ENGINEERING MATHEMATICS

UNIT-3 : *Statistical Techniques – I*

Lec-4

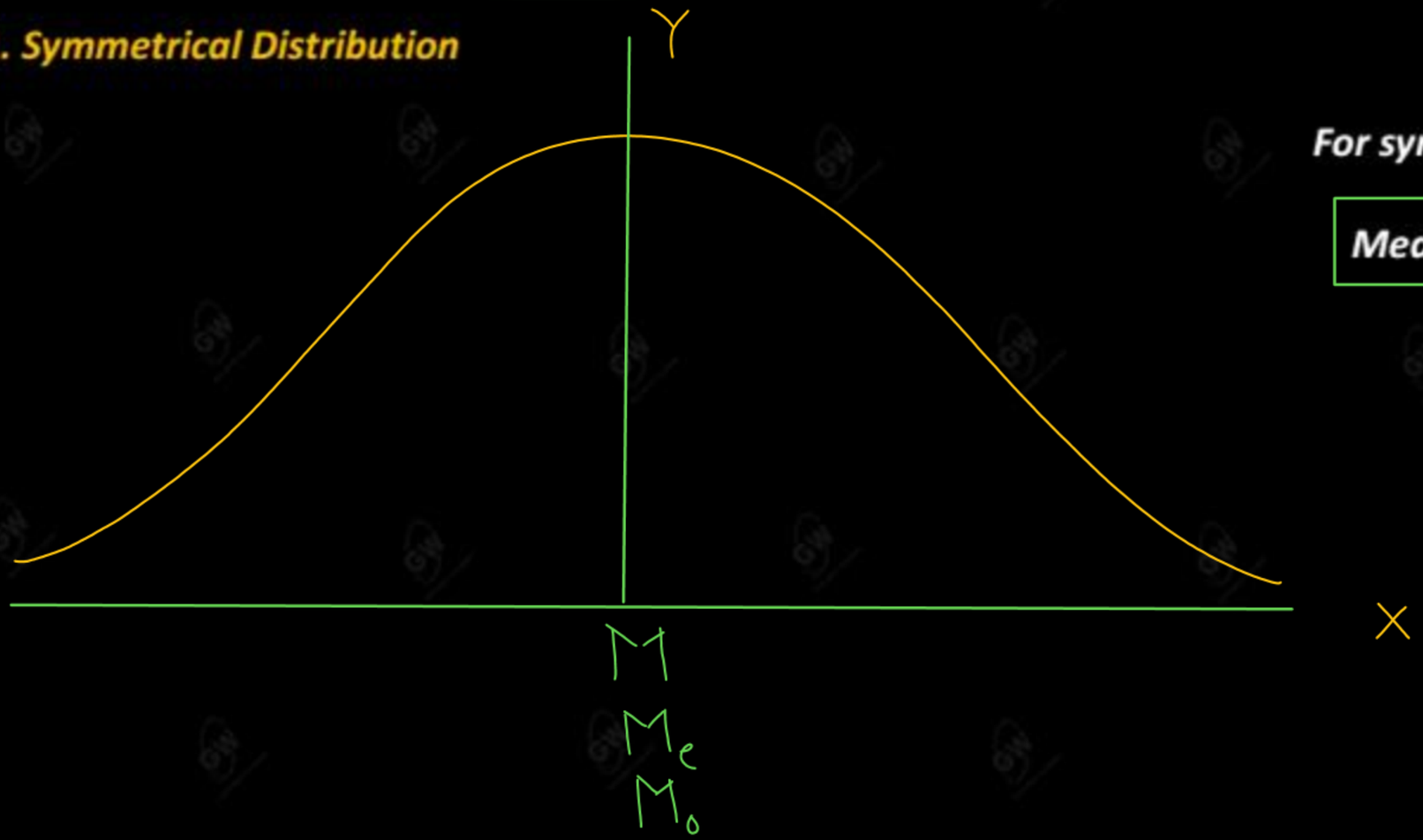
Today's Target

- ***Skewness***
- PYQ
- DPP

Skewness

Skewness means lack of symmetry in frequency distribution.

