ENGINEERING MATHEMATICS

UNIT-3 : *Statistical Techniques – I*

Lec-8

Today's Target

- **Important Questions based on Curve Fitting**
- PYQs
- DPP

1. Fitting a Straight Line $y = a + bx$

Normal Equations

$$\sum y = na + b\sum x$$

$$\sum xy = a\sum x + b\sum x^2$$

2. Fitting of second degree parabola $y = a + bx + cx^2$

Normal Equations

$$\sum y = na + b\sum x + c\sum x^2$$

$$\sum xy = a\sum x + b\sum x^2 + c\sum x^3$$

$$\sum x^2y = a\sum x^2 + b\sum x^3 + c\sum x^4$$

3. Fitting of an Exponential Curve $y = ae^{bx}$

$$Y = A + Bx$$

Normal Equations

$$\sum Y = nA + B\sum x$$

$$\sum xY = A\sum x + B\sum x^2$$

$$y = a e^{bx}$$

Taking log both side

$$\log_{10} y = \log_{10} a e^{bx}$$

$$\log_{10} y = \log_{10} a + \log_{10} e^{bx}$$

$$\log_{10} y = \log_{10} a + (b \log_{10} e)x$$

$$Y = A + Bx$$

$$\log mn = \log m + \log n$$

$$\log m^n = n \log m$$

4. Fitting of The Curve $y = ax + bx^2$

Normal Equations

$$\sum xy = a\sum x^2 + b\sum x^3$$

$$\sum x^2y = a\sum x^3 + b\sum x^4$$

Q. 1 :- By the method of least squares, find the curve $y = ax + bx^2$ that best fits the following data: **(UPTU – 2014)**

x	1	2	3	4	5
y	1.8	5.1	8.9	14.1	19.8

Equation of curve

$$y = ax + bx^2 \quad \text{—— (1)}$$

Normal Equations

$$\Sigma xy = a\Sigma x^2 + b\Sigma x^3 \quad \text{— (2)}$$

$$\Sigma x^2y = a\Sigma x^3 + b\Sigma x^4 \quad \text{— (3)}$$

x	y	x^2	x^3	x^4	xy	x^2y
1	1.8	1	1	1	1.8	1.8
2	5.1	4	8	16	10.2	20.4
3	8.9	9	27	81	26.7	80.1
4	14.1	16	64	256	56.4	225.6
5	19.8	25	125	625	99	495
		$\Sigma x^2 =$ 55	$\Sigma x^3 =$ 225	$\Sigma x^4 =$ 979	$\Sigma xy =$ 194.1	$\Sigma x^2y =$ 822.9

Put these valus in ② and ③

$$55a + 225b = 194.1 \quad -④$$

$$225a + 979b = 822.9 \quad -⑤$$

solve eqn ④ and ⑤

$$a = 1.51$$

$$b = 0.49$$

Put a and b in ①

$$y = an + bn^2$$

$$\boxed{y = 1.51n + 0.49n^2}$$

Required parabola

5. Fitting of The Curve $y = \frac{c_0}{x} + c_1\sqrt{x}$

Normal Equations

$$\Sigma \frac{y}{x} = c_0 \Sigma \frac{1}{x^2} + c_1 \Sigma \frac{1}{\sqrt{x}}$$

$$\Sigma y\sqrt{x} = c_0 \Sigma \frac{1}{\sqrt{x}} + c_1 \Sigma x$$

$$\frac{y}{n} = \frac{c_0}{n^2} + c_1 \frac{\sqrt{n}}{n}$$

$$\frac{y}{n} = \frac{c_0}{n^2} + \frac{c_1}{\sqrt{n}}$$

$$y\sqrt{n} = \frac{c_0\sqrt{n}}{n} + c_1\sqrt{n} \times \sqrt{n}$$

$$y\sqrt{n} = \frac{c_0}{\sqrt{n}} + c_1 n$$

Q. 2 :- ***Use the method of Least squares to fit the curve:*** $y = \frac{c_0}{x} + c_1\sqrt{x}$ ***to the following table of values:***

(AKTU – 2016 & 2018)

x:	0.1	0.2	0.4	0.5	1	2
y:	21	11	7	6	5	6

Equation of curve

$$y = \frac{c_0}{x} + c_1\sqrt{x} \quad \text{— (1)}$$

Normal Equations

$$\sum \frac{y}{x} = c_0 \sum \frac{1}{x^2} + c_1 \sum \frac{1}{\sqrt{x}} \quad \text{— (2)}$$

$$\sum y\sqrt{x} = c_0 \sum \frac{1}{\sqrt{x}} + c_1 \sum x \quad \text{— (3)}$$

Put these values in (2) an (3)

$$136.5\, c_0 + 10.10081\, c_1 = 302.5 \quad \text{— (4)}$$

$$10.100081\, c_0 + 4.2\, c_1 = 33.71524 \quad \text{— (5)}$$

Solve eqn (4) and (5)

$$\boxed{c_0 = 1.973267}$$

x	y	$\frac{y}{x}$	$y\sqrt{x}$	$\frac{1}{\sqrt{x}}$	$\frac{1}{x^2}$
0.1	21	210	6.64078	3.16228	100
0.2	11	55	4.91935	2.23607	25
0.4	7	17.5	4.42719	1.58114	6.25
0.5	6	12	4.24264	1.41421	4
1	5	5	5	1	1
2	6	3	8.48528	0.70711	0.25
$\Sigma x = 4.2$		$\Sigma y/x =$ 302.5	$\Sigma y\sqrt{x} =$ 33.71524	$\Sigma 1/\sqrt{x} =$ 10.10081	$\Sigma 1/x^2 =$ 136.5

