red and 8 black balls. If four balls are drawn, one by one, with replacement. what is the probability that

(i) none is white

(ii) all are white

(iii) at least on is white

(iv) only 2 are white ?

Q.3 Five cards are drawn successively with repelacement from a wellshuffled deck of 52 cards. What is the probability that

(i) All the five cards are spades

(ii) only three are spades ?

(iii) none is spade ?

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DPP-5 Maths - IV Unit - 4

Topic - Binomial Distribution - 2

Q-1 - Out of 800 families with 4 children each, how many families would be expected to have i) 2 boys 2 girls ii) at least one boy iii) no girl iv) almost two girls ?

Sol - Since probabilities for boys and girls are equal

p = probability of boy = $\frac{1}{2}$

q = probability of girl = $\frac{1}{2}$

$n = 4$

$N = 800$

i) The expected number of families having 2 boys and 2 girls

$$= 800 \times \left[{}^4C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^2 \right]$$

$$= 800 \times 6 \times \frac{1}{16}$$

$$= 300$$

ii) The expected number of families having atleast one boy

$$NP(r \geq 1) = 800 \left[{}^4C_1 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right) + {}^4C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^2 + {}^4C_3 \left(\frac{1}{2}\right) \left(\frac{1}{2}\right)^3 + {}^4C_4 \left(\frac{1}{2}\right)^4 \right]$$

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$= 800 \times \frac{1}{16} [4+6+4+1]$

$= 750$

iii) The expected no. of families having no girl i.e, having 4 boys.

$NP(r=4) = 800 \times {}^4C_4 \left(\frac{1}{2}\right)^4$

$= 800 \times \frac{1}{16}$

$= 50$

iv) The expected number of families having atmost two girls, having at least 2 boy.

$NP(r \geq 2) = 800 \left[{}^4C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^2 + {}^4C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right) + {}^4C_4 \left(\frac{1}{2}\right)^4 \right]$

$= 800 \times \frac{1}{16} [6+4+1]$

$= 550$

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Q-2 : A bag contains 5 white, 7 red, 8 black balls. If four balls are drawn, one by one, with replacement what is probability that

i) none is white
ii) all are white
iii) at least one is white
iv) only two are white

Sol - $n = 4$

$$p(\text{drawn white ball}) = \frac{5}{20} = \frac{1}{4}$$

$$q = 1 - \frac{1}{4} = \frac{3}{4}$$

i) none is white $(X = 0) = {}^4C_0 \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^4$

$$= \frac{81}{256}$$

ii) all are white $(X = 4) = {}^4C_4 \left(\frac{1}{4}\right)^4 \left(\frac{3}{4}\right)^0$

$$= \frac{1}{256}$$

iii) atleast one is white = 1 - no white

$$= 1 - \frac{81}{256}$$

$$= \frac{175}{256}$$

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iv) Only two are white $(X=2) = {}^4C_2 \left(\frac{1}{4}\right)^2 \left(\frac{3}{4}\right)^2$

$$= \frac{4!}{2!\times 2!} \times \frac{1}{16} \times \frac{9}{16}$$

$$= \frac{\cancel{4} \times 3 \times 9}{\cancel{2} \times \cancel{16}^{8} \times 16}$$

$$= \frac{27}{128}$$

Q-3- Five cards are drawn successively with replacement from a well-shuffled deck of 52 cards. What is the probability that

i) all the five cards are spade
ii) only three are spades
iii) none is spade

Sol- In a well shuffled deck of 52 cards, there are 13 cards.

$$p = \frac{13}{52} = \frac{1}{4}$$

$$q = 1 - \frac{1}{4} = \frac{3}{4}$$

$$n = 5$$

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i) $P(r=5) = {}^5C_5 \left(\frac{3}{4}\right)^0 \left(\frac{1}{4}\right)^5$

$= \left(\frac{1}{4}\right)^5$

ii) $P(r=3) = {}^5C_3 \left(\frac{3}{4}\right)^2 \left(\frac{1}{4}\right)^3$

$= 10 \times \frac{9}{16} \times \frac{1}{4^3}$

$= 90 \times \left(\frac{1}{4}\right)^5$

iii) $P(r=0) = {}^5C_0 \left(\frac{3}{4}\right)^5 \left(\frac{1}{4}\right)^0$

$= \frac{243}{1024}$

Ans

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DPP- 06

Topic :*Poisson Probability Distribution (Part – I)*

Q.1 In a poisson distribution, $P(r)$ for $r = 0$ is 10%, find the mean.

