# ENGINEERING MATHEMATICS

## UNIT-5 : *Statistical Techniques – III*

## *(Testing a Hypothesis and Statistical Quality Control)*

### *Lec-07*

**Today's Target**

- $\chi^2 - test\ of\ as\ a\ test\ of\ independence$
- PYQ
- DPP

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# $\chi^2 - test$

$\chi^2 - test\ of\ goodness\ of\ FIT$

L-5, L-6

$\chi^2 - test\ as\ a\ test\ of\ independence$

L-7

## $\chi^2 - test\ as\ a\ test\ of\ independence$

*Two attributes A and B are given with size r and S respectively.*

### *Steps to solve questions*

**Step – 1 :** *Null Hypothesis* $(H_o)$

$(H_o)$ : *Two attributes are independent*

**Step – 2: Contingency table : Simple data is given is tabular form, called contingency table**

| | |
|---|---|
| $a$ $O_1$ | $b$ $O_2$ |
| $c$ $O_3$ | $d$ $O_4$ |

**The expected frequency for any cell in a contingency table can be calculated by using formula**

$$\text{Expected frequency} = \frac{\text{Row total X column total}}{\text{grand total}}$$

**Observed frequency**

| | | |
|---|---|---|
| $O_1$ $a$ | $O_2$ $b$ | $a+b$ |
| $O_3$ $c$ | $O_4$ $d$ | $c+d$ |
| $a+c$ | $b+d$ | $N = a+b+c+d$ |

↑ grand total

**Expected frequency**

| | | |
|---|---|---|
| $E_1 = \frac{(a+b)(a+c)}{N}$ | $E_2 = \frac{(a+b)(b+d)}{N}$ | $a+b$ |
| $E_3 = \frac{(c+d)(a+c)}{N}$ | $E_4 = \frac{(c+d)(b+d)}{N}$ | $c+d$ |
| $a+c$ | $b+d$ | $N = a+b+c+d$ |

**Step – 3 : Calculate $\chi^2$ under $H_0$**

$$\chi^2 = \sum\left[\frac{(O_i - E_i)^2}{E_i}\right]$$

**Step – 4 : Degree of freedom $= (r-1)(S-1)$**

**Level of significance$(\propto)$**

From $\chi^2$-table $\propto = 5\% = 0.05$

$\chi^2_{cal} = 7$

**Step – 5 : Conclusion**

**(ii) If $\chi^2 < \chi^2_{0.05}$, then $H_0$ accepted**

**Two attributes A and B are independent**

**(ii) If $\chi^2 > \chi^2_{0.05}$, then $H_0$ is rejected.**

**Two attributes A and B are not independent**

**Q. 1** ***From the following table regarding the colour of eyes of father and son, test if the colour of son's eye is associated with that of the father.*** **(A. K. T. U. 2021)**

| | Eye colour of son | |
|---|---|---|
| | Light | Not light |
| Light | 471 | 51 |
| Not light | 148 | 230 |

***Eye colour of father***

Null Hypothesis($H_0$)

$H_0$: The colour of son's eye is not associated with that of father

contingency table

Observed Frequency

| | | |
|---|---|---|
| 471 $O_1$ | 51 $O_2$ | 522 |
| 148 $O_3$ | 230 $O_4$ | 378 |
| 619 | 281 | 900 |

↑ Grand total

Expected Frequency

| | | |
|---|---|---|
| $E_1 = \frac{522 \times 619}{900}$ <br> $= 359.02$ | $E_2 = \frac{522 \times 281}{900}$ <br> $= 162.98$ | 522 |
| $E_3 = \frac{378 \times 619}{900}$ <br> $= 259.98$ | $E_4 = \frac{378 \times 281}{900}$ <br> $= 118.02$ | 378 |
| 619 | 281 | 900 |

$$\chi^2 = \sum \left[ \frac{(O_i - E_i)^2}{E_i} \right]$$

$$\chi^2 = \frac{(O_1 - E_1)^2}{E_1} + \frac{(O_2 - E_2)^2}{E_2} + \frac{(O_3 - E_3)^2}{E_3} + \frac{(O_4 - E_4)^2}{E_4}$$

