ENGINEERING MATHEMATICS

UNIT-5 : *Statistical Techniques – III*

(Testing a Hypothesis and Statistical Quality Control)

Lec-05

Today's Target

- $\chi^2 - test\ of\ goodness\ of\ FIT\ (Part - 1)$
- PYQ
- DPP

Hypothesis Testing

Test of significance

(i) t – test

(ii) z – test

(iii) F – test

(iv) chi – square test (χ^2 – test) v. imp

Hpothesis Testing

Parametric test

(Sample drawn from normal population)

(i) t – test

(ii) z – test

(iii) F – test

Non – Parametric test

(No information about population)

(i) χ^2 – test

Chi – Square test ($\chi^2 - test$)

$\chi^2 - test$

- $\chi^2 - test$ of goodness of FIT — L – 5, L – 6
- $\chi^2 - test$ as a test of independence — L – 7

$\chi^2 - test$ of goodness of Fit : Given Data can be fitted in

(i) Poisson Distribution

(ii) Binomial Distribution

(iii) Normal Distribution

Fitting is good or not is test by $\chi^2 - test$.

In $\chi^2 - test$ $o_1, o_2 \ldots . o_n$ is a set of observed frequencies (Given in Question) and $E_1, E_2, E_3 \ldots E_n$ are the corresponding (Find) set of expected frequencies such that $\sum o_i = \sum E_i$

Conditions for applying χ^2 – test

(i) Sum of frequencies must be greater than equal to 50.

(ii) The expected frequency E_i of any item must be greater than 5.

Note : If any item has frequency less than 5 then it should be combined with the next or preceding item until the total frequency exceed 5.

Steps to solve questions

Step – 1 : Null Hypothesis (H_o)

(H_o): There is no significant difference between observed and expected frequency.

Step – 2 : Calculate χ^2 under H_o

$$\chi^2 = \sum \left[\frac{(O_i - E_i)^2}{E_i}\right]$$

Where O_i = observed frequency

E_i = Expected frequency

Step – 3 : Degree of freedom = Number of variables − number of constraints

$$\vartheta = n - k = n - 1$$

Level of significance $\propto = 5\% = 0.05$

Step – 4 : From χ^2 – table

Calculate $\chi^2_{0.05}$ at the given level of significance for the given degree of freedom

Step – 5 : conclusion

(i) If $\chi^2 < \chi^2_{0.05}$, then H_o accepted

- There is no significants difference between observed frequency and expected frequency.
- Fit of the data is considered to be good.

(ii) $\chi^2 > \chi^2_{0.05}$, then H_o is rejected.

- There is significants difference between observed frequency and expected frequency.
- Fit of the data is not good.

Q.1 ***A die is thrown 276 times and the results of these throws are given below:***

No. appeared on the die	1	2	3	4	5	6
Frequency	40	32	29	59	57	59

Test whether the die is biased or not. **(A.K.T.U. 2019)**

Null Hypothesis (H_0)

H_0: The die is an unbiased die

Calculation of χ^2, under H_0

$$E_i = \frac{\sum O_i}{n} = \frac{276}{6}$$

$$E_i = 46$$

O_i	E_i	$O_i - E_i$	$(O_i - E_i)^2$
40	46	−6	36
32	46	−14	196
29	46	−17	289
59	46	13	169
57	46	11	121
59	46	13	169
$\sum O_i = 276$	$\sum E_i = 276$		$\sum (O_i - E_i)^2 = 980$

$$\chi^2 = \sum \left[\frac{(O_i - E_i)^2}{E_i} \right]$$

$$\chi^2 = \frac{\sum (O_i - E_i)^2}{E_i}$$

$$= \frac{980}{46}$$

$$\boxed{\chi^2 = 21.30}$$

Degree of Freedom $= n - 1$

$= 6 - 1$

$= 5$

Level of significance α

$\alpha = 5\% = 0.05$

From χ^2-test table

$$\chi^2_{0.05} = 11.070$$

conclusion

$\chi^2 = 21.30$, $\chi^2 = 11.070$

$\because \chi^2 > \chi^2_{0.05}$

$\therefore H_0$ is rejected

Hence,

Given die is a biased die

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Table 4 : CHI-SQUARE

Significant Values $\chi^2(\alpha)$ of Chi-Square Distribution Right Tail Areas for Given Probability α,

$P = P_r(\chi^2 > \chi^2(\alpha)) = \alpha$

And v is Degrees of Freedom (d.f.)