Q.2 If X poisson variate such that $P(X = 1) = P(X = 2)$. Find (i) Mean (ii) $P(X = 4)$

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DPP-6 Maths-4 Unit-4

Topic- Poisson Distribution

Q-1- In a poisson distribution, $P(r)$ for $r=0$ is 10%. find the mean.

Sol- Probability of '0' success in poisson distribution = 10%.

$$P(r=0) = 0.1$$

By Poisson distribution,

$$P(r) = \frac{\lambda^r e^{-\lambda}}{r!}$$

$$P(r=0) = \frac{\lambda^0 e^{-\lambda}}{0!} = 0.1$$

$$e^{-\lambda} = 0.1$$

$$-\lambda = \log_e 0.1$$

$$\lambda = -\log_e 0.1$$

$$\lambda = -\log_e \frac{1}{10}$$

$$\boxed{\lambda = \log_e 10} = \text{Mean}$$

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Q-2- If X poisson variate such that $P(X=1) = P(X=2)$. find i) mean ii) $P(X=4)$

Sol- For Poisson Distribution

$$P(X=r) = \frac{e^{-\lambda}\lambda^r}{r!}$$

$$P(X=1) = P(X=2)$$

$$\frac{e^{-\lambda}\lambda}{1!} = \frac{e^{-\lambda}\lambda^2}{2!}$$

$$2\lambda = \lambda^2$$

$$\lambda^2 - 2\lambda = 0$$
$$\lambda(\lambda-2) = 0$$
$$\lambda = 0, 2$$

$$\boxed{\text{Mean} = \lambda = 2}$$

$$P(X=4) = \frac{e^{-\lambda}\lambda^4}{4!}$$
$$= \frac{e^{-2} \times 2^4}{24}$$
$$= e^{-2} \times 0.67$$
$$= 0.1353 \times 0.67$$
$$\boxed{P(X=4) = 0.0906}$$

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DPP- 07

Topic : *Poisson Probability Distribution (Part – II)*

Q. 1 In a certain factory manufacturing razor blades, there is a small chance of 0.002 for any blade to be defective. The blades are supplied in packets of 10. Use suitable distribution to calculate the approximate number of packets containing no defective, one defective and two defective blades respectively in a consignment of 50,000 packets. (AKTU 2018)

Q. 2 The distribution of the number of road accidents per day in a city is Poisson with mean 4. Find the number of days out of 100 days when there will be (i) no accident (ii) at least 2 accidents (iii) at most 3 accidents (iv) between 2 and 5 accidents.

Q. 3 Fit a Poisson distribution to the following data and calculate theoretical frequencies.

Deaths:	0	1	2	3	4
Frequencies:	122	60	15	2	1

Q. 4 Fit a Poisson distribution to the following data given the number of yeast cells per square for 400 squares:

No. of cells per sq. :	0	1	2	3	4	5	6	7	8	9	10
No. of squares :	103	143	98	42	8	4	2	0	0	0	0

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DPP-7 Maths-4 Unit-4

Topic - Poisson Probability Distribution

Q-1- In a certain factory manufacturing razor blades, there is a small chance of 0.002 for any blade to be defective. The blades are supplied in packets of 10. Use suitable distribution to calculate the approximate no. of packets containing no defective, one defective and two defective blades in a consignment of 50,000 packets.