$$= \frac{(471 - 359.02)^2}{359.02} + \frac{51 - 162.98}{162.98} + \frac{(148 - 259.98)^2}{259.98}$$

$$+ \frac{(230 - 118.02)^2}{118.02}$$

$$= 34.9290 + 76.9390 + 48.2326 + 106.3497$$

$$\chi^2_{(cal)} = 266.3497$$

Degree of freedom

$d.f = (r-1)(s-1)$

$= (2-1)(2-1)$

$= 1$

Level of significance

$\alpha = 5\% = 0.05$

From $\chi^2$-table

$$\chi^2_{tab} = 3.841$$

conclusion

$\chi^2_{cal} = 266.3497$, $\chi^2_{tab} = 3.841$

$\therefore \chi^2_{cal} > \chi^2_{tab}$

$\therefore H_0$ is rejected

Hence

The colour of son's eye is associated with that of father's eye

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# Table 4 : CHI-SQUARE

## Significant Values $\chi^2(\alpha)$ of Chi-Square Distribution Right Tail Areas for Given Probability $\alpha$,

$$P = P_r(\chi^2 > \chi^2(\alpha)) = \alpha$$

## And $v$ is Degrees of Freedom (d.f.)

| *Degree of freedom (v)* | *Probability (Level of Significance)* | | | | | | |
|---|---|---|---|---|---|---|---|
| | *0.99* | *0.95* | *0.50* | *0.10* | *0.05* | *0.02* | *0.01* |
| 1 | .000157 | .00393 | .455 | 2.706 | 3.841 | 5.214 | 6.635 |
| 2 | .0201 | .103 | 1.386 | 4.605 | 5.991 | 7.824 | 9.210 |
| 3 | .115 | .352 | 2.366 | 6.251 | 7.815 | 9.837 | 11.341 |
| 4 | .297 | .711 | 3.357 | 7.779 | 9.488 | 11.668 | 13.277 |
| 5 | .554 | 1.145 | 4.351 | 9.236 | 11.070 | 13.388 | 15.086 |
| 6 | .872 | 2.635 | 5.348 | 10.645 | 12.592 | 15.033 | 16.812 |
| 7 | 1.239 | 2.167 | 6.346 | 12.017 | 14.067 | 16.622 | 18.475 |
| 8 | 1.646 | 2.733 | 7.344 | 13.362 | 15.507 | 18.168 | 20.090 |
| 9 | 2.088 | 3.325 | 8.343 | 14.684 | 16.919 | 19.679 | 21.669 |
| 10 | 2.558 | 3.940 | 9.340 | 15.987 | 18.307 | 21.161 | 23.209 |
| 11 | 3.053 | 4.575 | 10.341 | 17.275 | 19.675 | 22.618 | 24.725 |
| 12 | 3.571 | 5.226 | 11.340 | 18.549 | 21.026 | 24.054 | 26.217 |
| 13 | 4.107 | 5.892 | 12.340 | 19.812 | 22.362 | 25.472 | 27.688 |
| 14 | 4.660 | 6.571 | 13.339 | 21.064 | 23.685 | 26.873 | 29.141 |
| 15 | 4.229 | 7.261 | 14.339 | 22.307 | 24.996 | 28.259 | 30.578 |
| 16 | 5.812 | 7.962 | 15.338 | 23.542 | 26.296 | 29.633 | 32.000 |
| 17 | 6.408 | 8.672 | 15.338 | 24.769 | 27.587 | 30.995 | 33.409 |
| 18 | 7.015 | 9.390 | 17.338 | 25.989 | 28.869 | 32.346 | 34.805 |
| 19 | 7.633 | 10.117 | 18.338 | 27.204 | 30.144 | 33.687 | 36.191 |
| 20 | 8.260 | 10.851 | 19.337 | 28.412 | 31.410 | 35.020 | 37.566 |
| 21 | 8.897 | 11.591 | 20.337 | 29.615 | 32.671 | 36.343 | 38.932 |
| 22 | 9.542 | 12.338 | 21.337 | 30.813 | 33.924 | 37.659 | 40.289 |
| 23 | 10.196 | 13.091 | 22.337 | 32.007 | 35.172 | 38.968 | 41.638 |
| 24 | 10.856 | 13.848 | 23.337 | 32.196 | 36.415 | 40.270 | 42.980 |
| 25 | 11.524 | 14.611 | 24.337 | 34.382 | 37.65 | 41.566 | 44.314 |
| 26 | 12.198 | 15.379 | 25.336 | 35.363 | 38.885 | 41.856 | 45.642 |
| 27 | 12.879 | 16.151 | 26.336 | 36.741 | 40.113 | 41.140 | 46.963 |
| 28 | 13.565 | 16.928 | 27.336 | 37.916 | 41.337 | 45.419 | 48.278 |
| 29 | 14.256 | 17.708 | 28.336 | 39.087 | 42.557 | 46.693 | 49.588 |
| 30 | 14.933 | 18.493 | 29.336 | 40.256 | 43.773 | 47.962 | 50.892 |