1. Symmetrical Distribution

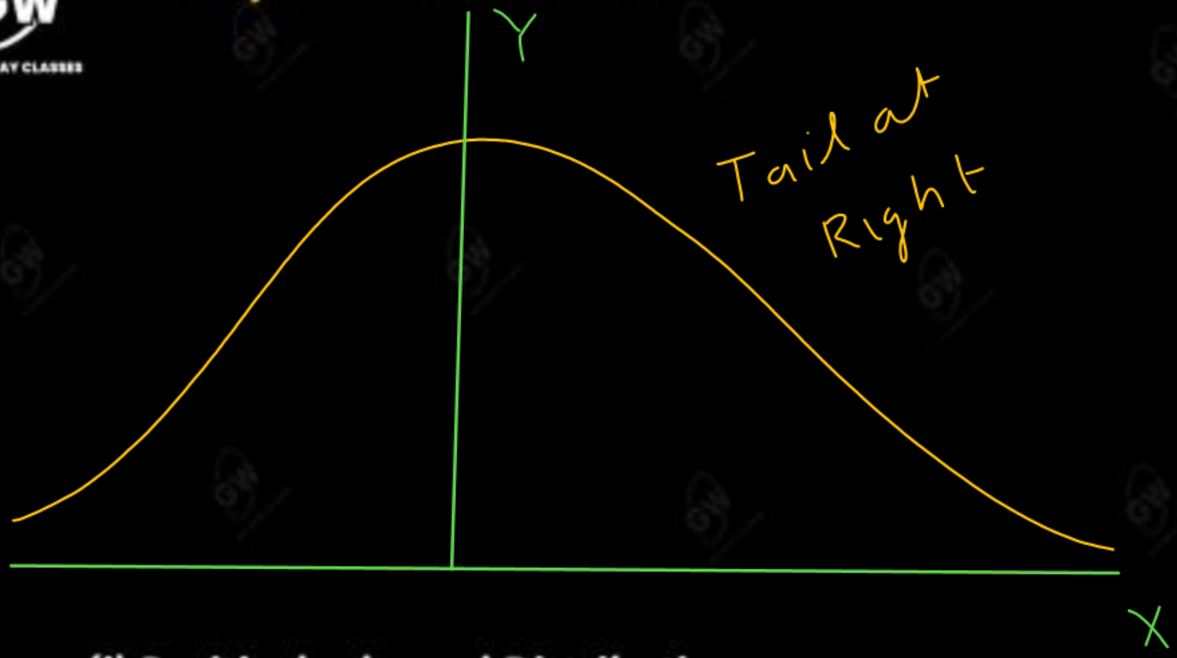


For symmetrical Distribution

Mean = Median = Mode

Note:- If Mean = Median = Mode, then there is no skewness in the distribution

2. Unsymmetrical Distribution



(i) Positively skewed Distribution

Mean > Mode



(ii) Negatively skewed Distribution

Mean < Mode

Methods of Measuring Skewness

(i) Karl Pearson's Method

V.Imp (ii) Method of Moments

Note:- Relative measures of skewness are called coefficient of skewness

Karl Pearson's Method

Karl Pearson's coefficient of skewness (SK_p)

(i) $$SK_p = \frac{Mean - Mode}{S.D}$$

Mode = 3 Median – 2 Mean

(ii) $$SK_p = \frac{3(Mean - Median)}{S.D}$$

S.D = Standard Deviation (σ)

Note:- (i) If Skp = 0

Mean = Mode

Distribution is symmetrical

(ii) If Skp > 0

Mean > Mode

Distribution is positively skewed

(iii) If Skp < 0

Mean < Mode

Distribution is negatively skewed

Standard Deviation

$$S.D = \sqrt{variance}$$

$$variance = \mu_2$$

$$\boxed{S.D = \sqrt{\mu_2}}$$

Relation between μ_r and μ'_r

$$\mu_1 = 0$$

$$\mu_2 = \mu'_2 - \mu'^2_1$$

$$\mu_3 = \mu'_3 - 3\mu'_2\mu'_1 + 2\mu'^3_1$$

$$\mu_4 = \mu'_4 - 4\mu'_3\mu'_1 + 6\mu'_2\mu'^2_1 - 3\mu'^4_1$$

Relation between μ_r and ν_r

$$\nu_1 = \bar{x}$$

$$\nu_2 = \mu_2 + \bar{x}^2$$

$$\nu_3 = \mu_3 + 3\mu_2\,\bar{x} + \bar{x}^3$$

$$\nu_4 = \mu_4 + 4\mu_3\,\bar{x} + 6\mu_2\bar{x}^2 + \bar{x}^4$$

Q.1 For a moderately skewed data, the arithmetic mean is 100, the variance is 35 and Karl Pearson's coefficient of skewness is 0.2. Find its mode and median.