$c_0 = 1.973267$

$c_1 = 3.281819$

Put c_0 and c_1 in (1)

$$y = \frac{1.9733}{x} + 3.2818\sqrt{x}$$

6. Fitting of The Curve $y = ax + \frac{b}{x}$

$$xy = ax^2 + b$$

Normal Equations

$$\sum xy = a\sum x^2 + nb$$

$$\sum \frac{y}{x} = na + b\sum \frac{1}{x^2}$$

$$\frac{y}{x} = a + \frac{b}{x^2}$$

Q. 3:- *Fit a relation* $y = ax + \frac{b}{x}$ *which satisfies the following data, using method of least squares:*

x	1	2	3	4	5	6	7	8
y	5.4	6.2	8.2	10.3	12.6	14.8	17.2	19.5

Equation of the curve

$$y = ax + \frac{b}{x} \quad \text{—①}$$

Normal Equations

$$\Sigma xy = a\Sigma x^2 + nb \quad \text{—②}$$

$$\Sigma \frac{y}{x} = na + b\Sigma \frac{1}{x^2} \quad \text{—③}$$

$$n = 8$$

Put these values in eqⁿ ② and ③

$$204a + 8b = 511.8 \quad \text{—④}$$

$$8a + 1.5274 = 23.705 \quad \text{—⑤}$$

Solve eqn ④ and ⑤

$$a = 2.39139$$

$$b = 2.99455$$

x	y	xy	x^2	y/x	$1/x^2$
1	5.4	5.4	1	5.4	1
2	6.2	12.4	4	3.1	0.25
3	8.2	24.6	9	2.74	0.1111
4	10.3	41.2	16	2.575	0.0625
5	12.6	63	25	2.52	0.04
6	14.8	88.8	36	2.47	0.0278
7	17.2	120.4	49	2.46	0.0204
8	19.5	156	64	2.44	0.0156
		$\Sigma xy =$ 511.8	$\Sigma x^2 =$ 204	$\Sigma y/x =$ 23.705	$\Sigma 1/x^2 =$ 1.5274

Put a and b in (1)

$$y = 2.39139x + \frac{2.99455}{x}$$

7. Fitting of The Curve $y = ab^x$

$$Y = A + Bx$$

Normal Equations

$$\sum Y = nA + B\sum x$$

$$\sum xY = A\sum x + B\sum x^2$$

$$y = ab^x$$

Taking log both side

$$\log_{10} y = \log_{10} ab^x$$

$$\log_{10} y = \log_{10} a + \log_{10} b^x$$

$$\log_{10} y = \log_{10} a + (\log_{10} b)x$$

$$\boxed{Y = A + Bx}$$

Q. 4:- Obtain a relation of the form $y = ab^x$ for the following data by the method of least squares: **(GBTU – 2019)**

x	2	3	4	5	6
y	8.3	15.4	33.1	65.2	127.4

Equation of curve

$$y = ab^x \quad \text{— (1)}$$

Taking log both side

$$\log_{10} y = \log_{10} ab^x$$

$$\log_{10} y = \log_{10} a + \log_{10} b^x$$

$$\log_{10} y = \log_{10} a + (\log_{10} b)x$$

Put

$$\log_{10} y = Y \quad \text{— (2)}$$

$$\log_{10} a = A \quad \text{— (3)}$$

$$\log_{10} b = B \quad \text{— (4)}$$

$$Y = A + Bx$$

Normal Equation

$$\Sigma Y = nA + B\Sigma x \quad \text{— (5)}$$

$$\Sigma xY = A\Sigma x + B\Sigma x^2 \quad \text{— (6)}$$

$n = 5$

x	y	$Y = \log_{10} y$	x^2	xY
2	8.3	0.9191	4	1.8382
3	15.4	1.1872	9	3.5616
4	33.1	1.5198	16	6.0792
5	65.2	1.8142	25	9.0710
6	127.4	2.1052	36	12.6312
$\Sigma x = 20$		$\Sigma Y =$ 7.5455	$\Sigma x^2 =$ 90	$\Sigma xY =$ 33.1812

Put these values in (5) and (6)

