Engineering Mathematics-IV

Unit: Partial Differential Equations (PDE)

Today's Target

- Introduction to Partial Differential Equations
- Univ. Questions
- DDPs



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Differential Equation

An equation containing the dependent variable, independent variable and the derivative of the dependent variable with respect to the independent variable is known as a differential equation.

$y = f(x)$

$\frac{dy}{dx}$

$\frac{d^2y}{dx^2}$

$\frac{d^ny}{dx^n}$

$z = f(x, y)$

$\frac{\partial z}{\partial x}$

$\frac{\partial z}{\partial y}$

$\frac{\partial^2 z}{\partial x^2}$

$\frac{\partial^2 z}{\partial y^2}$

$\frac{\partial^2 z}{\partial x \partial y}$

Differential Equation

Ordinary Differential Equation

Partial Differential Equation

Ordinary Differential Equation (ODE)

A differential equation containing ordinary derivatives :

(i) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$

(ii) $\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = x^2$

Partial Differential Equation (PDE)

A differential equation containing partial derivatives :

(i) $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = x$

(ii) $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = xy$

(iii) $\left(\frac{\partial z}{\partial y}\right)^2 = 2x\frac{\partial^3 z}{\partial x^3}$

(iv) $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = xyz$

Note :

$$\frac{\partial z}{\partial x} = p$$

$$\frac{\partial z}{\partial y} = q$$

Partial Derivative of first order

$$\frac{\partial^2 z}{\partial x^2} = r$$

$$\frac{\partial^2 z}{\partial x \partial y} = s$$

$$\frac{\partial^2 z}{\partial y^2} = t$$

Partial Derivative of second order

Order and Degree of a PDE

Order of PDE

Order of PDE is the order of highest order derivatives :

(i) $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$ $\quad O = 1$

(ii) $\frac{\partial^2 z}{\partial x^2} + x\frac{\partial z}{\partial y}$ $\quad O = 2$

(iii) $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = xy$ $\quad O = 2$

(iv) $\left(\frac{\partial z}{\partial y}\right)^2 = 2x\frac{\partial^3 z}{\partial x^3}$ $\quad O = 3$

Degree of PDE

Degree of PDE is degree of highest order derivatives in equation, provided it is free from radical and fractional power.

(i) $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = xy$ — $O = 1$, $D = 1$

(ii) $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^3 z}{\partial y^3} = 2x \frac{\partial z}{\partial x}$ — $O = 3$, $D = 1$

(iii) $\frac{\partial^2 z}{\partial x^2} = \left(1 + \frac{\partial z}{\partial y}\right)^{1/3}$ — $O = 2$, $D = 3$

(iv) $\frac{\partial^3 z}{\partial y^3} = \left(1 - \frac{\partial z}{\partial x}\right)^{1/2}$ — $O = 3$, $D = 2$

$\left(\frac{\partial^2 z}{\partial x^2}\right)^3 = \left(1 + \frac{\partial z}{\partial y}\right)^{3/3}$

$\left(\frac{\partial^2 z}{\partial x^2}\right)^3 = 1 + \frac{\partial z}{\partial y}$

$\left(\frac{\partial^3 z}{\partial y^3}\right)^2 = \left(1 - \frac{\partial z}{\partial x}\right)^{2/2}$

Linear PDE

A PDE is said to be linear, if dependent variable and its partial derivatives present only in first degree and not multiplied together.

Example

(A) $\boxed{\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = xyz}$ *Linear*

(B) $\boxed{yp + xq = x}$ *Linear*

$p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$

(C) $\boxed{\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = z + xy}$ *Linear*

$$\frac{\partial^2 u}{\partial x^2} + y\frac{\partial^2 u}{\partial y^2} + z\frac{\partial^2 u}{\partial z^2} = xyz$$

Non-Linear PDE

A PDE which is not linear is called non-linear PDE

Example

(1) $\left(\frac{\partial z}{\partial x}\right)^2 + \frac{\partial^3 z}{\partial y^3} = 2x\left(\frac{\partial z}{\partial x}\right)$ — **Non-Linear**

(2) $z\frac{\partial z}{\partial x} + \frac{\partial^2 z}{\partial y^2} = x$ — **Non-Linear**

(3) $u\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = x$ — **Non-Linear**

$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = xy + z^2$

Types of first order PDE

(i) Linear
(ii) Semi – Linear
(iii) Quasi – Linear

$p = \frac{\partial z}{\partial x}$

$q = \frac{\partial z}{\partial y}$

(1) Linear :

A first order PDE $f(x, y, z, p, q) = 0$ is known as linear if it is linear in p, q, and z and is of the form

$$P(x,y)\,p + Q(x,y)\,q = R(x,y)\,z + s(x,y)$$

Example

$$xp + yq = z + xy$$

$$y^2x^2\,p + x^3y^2\,q = xyz + x^3y^3$$

(2) Semi – Linear:

A first order PDE $f(x, y, z, p, q) = 0$ is known as semi-linear if it is linear in p, q, and is of the form

$$P(x, y)\,p + Q(x, y)q = R(x, y, z)$$

Example

$$x^2 y p + x^2 y^2 q = x^2 y^2 z^2$$

(3) Quasi – Linear :

A first order PDE $f(x, y, z, p, q) = 0$ is known as Quasi-linear if it is linear in p and q and is of the form

$$P(x, y, z)\, p + Q(x, y, z)\, q = R(x, y, z)$$

Example

$$x^2 z^2 p + y^2 x q = x y z$$

$$(x^2 - yz)\, p + (y^2 - zx)\, q = z^2 - x y$$

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Thank You