Sol- Probability of defective blade

$P = 0.002$
$n = 10$
$N = 50,000$

$\lambda = np = 0.002 \times 10$

$\boxed{\lambda = 0.02}$

By poisson distribution

$$P(X = r) = \frac{e^{-\lambda} \lambda^r}{r!}$$

i) Number of packets containing no defective blades $\Rightarrow P(X=0) = \frac{e^{-0.02} \times (0.02)^0}{01}$

$= e^{-0.02}$

$= 0.9802$

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Number of packets = $50,000 \times 0.9802$
$= 49,010$

ii) $P(X=1) = \frac{e^{-0.02} \times (0.02)^1}{1!}$

$P(X=1) = 0.0196$

No. of packets having 1 blade = $0.0196 \times 50,000$
$= 980$ packets

iii) $P(X=2) = \frac{e^{-0.02} \times (0.02)^2}{2!}$
$= 0.000196$

No. of packets having 2 defective blades = $0.000196 \times 50,000$
$= 9.8$
$= 10$ packets

Q-2 - The distribution of the number of road accidents per day in a city is poisson with mean 4. Find the no. of days out of 100 days there will be i) no accident ii) at least two accident iii) at most 3 accidents iv) b/w 2 and 5 accidents?

Sol - $\lambda = 4$, $N = 100$ days

By poisson distribution

$$P(X=r) = \frac{e^{-\lambda} \lambda^r}{r!}$$

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i) $P(X=0) = \frac{e^{-4} \times (4)^0}{0!} = e^{-4}$

$P(X=0) = e^{-4} = 0.0183$

No. of days Having no accident = 0.018×100
$= 1.8$
≈ 2 days

ii) $P(X \geq 2) \Rightarrow$
$1 - [P(X=0) + P(X=1)]$

$= 1 - \left[e^{-4} + \frac{e^{-4} \times 4}{1!} \right]$

$= 1 - e^{-4} \times 5$
$= 1 - 0.0183 \times 5$
$= 1 - 0.091$
$= 0.909$

No. of days having atleast two accidents = 0.909×100
$= 90.9$
≈ 91 days

iii) atmost 3 accidents $\Rightarrow P(X=0) + P(X=1) + P(X=2) + P(X=3)$

$= e^{-4} + e^{-4} \times 4 + \frac{e^{-4} \times 16}{2!} + \frac{e^{-4} \times 64}{3!}$

$= e^{-4} \left[1 + 4 + 8 + \frac{32}{3} \right]$

$= e^{-4} \times 23.66$
$= 0.432$

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No. of days having at most 3 accidents = 100×0.432
$= 43.2$
≈ 43 days

iv) between 2 and 5 accidents $\Rightarrow P(X=3) + P(X=4)$

$$= \frac{e^{-4} \times (4)^3}{3!} + \frac{e^{-4} \times (4)^4}{4!}$$

$$= e^{-4} \left[\frac{64}{6} + \frac{256}{24} \right]$$

$$= e^{-4} \times \frac{64}{3}$$

$$= e^{-4} \times 21.33$$

$$= 0.3903$$

No. of days having accidents between 2 and 5 = 0.390×100
$= 39$ days

Q-3- Fit a poisson distribution for the following data and calculate theoretical frequencies

Deaths	0	1	2	3	4
Frequencies	122	60	15	2	1

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x	f	fx	Theoritical frequencies
0	122	0	121.306
1	60	60	60.653
2	15	30	15.163
3	2	6	2.527
4	1	4	0.315
	$\Sigma f = 200$	$\Sigma fx = 100$	

$$\text{Mean} = \lambda = \frac{\Sigma fx}{\Sigma f} = \frac{100}{200}$$

$$\boxed{\lambda = 0.5}$$

$$\boxed{N = 200}$$

$$\text{Poisson Distribution} = \frac{N e^{-\lambda} \lambda^r}{r!}$$

When $r = 0$, $\frac{200 \times e^{-0.5} \times (0.5)^0}{0!}$

$= 200 \times 0.608$

$= 121.306$

When $r = 1$, $\frac{200 \times e^{-0.5} \times (0.5)^1}{1!}$

$= 200 \times 0.608 \times 0.5$

$= 60.653$

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When $r=2$, $\frac{200 \times e^{-0.5} \times (0.5)^2}{2!}$

$= 15.163$

When $r=3$, $\frac{200 \times e^{-0.5} \times (0.5)^3}{3!}$

$= 2.527$

When $r=4$, $\frac{200 \times e^{-0.5} \times (0.5)^4}{4!}$

$= 0.315$

Q-4- Fit a poisson distribution to the following data given the number of yeast cells per square for 400 squares.

