Engineering Mathematics

UNIT : Partial Differential Equation

Today's Target

- Equation Reducible to Linear PDE with constant coefficient
- PYQs
- DPP

Course Details(Paid) : All Subjects

1	Recorded Video Lectures (100 % Syllabus Coverage)
2	Pdf Notes
3	Lecture wise DPP with Answers
4	DPP Solutions
5	Unit wise set of PYQs

Paid Courses are available in Gateway Classes Application
Link in Description

Equation Reducible to Linear PDE with constant coefficient

General Form

$$a_0 x^2 \frac{\partial^2 z}{\partial x^2} + a_1 xy \frac{\partial^2 z}{\partial x \partial y} + a_2 y^2 \frac{\partial^2 z}{\partial y^2} + b_1 \frac{x \partial z}{\partial x} + b_2 y \frac{\partial z}{\partial y} = F(x, y)$$

Standard substitution to convert above equation in Linear PDE with constant coefficient

(1) Put $x = e^u \quad \Rightarrow \quad u = \log x$

$y = e^v \quad \Rightarrow \quad v = \log y$

(2) Put $x \frac{\partial z}{\partial x} = DZ$,

$$x^2 \frac{\partial^2 z}{\partial x^2} = D(D-1)\, Z,$$

where, $D \equiv \frac{\partial}{\partial u}$

$$x^3 \frac{\partial^2 z}{\partial x^3} = D(D-1)\,(D-2) Z$$

(3) : $y\frac{\partial z}{\partial y} = D'Z$

$y^2\frac{\partial^2 z}{\partial y^2} = D'(D'-1)\,Z,$ where $D' \equiv \frac{\partial}{\partial v}$

$y^3\frac{\partial^3 z}{\partial y^3} = D'(D'-1)\,(D'-2)Z$

(4) : $xy\frac{\partial^2 z}{\partial x\,\partial y} = DD'Z$

(5): After above substitution given PDE is converted in to Linear PDE with constant coefficient

Linear Partial Differential Equation with constant coefficient

Linear PDE with constant coefficient

Homogeneous Linear PDE with constant coefficient

(All partial derivatives are of same order)

Complete solution = CF + PI

Non-Homogeneous Linear PDE with constant coefficient

(All partial derivatives are not of same order)

Complete solution = CF + PI

Q.1 Solve the differential equation : $x^2\frac{\partial^2 z}{\partial x^2} - y^2\frac{\partial^2 z}{\partial y^2} = xy$. AKTU-2022-23

$$x^2\frac{\partial^2 z}{\partial x^2} - y^2\frac{\partial^2 z}{\partial y^2} = xy$$

Put $x = e^u \Rightarrow u = \log x$

$y = e^v \Rightarrow v = \log y$

$$x^2\frac{\partial^2 z}{\partial x^2} = D(D-1)z$$

$$y^2\frac{\partial^2 z}{\partial y^2} = D'(D'-1)z$$

$$D(D-1)z - D'(D'-1)z = e^u \times e^v$$

$$[D(D-1) - D'(D'-1)]z = e^{u+v}$$

$$(D^2 - D - D'^2 + D')z = e^{u+v}$$

$$(D^2 - D'^2 + D' - D)z = e^{u+v}$$

$$[(D-D')(D+D') - (D-D')]z = e^{u+v}$$

$$(D-D')(D+D'-1)z = e^{u+v}$$

$$(D-D')(D+D'-1)z = e^{u+v}$$

$$CF = e^{0u} f_1(v+u) + e^{u} f_2(v-u)$$

$$CF = f_1(v+u) + e^{u} f_2(v-u)$$

$$CF = f_1(\log y + \log x) + x f_2(\log y - \log x)$$

$$CF = f_1\{\log xy\} + x f_2(\log y/x)$$

$$\boxed{CF = g_1(xy) + x g_2(y/x)}$$

$$PI = \frac{1}{(D-D')(D+D'-1)} e^{u+v}$$

$$PI = \frac{1}{(D-D')}\left[\frac{1}{(D+D'-1)} e^{u+v}\right]$$

$$PI = \frac{1}{D-D'}\left[\frac{1}{1+1-1} e^{u+v}\right]$$

$$PI = \frac{1}{D-D'} e^{u+v}$$

case failure

$$PI = \frac{1}{D-D'} e^{u+v}$$

$$PI = u \times \frac{1}{1-0} e^{u+v}$$

$$PI = u e^{u+v}$$

$$PI = u \times e^{u} \times e^{v}$$

$$\boxed{PI = nylogn}$$

complete solution

$$z = CF + PI$$

$$\boxed{z = g_1(ny) + n\, g_2(y/n) + nylogn}$$

Q.2 Solve the linear partial differential equation : $x^2\frac{\partial^2 z}{\partial x^2} - 4xy\frac{\partial^2 z}{\partial x \partial y} + 4y^2\frac{\partial^2 z}{\partial y^2} + 6y\frac{\partial z}{\partial y} = x^3y^4$.