Given data

Mean($\bar{x}$) = 100

Variance = 35

SK_p = 0.2

We know that

$S.D = \sqrt{\text{Variance}}$

$S.D = \sqrt{35}$

Karl pearson's coefficient of skewness

$$SK_p = \frac{\text{Mean} - \text{Mode}}{SD}$$

$$0.2 = \frac{100 - \text{Mode}}{\sqrt{35}}$$

$$\text{Mode} = 100 - 0.2 \times \sqrt{35}$$

$$\boxed{\text{Mode} = 98.816}$$

$$\text{Mode} = 3\,\text{Median} - 2\,\text{Mean}$$

$$98.816 = 3\,\text{Median} - 2 \times 100$$

$$\boxed{\text{Median} = 99.61}$$

Q.2 *The sum of 20 observations is 300 and sum of their squares is 5000. The median is 15. Find the Karl Pearson's coefficient of skewness.*

Given Data

$n = 20$

$\sum x = 300$

$\sum x^2 = 5000$

Median = 15

We know that

$$\text{Mean} = \frac{\sum x}{n}$$

$$\text{Mean} = \frac{300}{20}$$

Mean = 15

$$\mu_2 = \nu_2 - \bar{x}^2$$

$$\mu_2 = \frac{\sum x^2}{n} - \bar{x}^2$$

$$= \frac{5000}{20} - (15)^2$$

$$= 250 - 225$$

$$\mu_2 = 25$$

$$S.D = \sqrt{\mu_2} = \sqrt{25}$$

SD = 5

$$Sk_p = \frac{3(\text{Mean} - \text{Median})}{S.D}$$

$$Sk_p = \frac{3(15-15)}{5}$$

$Sk_p = 0$ Symmetrical Distribution

2. Method of Moments

μ_2 and μ_3

In this Method, second and third central moments of the distribution are used.

This measure of skewness is called the moment coefficient of skewness

Moment coefficient of skewness (SK_M) OR γ_1

$$SK_M = \frac{\mu_3}{\sqrt{\mu_2^3}}$$

Note:- *We know that*

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3}$$

$$\sqrt{\beta_1} = \frac{\mu_3}{\sqrt{\mu_2^3}}$$

$$SK_M = \sqrt{\beta_1}$$

$$SK_M = \sqrt{\beta_1} = \gamma_1$$

Q.3 The first three central moments of a distribution are 0, 15, - 31. Find the moment coefficient of skewness.

Given Data

$\mu_1 = 0$

$\mu_2 = 15$

$\mu_3 = -31$

$$SK_M = \frac{\mu_3}{\sqrt{\mu_2^3}}$$

$$= \frac{-31}{\sqrt{(15)^3}}$$

$$\boxed{SK_M = -0.53}$$

Negatively skewed Distribution

Q.4 *The first four moments of a distribution about the value 5 of the variable are 2, 20, 40 and 50. calculate the moment coefficient of skewness.*

Given Data

$A = 5$

$\mu_1' = 2$

$\mu_2' = 20$

$\mu_3' = 40$

$\mu_4' = 50$

We know that

$$\mu_2 = \mu_2' - \mu_1'^2$$

$$= 20 - (2)^2$$

$$\boxed{\mu_2 = 16}$$

$$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3$$

$$= 40 - 3 \times 20 \times 2 + 2(2)^3$$

$$\mu_3 = 40 - 120 + 16$$

$$\boxed{\mu_3 = -64}$$

$$SK_M = \frac{\mu_3}{\sqrt{\mu_2^3}}$$

$$= \frac{-64}{\sqrt{16^3}} = -\frac{64}{16 \times 4}$$

$$\boxed{SK_M = -1}$$

Negatively skewed Distribution

DPP- 4

Topic : *Skewness*

Q.1 (i) Define skewness of a distribution. (UPTU-2015)

(ii) Define the coefficients of skewness. (MTU 2012, 2013)

Q.2. Karl Pearson's coefficient of skewness of a distribution is 0.32, its standard deviation is 6.5 and mean is 29.6. Find the mode of the distribution.

Q.3. The first-three central moments of a distribution are 0, 2.5, 0.7. Find the value of the moment coefficient of skewness. (UPTU-2015)

Q.4 In a certain distribution, the first four moments about the point 4 are – 1.5, 17, - 30 and 308. Calculate the moment coefficient of skewness. (UPTU-2015)

GATEWAY CLASSES

AKTU Full Courses (Paid)

Download Gateway Classes Application

From Google Play store

All Subjects

Link in Description

Thank You

GATEWAY CLASSES