No. of cells per sq:	0	1	2	3	4	5	6	7	8	9	10
No. of Square:	103	143	98	42	8	4	2	0	0	0	0

r	f	f.r	Theoritical frequency
0	103	0	107
1	143	143	141
2	98	196	93
3	42	126	41
4	8	32	13
5	4	20	4
6	2	12	1
7	0	0	0
8	0	0	0
9	0	0	0
10	0	0	0

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$\Sigma f = N = 400$

$\Sigma fr = 529$

$\text{Mean} = \lambda = \frac{529}{400} = 1.322$

$\text{Poisson Distribution} = N \frac{e^{-\lambda} \times \lambda^r}{r!}$

$= 400 \times \frac{e^{-1.322} \times \lambda^r}{r!}$

$= 107.2 \times \frac{\lambda^r}{r!}$

$r = 0, \quad = 107.2 \times \frac{\lambda^r}{r!}$

$= 107.2 \times \frac{(1.32)^0}{0!}$

$= 107.2 \approx 107$

$r = 1, \Rightarrow 107.2 \times \frac{1.322}{1!}$

$= 141.5 \approx 141$

$r = 2 \Rightarrow 107.2 \times \frac{(1.322)^2}{2!}$

$= 93.4$

≈ 93

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$r=3 \Rightarrow 107.2 \times \frac{(1.322)^3}{3!}$

$= 41.2$

≈ 41

$r=4 \Rightarrow 107.2 \times \frac{(1.322)^4}{4!}$

≈ 13

$r=5 \Rightarrow 107.2 \times \frac{(1.322)^5}{5!}$

$= 4$

$r=6 \Rightarrow 107.2 \times \frac{(1.322)^6}{6!}$

$= 0.80$

≈ 1

$r=7 \Rightarrow 107.2 \times \frac{(1.322)^7}{7!}$

$= 0.14$

≈ 0

$r=8 \Rightarrow 107.2 \times \frac{(1.322)^8}{8!} \approx 0$

$r=9 \Rightarrow 107.2 \times \frac{(1.322)^9}{9!} \approx 0$

$r=10 \Rightarrow 107.2 \times \frac{(1.322)^{10}}{10!} \approx 0$

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DPP- 08

Topic :*Normal Probability Distribution (Part – I)*

Q.1 Define:

(i) Binomial distribution

(ii) Poisson distribution

(iii) Normal distribution. *(AKTU 2018)*

Q.2 If X is a normal variate with mean 30 and S.D. 5, find the probabilities that

(i) $26 \leq X \leq 40$

(ii) $X \geq 45$ *and*

(iii) $[X - 30 | > 5$

Ans. 2*(i)* 0.7653 *(ii)* 0.00135 *(iii)* 0.3174

Q.3 If Z is a standard normal variable, find the following probabilities:

(i) $P(Z < 1.2)$

(ii) $P(Z > -1.2)$

(iii) $P(-1.2 < Z < 1.3)$.

Ans. 3*(i)* 0.8849 *(ii)* 0.8849 *(iii)* 0.7881

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DPP-8 Maths-4 Unit-4

Topic- Normal Distribution

Q-1- Define

i) Binomial Distribution- Let a random variable X denote the number of successes in n independent trials in an experiment.
Let p be the probability of success and q be the probability of failure in a single trial so that

$$p+q=1$$

r successes can be obtained in n trials in ${}^nC_r$ ways.