$5A + 20B = 7.5455$

$20A + 90B = 33.1812$

$A = 0.3094$

$B = 0.2999$

Put A in ③

$\log_{10} a = 0.3094$

$a = \text{Antilog}(0.3094)$

$\boxed{a = 2.0389}$

Put B in ④

$\log_{10} b = 0.2999$

$b = \text{Antilog}(0.2999)$

$\boxed{b = 1.9948}$

Put a and b in ①

$\boxed{y = 2.0389(1.9948)^{x}}$

8. Fitting of The Curve $pv^{\gamma} = c$

$$Y = A + BX$$

Normal Equations

$$\sum Y = nA + B\sum X$$

$$\sum XY = A\sum X + B\sum X^2$$

$$PV^{\gamma} = c$$

$$P = \frac{c}{V^{\gamma}}$$

$$P = cV^{-\gamma}$$

Taking log both side

$$\log P = \log cV^{-\gamma}$$

$$\log P = \log c + \log V^{-\gamma}$$

$$\log P = \log c - \gamma \log V$$

Put

$$\log P = Y$$

$$\log c = A$$

$$\log V = X$$

$$-\gamma = B$$

$$Y = A + BX$$

Q. 5:- ***The pressure of the gas correspomding to various volumes V is measured, given by the following data:***

$V\ (cm^3)$:	50	60	70	90	100
$P\ (kg/cm^2)$	64.7	51.3	40.5	25.9	78

Fit the data to the equation $\boxed{PV^\gamma = C}$

Equation of the curve

$$PV^\gamma = C \quad \text{—— (1)}$$

$$P = \frac{C}{V^\gamma}$$

$$P = CV^{-\gamma}$$

Taking log both side

$$\log_{10} P = \log_{10} CV^{-\gamma}$$

$$\log_{10} P = \log_{10} C - \gamma \log_{10} V$$

Put

$$\log_{10} P = Y \quad \text{— (2)}$$

$$\log_{10} C = A \quad \text{— (3)}$$

$$\log_{10} V = X \quad \text{— (4)}$$

$$-\gamma = B \quad \text{— (5)}$$

$$\boxed{Y = A + BX}$$

Normal Equations

$$\Sigma Y = nA + B\Sigma X \quad \text{— (6)}$$

$$\Sigma XY = A\Sigma X + B\Sigma X^2 \quad \text{— (7)}$$

$n = 5$

V	P	$X = \log_{10} V$	$Y = \log_{10} P$	XY	X^2
50	64.7	1.69897	1.81090	3.07666	2.886 50
60	51.3	1.77815	1.71012	3.04085	3.16182
70	40.5	1.84510	1.60746	2.96592	3.40439
90	25.9	1.95424	1.41330	2.76193	3.81905
100	78	2	1.89209	3.78418	4
		$\Sigma X =$ 9.27646	$\Sigma Y =$ 8.43387	$\Sigma XY =$ 15.62954	$\Sigma X^2 =$ 17.27176

Put these values in ⑥ and ⑦

$5A + 9.27646B = 8.43387$ ⑧

$9.27646A + 17.27176 =$

15.62954 ⑨

Solve ⑧ and ⑨

$A = 2.2248$

$B = -0.2899$

Put A in ③

$\log_{10} c = 2.2248$

$c = \text{Antilog}(2.2248)$

$\boxed{c = 167.8031}$

Put B in ⑤

$-\gamma = -0.2899$

$\boxed{\gamma = 0.2899}$

Put c and γ in ①

$\boxed{PV^{0.2899} = 167.8031}$

9. Fitting of The Curve $xy = b + ax$

$$y = a + \frac{b}{x}$$

Normal Equations

$$\sum y = na + b\sum \frac{1}{x}$$

$$\sum \frac{y}{x} = a\sum \frac{1}{x} + b\sum \frac{1}{x^2}$$

10. Fitting of The Curve $y = ax^2 + \frac{b}{x}$

$$x^2 y = a x^4 + b x$$

Normal Equations

$$\sum x^2 y = a \sum x^4 + b \sum x$$

$$\sum \frac{y}{x} = a \sum x + b \sum \frac{1}{x^2}$$

$$\frac{y}{x} = ax + \frac{b}{x^2}$$

11. Fitting of The Curve $y = a + \frac{b}{x} + \frac{c}{x^2}$

Normal Equations

$$\sum y = na + b\sum \frac{1}{x} + c\sum \frac{1}{x^2}$$

$$\sum \frac{y}{x} = a\sum \frac{1}{x} + b\sum \frac{1}{x^2} + c\sum \frac{1}{x^3}$$

$$\sum \frac{y}{x^2} = a\sum \frac{1}{x^2} + b\sum \frac{1}{x^3} + c\sum \frac{1}{x^4}$$

(12) $y = a x^b$

(13) $y = a + bx^2$

Q.1 *Given the following experimental values:*

x:	0	1	2	3
y:	2	4	10	15

Q.2 *Using method of least squares, derive the normal equations to fit the curve* $y = ax^2 + bx$ *. Hence fit this curve to the following data:*

x	1	2	3	4	5	6	7	8
y	1	1.2	1.8	2.5	3.6	4.7	6.6	9.1

Q.3 *Fit the curve* $pv^Y = k$ *to the following data:*

$p\ (kg/cm^2)$	0.5	1	1.5	2	2.5	3
$v\ (Liters)$	1620	1000	750	620	520	460

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Thank You