# Engineering Mathematics

**Unit:** Partial Differential Equations (PDE)

## Lec-2

**Today's Target**

- Formation of PDE [Part-1]
- Univ. Questions
- DDPs



By Gulshan sir

**Note :** $z = f(x, y)$

$$\frac{\partial z}{\partial x} = p$$

$$\frac{\partial z}{\partial y} = q$$

Partial Derivative of first order

$$\frac{\partial^2 z}{\partial x^2} = r$$

$$\frac{\partial^2 z}{\partial x \partial y} = s$$

$$\frac{\partial^2 z}{\partial y^2} = t$$

Partial Derivative of Second order

# Formation of PDE

*By elimination of arbitrary constants* | *By elimination of arbitrary functions*

## *By elimination of arbitrary constants*

Let $F(x, y, z, a, b) = 0$ ——— (1)

*be the given function, where a and b are arbitrary constants*

*Step 1 -* *Find* $\frac{\partial F}{\partial x} = 0$ ——— (2)

*Step 2 -* *Find* $\frac{\partial F}{\partial y} = 0$ ——— (3)

*Step 3 - Eliminate a, b (arbitrary constants) from equations* (1), (2) *and* (3)

*Now, we get required PDE*

## CASE - I

*If No. of arbitrary constants < No. of independent variables*

*Then we get more than one PDE of order one*

**Q.1 Find PDE by eleminating $a$ from $z = ax + y$.**

Given

$$z = ax + y \quad -(1)$$

Partially Diff. w.r.t $x$ and $y$

$$\frac{\partial z}{\partial x} = a \quad -(2)$$

$$\frac{\partial z}{\partial y} = 1 \quad -(3)$$

From eqn (1) and (2)

$$z = x\frac{\partial z}{\partial x} + y$$

$$\boxed{z = px + y}$$

$$\boxed{q = 1}$$

} Required PDE

## CASE - II

*If No. of arbitrary constants = No. of independent variables*

*Then we get only one PDE of order one*

## Q.2 Form partial differential equation by eliminating the arbitrary constants:

$$z = ax + a^2 + b^2 + by$$

Given

$$z = ax + a^2 + b^2 + by \quad \text{—①}$$

Partially diff. w.r.t $x$ and $y$

$$\frac{\partial z}{\partial x} = a + 0 + 0 + 0$$

$$\frac{\partial z}{\partial x} = a \quad \text{—②}$$

$$\frac{\partial z}{\partial y} = 0 + 0 + 0 + b$$

$$\frac{\partial z}{\partial y} = b \quad \text{—③}$$

From eqn ①, ② and ③

$$z = x\frac{\partial z}{\partial x} + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + y\frac{\partial z}{\partial y}$$

$$\boxed{z = px + p^2 + q^2 + qy}$$

## Q.3 Form partial differential equation by eliminating the arbitrary constants:

$$z = (x + a)(y + b)$$

Given

$z = (x+a)(y+b)$ — ①

Partially diff. w.r.t x and y

$\frac{\partial z}{\partial x} = (y+b)(1+0)$

$\frac{\partial z}{\partial x} = (y+b)$ — ②

$\frac{\partial z}{\partial y} = (x+a)(1+0)$

$\frac{\partial z}{\partial y} = (x+a)$ — ③

From ①, ② and ③

$z = \frac{\partial z}{\partial y} \times \frac{\partial z}{\partial x}$

$$\boxed{z = pq}$$

**Q.4 Form the partial differential equation by eliminating a and b from $z = (x^2 + a)(y^2 + b)$.** [2012]

Given

$$z = (x^2+a)(y^2+b) \quad -(1)$$

Partially diff. w.r.t x and y

$$\frac{\partial z}{\partial x} = (y^2+b)(2x+0)$$

$$\frac{\partial z}{\partial x} = 2x(y^2+b)$$

$$(y^2+b) = \frac{1}{2x} \times \frac{\partial z}{\partial x} \quad -(2)$$

$$\frac{\partial z}{\partial y} = (x^2+a)(2y+0)$$

$$\frac{\partial z}{\partial y} = 2y(x^2+a)$$

$$(x^2+a) = \frac{1}{2y} \times \frac{\partial z}{\partial y} \quad -(3)$$

From (1), (2) and (3)

$$z = \frac{1}{2y} \times \frac{\partial z}{\partial y} \times \frac{1}{2x} \times \frac{\partial z}{\partial x}$$

$$4xyz = \frac{\partial z}{\partial y} \times \frac{\partial z}{\partial x}$$

$$4xy = pq$$

**Q.5** ***Find the partial differential equation of all spheres whose centers lie on z-axis and given by equations*** $x^2 + y^2 + (z-a)^2 = b^2$ ***; a and b being constants.*** **[2017]**

Given

$$x^2+y^2+(z-a)^2 = b^2 \quad -(1)$$

Partially diff. w.r.t $x$ and $y$

$$2x + 0 + 2(z-a)\frac{\partial z}{\partial x} = 0$$

$$2x + 2(z-a)\frac{\partial z}{\partial x} = 0$$

$$2(z-a)\frac{\partial z}{\partial x} = -2x$$

$$(z-a)p = -x$$

$$(z-a) = -\frac{x}{p} \quad -(2)$$

$$0 + 2y + 2(z-a)\frac{\partial z}{\partial y} = 0$$

$$2(z-a)q = -2y$$

$$z-a = -\frac{y}{q} \quad -(3)$$

From equation (2) and (3)

$$+\frac{x}{p} = +\frac{y}{q}$$

$$\boxed{qx - py = 0}$$

## CASE - III

*If No. of arbitrary constants > No. of independent variables*

more than one

*Then we get higher order PDE*

**Q.6 Find PDE by eleminating a, b, c :** $z = ax + by + cxy$

$z = ax + by + cxy$ — (1)

Partially diff. w.r.t x

$\frac{\partial z}{\partial x} = a + 0 + cy$

$\frac{\partial z}{\partial x} = a + cy$ — (2)

Again partially diff. w.r.t x

$\frac{\partial^2 z}{\partial x^2} = 0$ — (3)

Partially diff. (1) w.r.t y

$\frac{\partial z}{\partial y} = 0 + b + cx$ — (4)

Again partially diff w.r.t y

$\frac{\partial^2 z}{\partial y^2} = 0$ — (5)

Multiply eqn (2) by x and (4) by y

$x\frac{\partial z}{\partial x} = ax + cxy$ — (6)

$y\frac{\partial z}{\partial y} = by + cxy$ — (7)

Adding (6) and (7)

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = ax + cxy + by + cxy$$

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = z + cxy \quad - (3)$$

From (2)

$$\frac{\partial z}{\partial x} = a + cy$$

Partially diff wrt y

$$\frac{\partial^2 z}{\partial x \partial y} = c$$

Put c in (3)

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = z + \frac{\partial^2 z}{\partial x \partial y} \times xy$$

$$\boxed{x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = z + xy\frac{\partial^2 z}{\partial x \partial y}}$$

For more Subject/pdf Notes
Download Gateway Classes Application
From Google Play store

Link in Description

# Thank You