Probability of r successes in n trials

$$\boxed{P(X=r) = {}^nC_r p^r q^{n-r}}$$

ii) Poisson Distribution- If the parameters n is very large and p is very small, application of binomial distribution is very labourious. If we assume $n \to \infty$ and $p \to 0$ such that np always remain finite, say λ, we get poisson approximation to the binomial distribution.

$$\boxed{P(X=r) = \frac{e^{-\lambda}\lambda^r}{r!} \quad (r=0,1,2,3\ldots)}$$

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Normal Distribution - The normal distribution is a continuous distribution based on continuous random variable. A continuous random variable X is said to have a normal distribution with parameters μ (Mean) and σ^2 (Variance) if the probability density function is given by

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

i) $f(x) \geq 0$

ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

Q-2 - If X is a normal variable with mean 30 and S.D. 5. find the probabilities that

i) $26 \leq X \leq 40$

$X_1 = 26$, $X_2 = 40$

$$Z_1 = \frac{x_1 - \mu}{\sigma}$$

$$Z_1 = \frac{26-30}{5} = -0.8$$

$$Z_2 = \frac{40-30}{5} = 2$$

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$P(26 \le X \le 40) = P(-0.8 \le Z \le 2)$

$= P(-0.8 \le Z \le 0) + P(0 \le Z \le 2)$

$= P(0 \le Z \le 0.8) + P(0 \le Z \le 2)$

$= 0.2881 + 0.4772$

$= 0.7653$

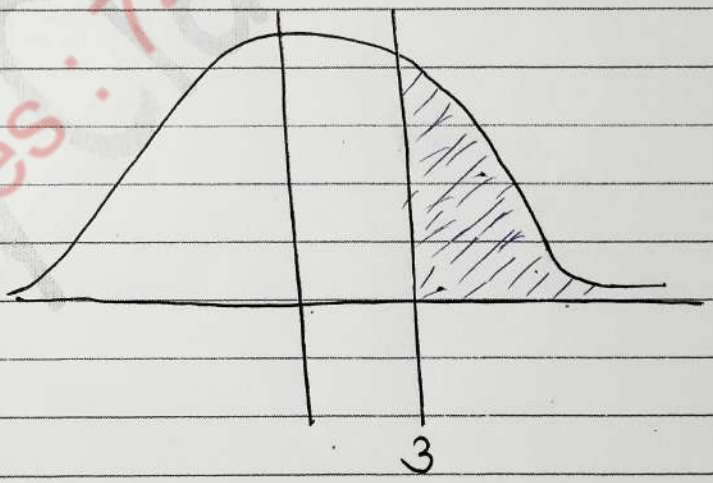
ii) $X \ge 45$

$Z = \frac{x-\mu}{\sigma} = \frac{45-30}{5} = 3$

$P(X \ge 45) = P(Z \ge 3)$

$= 0.5 - 0.4987$

$= 0.0013$



iii) $|X-30| > 5$

$-5 > X-30 > 5$

$25 > X > 35$

$x_1 \quad x_2$

$Z_1 = \frac{25-30}{5} = -1$

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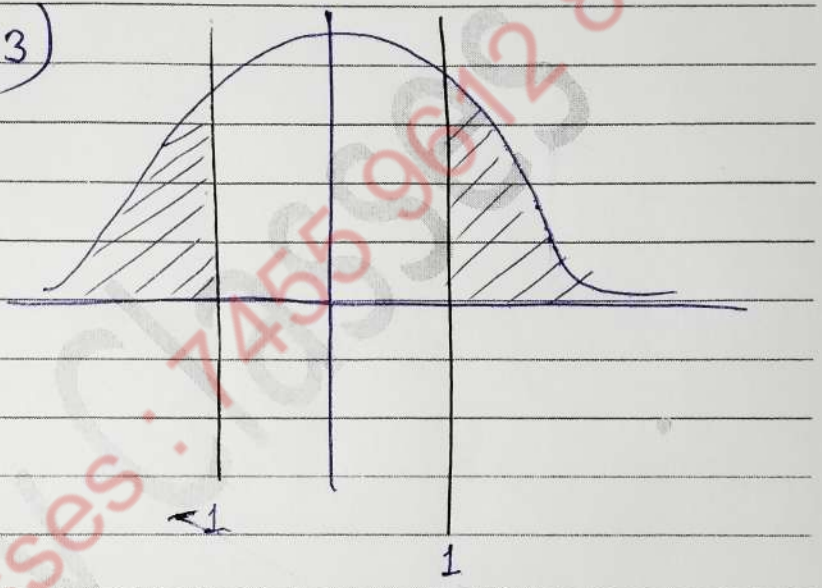
$Z_2 = \frac{35-30}{5} = 1$