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$$x^2\frac{\partial^2 z}{\partial x^2} - 4xy\frac{\partial^2 z}{\partial x \partial y} + 4y^2\frac{\partial^2 z}{\partial y^2} + 6y\frac{\partial z}{\partial y} = x^3y^4$$

Put $x = e^u \Rightarrow u = \log x$

$y = e^v \Rightarrow v = \log y$

$$x^2\frac{\partial^2 z}{\partial x^2} = D(D-1)z$$

$$xy\frac{\partial^2 z}{\partial x \partial y} = DD'z$$

$$y^2\frac{\partial^2 z}{\partial y^2} = D'(D'-1)z$$

$$y\frac{\partial z}{\partial y} = D'z$$

$$D(D-1)z - 4DD'z + 4D'(D'-1)z + 6D'z = (e^u)^3(e^v)^4$$

$$[D^2 - D - 4DD' + 4D'^2 - 4D' + 6D']z = e^{3u} \times e^{4v}$$

$$[D^2 - D - 4DD' + 4D'^2 + 2D']z = e^{3u+4v}$$

$$[(D^2 - 4DD' + 4D'^2) - D + 2D']z = e^{3u+4v}$$

$$\left[(D-2D')^2-(D-2D')\right]z = e^{3u+4v}$$

$$(D-2D')(D-2D'-1)z = e^{3u+4v}$$

$$CF = e^{0u} f_1(v+2u) + e^{u} f_2(v+2u)$$

$$CF = f_1(\log y+2\log x) + x f_2(\log y+2\log x)$$

$$CF = f_1(\log y+\log x^2) + x f_2(\log y+\log x^2)$$

$$CF = f_1(\log yx^2) + x f_2(\log yx^2)$$

$$\boxed{CF = g_1(yx^2) + x g_2(yx^2)}$$

$$PI = \frac{1}{(D-2D')(D-2D'-1)} e^{3u+4v}$$

Put $D=3$ and $D'=4$

$$PI = \frac{1}{(3-8)(3-8-1)} e^{3u+4v}$$

$$PI = \frac{1}{30} e^{3u} \times e^{4v}$$

$$PI = \frac{1}{30}(e^u)^3 \times (e^v)^4$$

$$\boxed{PI = \frac{1}{30} x^3 y^4}$$

complete solution

$$z = CF + PI$$

$$\boxed{z = g_1(yx^2) + x\, g_2(yx^2) + \frac{1}{30} x^3 y^4}$$

Q.3 Solve : $x^2r - y^2t + px - qy = \log x$

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$$x^2\frac{\partial^2 z}{\partial x^2} - y^2\frac{\partial^2 z}{\partial y^2} + x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y} = \log x$$

Put $x = e^u \Rightarrow u = \log x$

$y = e^v \Rightarrow v = \log y$

$$x^2\frac{\partial^2 z}{\partial x^2} = D(D-1)z$$

$$y^2\frac{\partial^2 z}{\partial y^2} = D'(D'-1)z$$

$$x\frac{\partial z}{\partial x} = Dz$$

$$y\frac{\partial z}{\partial y} = D'z$$

$$D(D-1)z - D'(D'-1)z + Dz - D'z = u$$

$$[D^2 - D - D'^2 + D' + D - D']z = u$$

$$(D^2 - D'^2)z = u$$

Aux. Eqn

$$D^2 - D'^2 = 0$$

Put $D = m, D' = 1$

$$m^2 - 1 = 0$$

$$m = \pm 1$$

$$CF = f_1(v+u) + f_2(v-u)$$

$$CF = f_1(\log y + \log x) + f_2(\log y - \log x)$$

$$CF = f_1(\log xy) + f_2\left(\log \frac{y}{x}\right)$$

$$\boxed{CF = g_1(xy) + g_2\left(\frac{y}{x}\right)}$$

$$PI = \frac{1}{(D^2 - D'^2)} u$$

$$PI = \frac{1}{(D^2 - D'^2)} (u + 0v)$$

Put $D = 1$, $D' = 0$ and $u + 0v = t$

$$PI = \frac{1}{1-0} \int\int t\, dt + dt$$

$$PI = \int \frac{t^2}{2} dt$$

$$PI = \frac{t^3}{6} = \frac{u^3}{6}$$

$$PI = \frac{(\log n)^3}{6}$$

Complete solution

$$z = CF + PI$$

$$\boxed{z = g_1(xy) + g_2(y/x) + \frac{1}{6}(\log n)^3}$$

PDE : DPP-15

➢ Topic : Non-Homogeneous Linear Partial Differential Equation with constant coefficient

Particular Integral (PI) : Case-3 and Case-4

Q .1 *Solve the linear partial differential equation :* $\left(x^2D^2 + 2xyDD' + y^2D'^2\right)z = x^m y^n$. *AKTU-2020-21*

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Thank You

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