Engineering Mathematics

UNIT : Partial Differential Equation

Today's Target

- Non-Homogeneous Linear Partial Differential Equation with constant coefficient

Complementary Function (CF)

- PYQs
- DPP

Course Details(Paid) : All Subjects

1	Recorded Video Lectures (100 % Syllabus Coverage)
2	Pdf Notes
3	Lecture wise DPP with Answers
4	DPP Solutions
5	Unit wise set of PYQs with Answers
6	Course will be completed by 'Last week of Nov 2023'
	Tentative AKTU Semester Exam- Jan/Feb

Paid Courses are available in Gateway Classes Application
Link in Description

Non - Homogeneous Linear Partial Differential Equation with constant coefficient

(All Partial Derivatives are not of same order)

Given PDE

$$\phi(D,\ D') = F(x,\ y) \text{ --------①}$$

Methods to find CF

CASE-1:- When $\phi(D, D')$ resolve in to linear factors

$$(D - m_1 D' - a_1)(D - m_2 D' - a_2)(D - m_3 D' - a_3)z = F(x, y)$$

$$CF = e^{a_1 x} f_1(y + m_1 x) + e^{a_2 x} f_2(y + m_2 x) + e^{a_3 x} f_3(y + m_3 x)$$

NOTE : (i) CF corresponding to D = $f_1(y)$ (ii)CF corresponding to D' = $f_1(x)$

Q.1 *Solve the linear partial differential equation* $(D + D' - 1)(D + 2D' - 2)z = 0.$

$$(D + D' - 1)(D + 2D' - 2)z = 0$$

$$CF = e^{x} f_1(y - x) + e^{2x} f_2(y - 2x)$$

$$PI = 0$$

Complete solution

$$z = CF + PI$$

$$\boxed{z = e^{x} f_1(y - x) + f_2(y - 2x)}$$

Q.2 Solve: $DD'(D + 2D' + 1)z = 0$.

$$DD'(D+2D'+1)z = 0$$

CF corresponding to $D = f_1(y)$

CF " " $D' = f_2(x)$

CF " $(D+2D'+1) = e^{-x} f_3(y-2x)$

$$CF = f_1(y) + f_2(x) + e^{-x} f_3(y-2x)$$

$$PI = 0$$

Complete solution

$$z = CF + PI$$

Q.3 Solve : $r - t + p - q = 0$. (UPTU-2013,2018)

$$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} + \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y} = 0$$

$$(D^2 - D'^2 + D - D')z = 0$$

$$[(D - D')(D + D') + (D - D')]z = 0$$

$$(D - D')(D + D' + 1)z = 0$$

$$(D - D' + 0)(D + D' + 1)z = 0$$

$$CF = e^{0x} f_1(y + x) + e^{-x} f_2(y - x)$$

$$CF = f_1(y + x) + e^{-x} f_2(y - x)$$

$$PI = 0$$

complete solution

$$z = CF + PI$$

$$\boxed{z = f_1(y + x) + e^{-x} f_2(y - x)}$$

CASE-2:-

When $\phi(D,\ D')$ contains repeated factors

$$(D - m\,D' - a)^3\ \ z = F(x, y)$$

$$CF = e^{ax}\, f_1(y + mx) + xe^{ax}\, f_2(y + mx) + x^2 e^{ax}\, f_3(y + mx)$$

Q.4 Solve : $(D + 4D' + 5)^2 z = 0$. (AKTU-2018)

$$(D + 4D' + 5)^2 z = 0$$

$$CF = e^{-5x} f_1(y - 4x) + x e^{-5x} f_2(y - 4x)$$

$$PI = 0$$

Complete solution

$$z = CF + PI$$

$$\boxed{z = e^{-5x} f_1(y - 4x) + x f_2(y - 4x)}$$

Q.5 Solve: $r + 2s + t + 2p + 2q + z = 0.$

$$r + 2s + t + 2p + 2q + z = 0$$

$$\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} + 2\frac{\partial z}{\partial x} + 2\frac{\partial z}{\partial y} + z = 0$$

$$\left[(D^2 + 2DD' + D'^2) + 2D + 2D' + 1\right] z = 0$$

$$\left[(D + D')^2 + 2(D + D') + 1\right] z = 0$$

$$(D + D' + 1)^2 z = 0$$

$$CF = e^{-x} f_1(y - x) + x e^{-x} f_2(y - x)$$

$$PI = 0$$

complete solution

$$z = CF + PI$$

$$z = e^{-x} f_1(y - x) + x e^{-x} f_2(y - x)$$

CASE-3:-

When Function $\phi(D,\ D')$ con not be factorized into linear factor

Take a trial solution

$$z = Ae^{hx+ky}$$

Where A, h, k *are constants.*

Q. 6 **Solve:** $(D^2 + D'^2 - p^2)z = 0.$

$$(D^2 + D'^2 - p^2)z \quad \text{——} (1)$$

consider a trial solution

$$z = Ae^{hx+ky}$$

From eqn (1)

$$(D^2+D'^2-p^2)Ae^{hx+ky} = 0$$

$$A\left[D^2 e^{hx+ky} + D'^2 e^{hx+ky} - p^2 e^{hx+ky}\right] = 0$$

$$A\left[h^2 e^{hx+ky} + k^2 e^{hx+ky} - p^2 e^{hx+ky}\right] = 0$$

$$Ae^{hx+ky}(h^2+k^2-p^2) = 0$$

But $Ae^{hx+ky} \neq 0$

$\therefore h^2+k^2-p^2 = 0$

$$CF = \sum Ae^{hx+ky}$$

$$z = CF + PI$$

Where $h^2+k^2-p^2 = 0$

$PI = 0$

Engineering Mathematics by Gulshan Sir

PDE : DPP-12

Topic : Non-Homogeneous Linear Partial Differential Equation with constant coefficient

Complementary Function (CF)

Q .1 *Solve* $2s + t - 3q = 0$

Q .2 Solve $DD'(2D + D' + 11)^2 z = 0$

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Thank You

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