Degree of freedom (v)	*Probability (Level of Significance)*						
	0.99	*0.95*	*0.50*	*0.10*	*0.05*	*0.02*	*0.01*
1	.000157	.00393	.455	2.706	3.841	5.214	6.635
2	.0201	.103	1.386	4.605	5.991	7.824	9.210
3	.115	.352	2.366	6.251	7.815	9.837	11.341
4	.297	.711	3.357	7.779	9.488	11.668	13.277
5	.554	1.145	4.351	9.236	11.070	13.388	15.086
6	.872	2.635	5.348	10.645	12.592	15.033	16.812
7	1.239	2.167	6.346	12.017	14.067	16.622	18.475
8	1.646	2.733	7.344	13.362	15.507	18.168	20.090
9	2.088	3.325	8.343	14.684	16.919	19.679	21.669
10	2.558	3.940	9.340	15.987	18.307	21.161	23.209
11	3.053	4.575	10.341	17.275	19.675	22.618	24.725
12	3.571	5.226	11.340	18.549	21.026	24.054	26.217
13	4.107	5.892	12.340	19.812	22.362	25.472	27.688
14	4.660	6.571	13.339	21.064	23.685	26.873	29.141
15	4.229	7.261	14.339	22.307	24.996	28.259	30.578
16	5.812	7.962	15.338	23.542	26.296	29.633	32.000
17	6.408	8.672	15.338	24.769	27.587	30.995	33.409
18	7.015	9.390	17.338	25.989	28.869	32.346	34.805
19	7.633	10.117	18.338	27.204	30.144	33.687	36.191
20	8.260	10.851	19.337	28.412	31.410	35.020	37.566
21	8.897	11.591	20.337	29.615	32.671	36.343	38.932
22	9.542	12.338	21.337	30.813	33.924	37.659	40.289
23	10.196	13.091	22.337	32.007	35.172	38.968	41.638
24	10.856	13.848	23.337	32.196	36.415	40.270	42.980
25	11.524	14.611	24.337	34.382	37.65	41.566	44.314
26	12.198	15.379	25.336	35.363	38.885	41.856	45.642
27	12.879	16.151	26.336	36.741	40.113	41.140	46.963
28	13.565	16.928	27.336	37.916	41.337	45.419	48.278
29	14.256	17.708	28.336	39.087	42.557	46.693	49.588
30	14.933	18.493	29.336	40.256	43.773	47.962	50.892

Q. 2 *The following table gives the number of accidents that took place in an industry during various days of the week. Test if accidents are uniformly distributed over the week.*

Day	Mon	Tue	Wed	Thu	Fri	Sat
No. of accidents	14	18	12	11	15	14

Null Hypothesis (H_0)

H_0: Accidents are uniformly distributed over the week

calculation of χ^2, under H_0

$$E_i = \frac{\sum O_i}{n} = \frac{84}{6} = 14$$

O_i	E_i	$O_i - E_i$	$(O_i - E_i)^2$
14	14	0	0
18	14	4	16
12	14	−2	4
11	14	−3	9
15	14	1	1
14	14	0	0
$\sum O_i =$ 84			$\sum (O_i - E_i)^2 =$ 30

$$\chi^2 = \sum \left[\frac{(O_i - E_i)^2}{E_i} \right]$$

$$= \frac{\sum (O_i - E_i)^2}{E_i}$$

$$= \frac{30}{14}$$

$$\boxed{\chi^2 = 2.1428}$$

Degree of freedom $= n - 1$

$= 6 - 1$

$= 5$

Level of significance (α)