**Q. 2** *To test the effectiveness of inoculation against cholera, the following table was obtained*

| | Attacked | Not attacked | Total |
|---|---|---|---|
| Inoculated | 30 $O_1$ | 160 $O_2$ | 190 |
| Not inoculated | 140 $O_3$ | 460 $O_4$ | 600 |
| Total | 170 | 620 | 790 |

*(The figures represent the number of persons.)* **(A.K.T.U. 2009, 2018)**

*Use $\chi^2$ – test to defend or refute the statement that the inoculation prevents attack from cholera.*

Null Hypothesis ($H_0$)

$H_0$: The inoculation does not prevent from cholera attack

contingency table

observed Frequency

| | | |
|---|---|---|
| 30 $O_1$ | 160 $O_2$ | 190 |
| 140 $O_3$ | 460 $O_4$ | 600 |
| 170 | 620 | 790 |

| | | |
|---|---|---|
| $E_1 = \frac{190 \times 170}{790}$ $= 40.89$ | $E_2 = \frac{190 \times 620}{790}$ $= 149.11$ | 190 |
| $E_3 = \frac{600 \times 170}{790}$ $= 129.11$ | $E_4 = \frac{600 \times 620}{790}$ $= 470.89$ | 600 |
| 170 | 620 | 790 |

$$\chi^2 = \sum \left[ \frac{(O_i - E_i)^2}{E_i} \right]$$

$$\chi^2 = \frac{(O_1 - E_1)^2}{E_1} + \frac{(O_2 - E_2)^2}{E_2} + \frac{(O_3 - E_3)^3}{E_3} + \frac{(O_4 - E_4)^2}{E_4}$$

$$\chi^2 = \frac{(30 - 40.89)^2}{40.89} + \frac{(160 - 149.11)^2}{149.11} + \frac{(140 - 129.11)^2}{129.11} + \frac{(460 - 470.89)^2}{470.89}$$