$P(-1 > Z > 1)$

$P(-1 > z) + P(z > 1)$

$= (0.5 - 0.3413) + (0.5 - 0.3413)$

$= 0.3174$



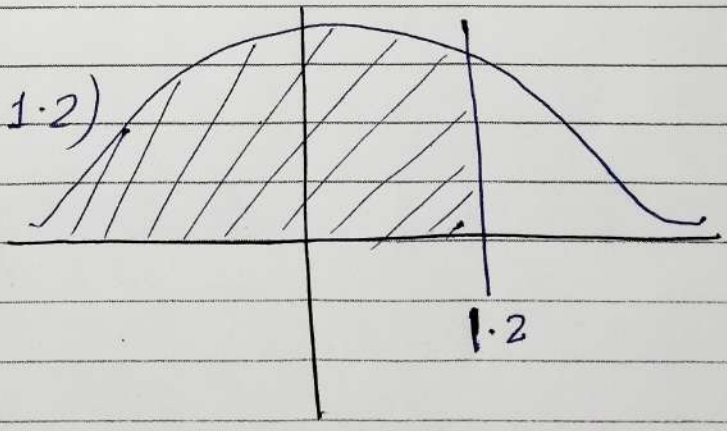
Q-3. If Z is a standard normal variable. Find the following probabilities.

i) $P(Z < 1.2)$

$= P(-\infty < Z < 0) + P(0 < Z < 1.2)$

$= 0.5 + 0.3849$

$= 0.8849$



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ii) $P(Z > -1.2)$

$P(Z < 1.2)$

$= P(-\infty < Z < 0) + P(0 < Z < 1.2)$

$= 0.5 + 0.3849$

$= 0.8849.$

iii) $P(-1.2 < Z < 1.3)$

$P(-1.2 < Z < 0) + P(0 < Z < 1.3)$

$P(0 < Z < 1.2) + P(0 < Z < 1.3)$

$= 0.3849 + 0.4032$

$= 0.7881$

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DPP- 09

Topic :*Normal Probability Distribution (Part – II)*

Q. 1 In a test on 2000 electric bulbs, it was found that the life of a particular make, was normally distributed with an average life of 2040 hours and S. D. of 60 hours, estimate the number of bulbs likely to burn for

(A. K. T. U. 2021)

(i) more than 2150 hours
(it) less than 1950 hours
(iii) more than 1920 hours but less than 2160 hours.

Ans. 1 (i) 67 (ii) 134 (iii) 1909

Q. 2 The marks X obtained in Mathematics by 1000 students are normally distributed with meas 78% and standard deviation 11% Determine:

Ans. 2 (i) 138 (ii) 63.92% (iii) Between 60 and 96

(i) how many students got marks above 90%?
(ii) What was the highest marks obtained by the lowest 10% of students?
(iii) Within what limits did the middle 90% of the students lie?

Q. 3 In a normal distribution exactly normal, 7% of the items are under 35 and 89% areunder 63.

What are the mean and standard deviation of the distribution?

Ans. 3 50.3 and 10.33

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DPP-9 Maths-4 Unit-4

Topic- Normal Distribution

Q-1- In a test on 2000 electric bulbs, it was found that the life of a particular make, was normally distributed with an average life of 2040 hours and S.D. of 60 hours, estimate the number of bulbs likely to burn for

i) more than 2150 hours
ii) less than 1950 hours
iii) more than 1920 but less than 2160 hours.

