DPP-12 Mathr-IV Unit-2

Topic- Fourier Sine Transformation

Q-1:- Find Fourier sine transform of $\frac{1}{x}$.

Sol- By Fourier sine transform

$$f_s(p) = \int_0^\infty F(x) \sin px \, dx$$

$$f_s(p) = \int_0^\infty \frac{1}{x} \sin px \, dx$$

Let $\sin px = y$

$p\,dx = dy$

$$f_s(p) = \int_0^\infty \frac{1}{(y/p)} \times \sin y \times \frac{dy}{p}$$

$$f_s(p) = \int_0^\infty \frac{\sin y}{y}$$

We know $\int_0^\infty \frac{\sin y}{y} = \frac{\pi}{2}$

$$f_s(p) = \frac{\pi}{2}$$

Ans

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Q-2:- Find the Fourier transform of Block Function $f(t)$ of height 1 and duration a defined by

$$f(t) = \begin{cases} \sin x, & 0 < x < a \\ 0, & x > a \end{cases}$$

Sol:- By Fourier sine transform

$$f_s(p) = \int_0^a f(t) \sin px \, dx + \int_a^\infty f(t) \sin px \, dx$$

$$= \int_0^a \sin x \sin px \, dx + 0$$

$$= \frac{1}{2} \int_0^a 2 \sin x \sin px \, dx$$

$$2 \sin A \sin B = \cos(A-B) - \cos(A+B)$$

$$= \frac{1}{2} \int_0^a \left(\cos(p-1)x - \cos(p+1)x \right) dx$$

$$= \frac{1}{2} \left[\frac{\sin(p-1)x}{p-1} - \frac{\sin(p+1)x}{p+1} \right]_0^a$$

$$f_s(p) = \frac{1}{2} \left[\frac{\sin(p-1)a}{p-1} - \frac{\sin(p+1)a}{p+1} \right]$$

Ans:-

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Q-3 - Find $F_s^{-1}\left(\frac{e^{-ap}}{p}\right)$ and hence evaluate $F_s^{-1}\left(\frac{1}{p}\right)$

Sol - By Inverse Fourier Sine Transform

$$F(x) = \frac{2}{\pi}\int_0^\infty F_s(p)\sin px\,dp$$

$$F(x) = \frac{2}{\pi}\int_0^\infty \frac{e^{-ap}}{p}\sin px\,dp = I \text{ (say)}$$

$$I = \frac{2}{\pi}\int_0^\infty \frac{e^{-ap}}{p}\sin px\,dp$$

differentiate wrt x

$$\frac{dI}{dx} = \frac{2}{\pi}\int_0^\infty \frac{e^{-ap}}{p}\frac{d}{dx}\sin px\,dp$$

$$\frac{dI}{dx} = \frac{2}{\pi}\int_0^\infty \frac{e^{-ap}}{\cancel{p}}\times\cancel{p}\times\cos px\,dp$$

$$\frac{dI}{dx} = \frac{2}{\pi}\left[\frac{e^{-ap}}{(-a)^2+x^2}(-a\cos px + x\sin px)\right]_0^\infty$$

$$= \frac{2}{\pi}\left[\frac{e^{-\infty}}{a^2+x^2}(-a\cos\infty + x\sin\infty) - \frac{e^0}{a^2+x^2}(-a\cos 0+\sin 0\right.$$

($e^{-\infty}\to 0$)

$$\boxed{\frac{dI}{dx} = \frac{2}{\pi}\times\frac{a}{a^2+x^2}}$$

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$$dI = \frac{2}{\pi} \times \frac{a}{a^2+x^2}\, dx$$

Integrate both side

$$I = \frac{2}{\pi} \int \frac{a}{a^2+x^2}\, dx$$

$$I = \frac{2}{\pi} \tan^{-1}\left(\frac{x}{a}\right) + C = F(x)$$

For $x = 0$

$$\boxed{C = 0}$$

$$\boxed{F(x) = F_S^{-1}\left(\frac{e^{-ap}}{\rho}\right) = \frac{2}{\pi} \tan^{-1}\left(\frac{x}{a}\right)}$$

Put $a = 0$

$$F_S^{-1}\left(\frac{1}{\rho}\right) = \frac{2}{\pi} \tan^{-1}\left(\frac{x}{0}\right)$$

$$F_S^{-1}\left(\frac{1}{\rho}\right) = \frac{2}{\pi} \tan^{-1}(\infty)$$

$$F_S^{-1}\left(\frac{1}{\rho}\right) = \frac{2}{\pi} \times \frac{\pi}{2}$$

$$\boxed{F_S^{-1}\left(\frac{1}{\rho}\right) = 1}$$

Ans

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