$\alpha = 5\% = 0.05$

From χ^2 - test table

$$\boxed{\chi^2_{0.05} = 11.070}$$

conclusion

$\chi^2 = 2.1428$, $\chi^2_{0.05} = 11.070$

$\therefore \chi^2 < \chi^2_{0.05}$

$\therefore H_0$ is accepted

Hence,

Accidents are uniformly distributed over the week

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	0.99	*0.95*	*0.50*	*0.10*	*0.05*	*0.02*	*0.01*
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Q.3 ***In experiments on pea breeding, the following frequencies of seeds were obtained:***

Round and yellow	*Wrinkled and yellow*	*Round and green*	*Wrinkled and green*	*Total*
315	101	108	32	556

Theory predicts that the frequencies should be in proportions **9 : 3 : 3 : 1.**

Examine the correspondence between theory and experiment.

Null Hypothesis (H_0)

H_0: Experimental Results support theory

Calculation of χ^2 under H_0

$$E_1 = \frac{9}{16} \times 556 = 312.75$$

O_i	E_i	$O_i - E_i$	$(O_i - E_i)^2$	$\frac{(O_i - E_i)^2}{E_i}$
315	312.75	2.25	5.0625	0.016187
101	104.25	−3.25	10.5625	0.101319
108	104.25	3.75	14.0625	0.134892
32	34.75	−2.75	7.5625	0.217626
$\Sigma O_i = 556$	$\Sigma E_i = 556$			$\sum\left[\frac{(O_i - E_i)^2}{E_i}\right] = ?$

$$E_2 = \frac{3}{16} \times 556 = 104.25$$

$$E_3 = \frac{3}{16} \times 556 = 104.25$$

$$E_4 = \frac{1}{16} \times 556 = 34.75$$

$$\chi^2 = \sum \left[\frac{(O_i - E_i)^2}{E_i} \right]$$

$$\boxed{\chi^2 = 0.470024}$$

Degree of freedom $= n - 1$

$= 4 - 1$

$= 3$

Level of significance (α)

$\alpha = 5\% = 0.05$

From χ^2-test table

$$\chi^2_{0.05} = 7.185$$

Conclusion

$\chi_{cal} = 0.470024$, $\chi_{tab} = 7.185$

$\therefore \chi_{cal} < \chi_{tab}$

$\therefore H_0$ is Accepted

Hence,

Experimental Results support theoritical results

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30	14.933	18.493	29.336	40.256	43.773	47.962	50.892

Topic : $\chi^2 - test\ of\ goodness\ of\ FIT\ (Part - 1)$

Q.1 $What\ is\ \chi^2 - test?$

$A\ die\ is\ thrown\ 90\ times\ with\ the\ following\ results:$ [AKTU 2010]

$Face$	:	1	2	3	4	5	6	$Total$
$Frequency$	:	10	12	16	14	18	20	90

$Use\ \chi^2 - test\ to\ test\ whether\ these\ data\ are\ consistent\ with\ the\ hypothesis\ that\ die\ is\ unbiased.$

Q.2 $The\ demand\ for\ a\ particular\ spare\ part\ in\ a\ factory\ was\ found\ to\ vary\ from\ day - to - day.$

$In\ a\ sample\ study, the\ following\ information\ was\ obtained:$

$Days:$	Mon	Tue	Wed	$Thurs$	Fri	Sat
$No.of\ parts\ demanded:$	1124	1125	1110	1120	1126	1115

$Test\ the\ hypothesis\ that\ the\ number\ of\ parts\ demanded\ does\ not\ depend\ on\ the\ day\ of\ the\ week.$

Q.3 $The\ theory\ predicts\ the\ proportion\ of\ beans\ in\ the\ four\ groups, G_1, G_2, G_3, G_4\ should\ be\ in\ the\ ratio\ 9:3:3:1.\ In\ an\ experiment\ with\ 1600\ beans\ the\ numbers\ in\ the\ four\ groups\ were\ 882, 313, 287\ and\ 118.\ Does\ the\ experimental\ result\ support\ the\ theory?$

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