$$\chi^2 = 2.90 + 0.795 + 0.918 + 0.252$$

$$\chi^2_{cal} = 4.865$$

Degree of freedom

$$d.f = (r-1)(s-1)$$

$$= (2-1)(2-1)$$

$$= 1$$

Level of significance

$$\alpha = 5\% = 0.05$$

From $\chi^2$-table

$$\chi^2_{tab} = 3.841$$

conclusion

$$\chi_{cal} = 4.865$$

$$\chi^2_{tab} = 3.841$$

$\because \chi^2_{cal} > \chi^2_{tab}$

$\therefore H_0$ is rejected

Hence

The inoculation prevent from cholera Attack

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# Table 4 : CHI-SQUARE

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| *Degree of freedom (v)* | *Probability (Level of Significance)* | | | | | | |
|---|---|---|---|---|---|---|---|
| | *0.99* | *0.95* | *0.50* | *0.10* | *0.05* | *0.02* | *0.01* |
| 1 | .000157 | .00393 | .455 | 2.706 | 3.841 | 5.214 | 6.635 |
| 2 | .0201 | .103 | 1.386 | 4.605 | 5.991 | 7.824 | 9.210 |
| 3 | .115 | .352 | 2.366 | 6.251 | 7.815 | 9.837 | 11.341 |
| 4 | .297 | .711 | 3.357 | 7.779 | 9.488 | 11.668 | 13.277 |
| 5 | .554 | 1.145 | 4.351 | 9.236 | 11.070 | 13.388 | 15.086 |
| 6 | .872 | 2.635 | 5.348 | 10.645 | 12.592 | 15.033 | 16.812 |
| 7 | 1.239 | 2.167 | 6.346 | 12.017 | 14.067 | 16.622 | 18.475 |
| 8 | 1.646 | 2.733 | 7.344 | 13.362 | 15.507 | 18.168 | 20.090 |
| 9 | 2.088 | 3.325 | 8.343 | 14.684 | 16.919 | 19.679 | 21.669 |
| 10 | 2.558 | 3.940 | 9.340 | 15.987 | 18.307 | 21.161 | 23.209 |
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| 13 | 4.107 | 5.892 | 12.340 | 19.812 | 22.362 | 25.472 | 27.688 |
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| 16 | 5.812 | 7.962 | 15.338 | 23.542 | 26.296 | 29.633 | 32.000 |
| 17 | 6.408 | 8.672 | 15.338 | 24.769 | 27.587 | 30.995 | 33.409 |
| 18 | 7.015 | 9.390 | 17.338 | 25.989 | 28.869 | 32.346 | 34.805 |
| 19 | 7.633 | 10.117 | 18.338 | 27.204 | 30.144 | 33.687 | 36.191 |
| 20 | 8.260 | 10.851 | 19.337 | 28.412 | 31.410 | 35.020 | 37.566 |
| 21 | 8.897 | 11.591 | 20.337 | 29.615 | 32.671 | 36.343 | 38.932 |
| 22 | 9.542 | 12.338 | 21.337 | 30.813 | 33.924 | 37.659 | 40.289 |
| 23 | 10.196 | 13.091 | 22.337 | 32.007 | 35.172 | 38.968 | 41.638 |
| 24 | 10.856 | 13.848 | 23.337 | 32.196 | 36.415 | 40.270 | 42.980 |
| 25 | 11.524 | 14.611 | 24.337 | 34.382 | 37.65 | 41.566 | 44.314 |
| 26 | 12.198 | 15.379 | 25.336 | 35.363 | 38.885 | 41.856 | 45.642 |
| 27 | 12.879 | 16.151 | 26.336 | 36.741 | 40.113 | 41.140 | 46.963 |
| 28 | 13.565 | 16.928 | 27.336 | 37.916 | 41.337 | 45.419 | 48.278 |
| 29 | 14.256 | 17.708 | 28.336 | 39.087 | 42.557 | 46.693 | 49.588 |
| 30 | 14.933 | 18.493 | 29.336 | 40.256 | 43.773 | 47.962 | 50.892 |

**Q. 3** *In an experiment on the immunisation of cattle from a disease, the following results are obtained. Derive your inferences on the efficiency of the vaccine.* **(A. K. T. U. 2021)**

| | *Affected* | *Unaffected* |
|---|---|---|
| *Inoculated with vaccine* | 12 $O_1$ | 28 $O_2$ |
| *Not incoculated* | 13 $O_3$ | 7 $O_4$ |

$$\chi^2_{cal} = 6.73$$

## Topic : $\chi^2$ – test of as a test of independence

**Q. 1** In an experiment on the immunisation of goats from anthrox, the following results were obtained. Derive your inferences on the efficiency of the vaccine.

| | Died anthrox | Survived |
|---|---|---|
| Inoculated with vaccine | 2 | 10 |
| Not incoulated | 6 | 6 |

**Q. 2** By using $\chi^2$ – test, find out whether there is any association between income level and type of schooling:

| Income | Public School | Govt. school |
|---|---|---|
| Low | 200 | 400 |
| High | 1000 | 400 |

**Q. 3** The following data is collected on two characters:

| | Smokers | Non smokers |
|---|---|---|
| Literate | 83 | 57 |
| Illiterate | 45 | 68 |

Based on this information can you say that there is no relation between habit of smoking and literacy.

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# Thank You

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