Sol- i) $P(X > 2150)$

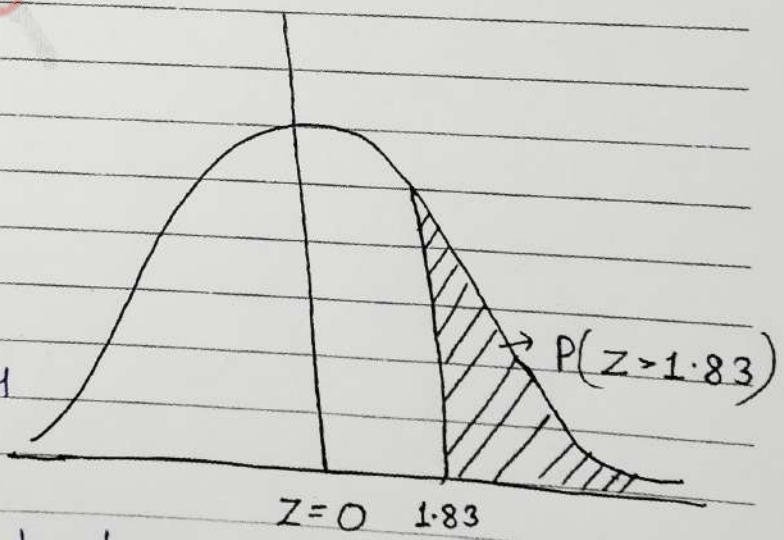
$\mu = 2040$, $\sigma = 60$

$$Z = \frac{X - \mu}{\sigma}$$

$$Z = \frac{2150 - 2040}{60}$$

$$Z = 1.833$$

$P(Z > 1.83) = 0.5 - 0.4664$
$= 0.0336$



The number of bulbs likely to burn more than 2150 hours
$= 0.0336 \times 2000$
$= 67.2$
≈ 67

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ii) $P(X < 1950)$

$$Z = \frac{1950 - 2040}{60}$$

$$Z = -1.5$$

$$P(Z < -1.5) = P(Z > 1.5)$$
$$= P(0 < Z < \infty) - P(0 < Z < 1.5)$$
$$= 0.5 - 0.4332$$
$$= 0.0668$$

No. of bulbs likely to burn for less than 1950 hours =
$$= 2000 \times 0.0668$$
$$= 133.6$$
$$\approx 134$$

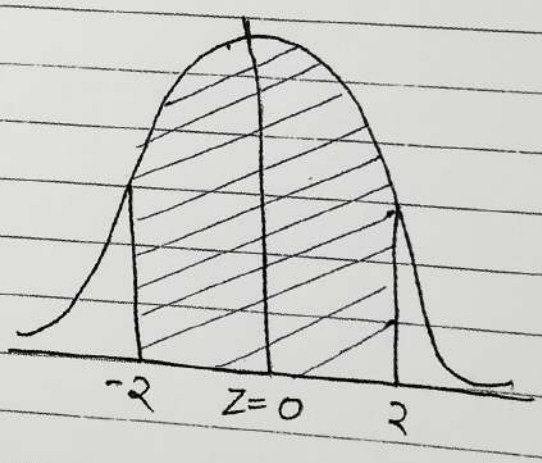
iii) $P(1920 < X < 2160)$

$$Z_1 = \frac{1920 - 2040}{60}$$

$$Z_1 = -2$$

$$Z_2 = \frac{2160 - 2040}{60}$$

$$Z_2 = 2$$



$$P(-2 < Z < 2) = P(-2 < Z < 0) + P(0 < Z < 2)$$
$$= P(0 < Z < 2) + P(0 < Z < 2)$$
$$= 0.4772 + 0.4772$$
$$= 0.9544$$

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No. of bulbs likely to burn between 1920 and 2160
$= 0.9544 \times 2000$
$= 1909$ bulbs

Q-2 - The marks X obtained in maths by 1000 students are normally distributed with mean 78% and S.D. 11%. Determine
i) how many students got marks above 90%?
ii) What was highest marks obtained by lowest 10% students?
iii) Within what limit did the middle 90% of student lie?

