DPP-13 Maths - IV Unit - 1

Topic - Non Homogeneous Partial Differential Equation with constant coefficient

Q-1 : Solve $(D^3 - 3DD' + D' + 4)z = e^{2x+y}$

$(D^3 - 3DD' + D' + 4)z = e^{2x+y}$

$(D^2 - 3DD' + D' + 4)z = 0$ — (1)

Here equation (1) cannot be resolved into linear factors in D and D'.

Consider a trial solution -

$$z = Ae^{hx+ky}$$

From eq (1)

$$(D^3 - 3DD' + D' + 4)Ae^{hx+ky} = 0$$

$$A\left[D^3 e^{hx+ky} - 3DD' e^{hx+ky} + D' e^{hx+ky} + 4e^{hx+ky}\right] = 0$$

$$A\left[h^3 e^{hx+ky} - 3hk e^{hx+ky} + k e^{hx+ky} + 4e^{hx+ky}\right] = 0$$

$$Ae^{hx+ky}\left[h^3 - 3hk + k + 4\right] = 0$$

$Ae^{hx+ky} \neq 0$

$\therefore h^3 - 3hk + k + 4 = 0$

Gateway Classes

Download Our App from Google PlayStore
Helpline Number : 7455961284
PDF Notes PYQs Solution Recorded

$\boxed{C.F. = \Sigma A e^{hx+Ky}}$, where $h^3-3hK+K+4=0$

$$PI = \frac{1}{D^3-3DD'+D'+4} e^{2x+y}$$

Replace $D=2$, $D'=1$

$$PI = \frac{1}{8-6+1+4} e^{2x+y}$$

$$\boxed{PI = \frac{1}{7} e^{2x+y}}$$

Complete solution is :-

$$Z = CF + PI$$

$$\boxed{Z = \Sigma A e^{hx+Ky} + \frac{1}{7} e^{2x+y}}, \text{ where } h^3-3hK+K+4=0$$

Q-2 :- Solve $(D^2-D'^2-3D+3D')z = e^{x-2y}$

$$(D^2-D'^2-3D+3D')z = e^{x-2y}$$

$$[(D-D')(D+D')-3(D-D')] = e^{x-2y}$$

$$(D-D')(D+D'-3) = e^{x-2y}$$

$$\boxed{C.F. = f_1(y+x) + e^{3x} f_2(y-x)}$$

Gateway Classes

Download Our App from Google PlayStore
Helpline Number : 7455961284
PDF Notes
PYQs Solution
Recorded

$$PI = \frac{1}{D^2 - D'^2 - 3D + 3D'} e^{x-2y}$$

Put $D = 1$, $D' = -2$

$$PI = \frac{1}{1 - 4 - 3 - 6} e^{x-2y}$$

$$\boxed{PI = \frac{-1}{12} e^{x-2y}}$$

Complete solution - $Z = CF + PI$

$$\boxed{Z = f_1(y+x) + e^{3x} f_2(y-x) - \frac{1}{12} e^{x-2y}}$$

Q-3:- Find the Particular Integral of $2s + t - 3q = 5\cos(3x - 2y)$

$$2s + t - 3q = 5\cos(3x - 2y)$$

$$(2DD' + D'^2 - 3D')Z = 5\cos(3x - 2y)$$

$$PI = \frac{1}{2DD' + D'^2 - 3D'} 5\cos(3x - 2y)$$

Replace $D^2 = -9$, $D'^2 = -4$, $DD' = 6$

$$PI = \frac{1}{2(6) - 4 - 3D'} 5\cos(3x - 2y)$$

Gateway Classes

GATEWAY CLASSES
Empowering learners, transforming futures

$$PI = \frac{1}{8-3D'} 5\cos(3x-2y)$$

Now multiply by conjugate,

$$PI = \frac{1}{8-3D'} \times \frac{8+3D'}{8+3D'} 5\cos(3x-2y)$$

$$PI = \frac{8+3D'}{64-9D'^2} 5\cos(3x-2y)$$

Again put $D'^2 = -4$

$$PI = \frac{8+3D'}{64-(-4)9} 5\cos(3x-2y)$$

$$PI = \frac{5}{100}(8+3D')\cos(3x-2y)$$

$$PI = \frac{1}{20}[8\cos(3x-2y) + 3D'\cos(3x-2y)]$$

$$PI = \frac{1}{20}[8\cos(3x-2y) + 3(-2)(-\sin(3x-2y)]$$

$$\boxed{PI = \frac{1}{10}[4\cos(3x-2y) + 3\sin(3x-2y)]}$$

Ans

Gatway Classes

GATEWAY CLASSES
Empowering learners, transforming futures

Q-4: Solve the following partial differential equation

$$(D^2 - DD' + D' - 1)z = \cos(x + 2y)$$

Sol: $(D^2 - DD' + D' - 1)z = \cos(x + 2y)$

$$[D^2 - 1 - D'(D-1)]z = \cos(x + 2y)$$

$$[(D+1)(D-1) - D'(D-1)]z = \cos(x + 2y)$$

$$(D-1)[D - D' + 1]z = \cos(x + 2y)$$

$$\boxed{C.F. = e^x f_1(y) + e^{-x} f_2(y+x)}$$

$$PI = \frac{1}{D^2 - DD' + D' - 1} \cos(x + 2y)$$

Put $D^2 = -1$, $D'^2 = -4$, $DD' = -2$

$$PI = \frac{1}{-1 - (-2) + D' - 1} \cos(x + 2y)$$

$$PI = \frac{1}{-1 + 2 + D' - 1} \cos(x + 2y)$$

$$PI = \frac{1}{D'} \cos(x + 2y)$$

$$\boxed{PI = \frac{\sin(x + 2y)}{2}}$$

Gateway Classes

Download Our App from Google PlayStore
Helpline Number : 7455961284
PDF Notes
PYQs Solution
Recorded Lectures

Complete solution ⇒ $z = CF + PI$

$$\boxed{z = e^{x} f_1(y) + e^{-x} f_2(y+x) + \frac{1}{2}\sin(x+2y)}$$

Ans

Gatway Classes