Sol - i) $P(X > 90)$

$$Z = \frac{x - \mu}{\sigma}$$

$$= \frac{90 - 78}{11}$$

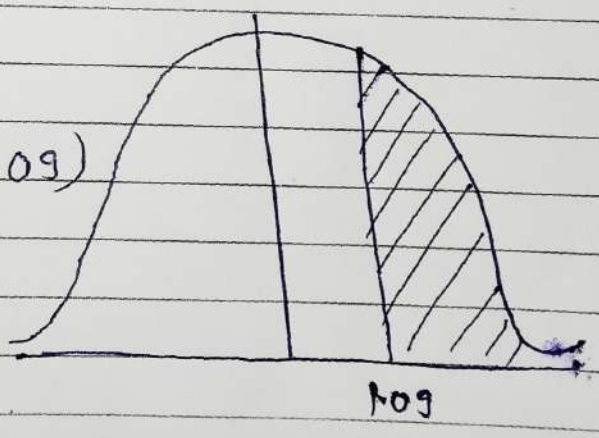
$$Z = 1.09$$

$P(Z > 1.09)$

$= P(0 < Z < \infty) - P(0 < Z < 1.09)$

$= 0.5 - 0.3621$

$= 0.1379$



No. of student got above 90% $= 0.1379 \times 1000$
$= 137.9$
≈ 138

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ii) What was the highest marks obtained by lowest 10% of student?

$Z = -1.28$

$-1.28 = \frac{Value - 78}{11}$

$Value = 63.92 \simeq 64$

Highest mark obtained by the lowest 10% of student = 64%.

iii) Within what limits did the middle of 90% student lie?

Sol- 5% to 95% students

$\Rightarrow Z = -1.645$ and $Z = 1.645$

$-1.645 = \frac{value - 78}{11}$

$Value = 59.9 \approx 60$

$1.645 = \frac{Value - 78}{11}$

$value = 96.1 \approx 96$

middle of 90% of the students lie b/w 60% to 96%.

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Q-3- In a normal distribution, 7% of the items are under 35 and 89% are under 63. Determine the mean and variance of the distribution.

Sol- Let μ be the mean and σ be standard deviation

$P(X<35) = 7\%$

$P(X<35) = 0.07$

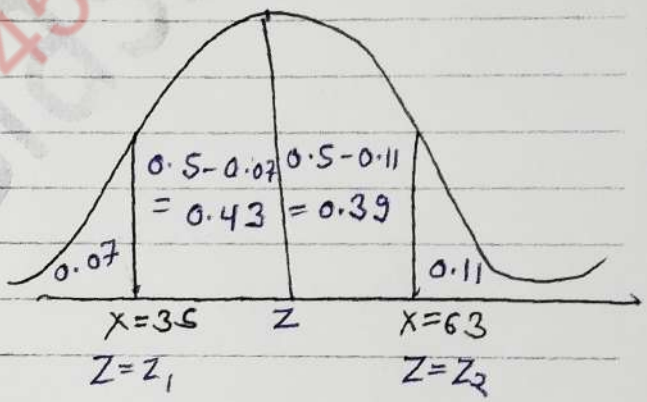
$P(X<63) = 89\%$

$P(X>63) = 1 - P(X<63)$

$= 1 - 0.89$

$P(X>63) = 0.11$

$$Z = \frac{x-\mu}{\sigma}$$



$x_1 = 35$

$P(-35 < x < 0) = 0.43$

$Z_1 = -1.48$

when $x_2 = 63$

$P(0 < x < 63) = 0.39$

$Z_2 = 1.23$

$$Z_1 = \frac{x_1 - \mu}{\sigma}$$

$$-1.48 = \frac{35-\mu}{\sigma}$$

$-1.48\sigma + \mu = 35 \quad -(1)$

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$$Z_2 = \frac{x_2 - \mu}{\sigma}$$

$$1.23 = \frac{63 - \mu}{\sigma}$$

$$1.23\sigma = 63 - \mu$$

$$1.23\sigma + \mu = 63 \quad - (2)$$

On solving eq (1) and (2)

$$1.23\sigma + \mu = 63$$
$$- 1.48\sigma + \mu = 35$$
$$+ \quad - \quad -$$

$$2.71\sigma = 28$$

$$\sigma = \frac{28}{2.71}$$

$$\boxed{\sigma = 10.33}$$

Similiarly

$$\boxed{\mu = 50.3}$$

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