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DPP-11 Topic - Particular Integral (PI)
Case-II and case-III

Q-1: Solve $[D^2+(a+b)DD'+abD'^2]z = xy$

$[D^2+(a+b)DD'+abD'^2]z = xy$

Put $D = m$, $D' = 1$

$[m^2+(a+b)m+ab]z = xy$

Auxiliary Equation $\Rightarrow m^2+(a+b)m+ab = 0$

$m(m+b)+a(m+b) = 0$

$(m+a)(m+b) = 0$

$m = -a, -b$

$$\boxed{C.F. = f_1(y-ax)+f_2(y-bx)}$$

$$PI = \frac{1}{D^2+(a+b)DD'+abD'^2}xy$$

$$PI = \frac{1}{D^2\left[1+(a+b)\frac{D'}{D}+ab\frac{D'^2}{D^2}\right]} \times xy$$

$$PI = \frac{1}{D^2}\left[1+\left((a+b)\frac{D'}{D}+ab\frac{D'^2}{D^2}\right)\right]^{-1} \times xy$$

Using expansion

$$PI = \frac{1}{D^2}\left[1-\left((a+b)\frac{D'}{D}+ab\frac{D'^2}{D^2}\right)+\left((a+b)\frac{D'}{D}+ab\frac{D'^2}{D^2}\right)^2-\cdots\right]xy$$

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$$PI = \frac{1}{D^2}\left[1-(a+b)\frac{D'}{D}\right]xy$$

$$PI = \frac{1}{D^2}\left[xy-(a+b)\frac{D'}{D}(xy)\right]$$

$$PI = \frac{1}{D^2}\left[xy-(a+b)\frac{x}{D}\right]$$

$$PI = \frac{1}{D^2}\left[xy-(a+b)\frac{x^2}{2}\right]$$

$$PI = \frac{1}{D}\left[\int\left[xy-(a+b)\frac{x^2}{2}\right]dx\right]$$

$$PI = \frac{1}{D}\left[\int xy\,dx-\int(a+b)\frac{x^2}{2}dx\right]$$

$$PI = \frac{1}{D}\left[\frac{x^2y}{2}-\frac{(a+b)}{2}\times\frac{x^3}{3}\right]$$

$$PI = \int\frac{x^2y}{2}dx-\int\frac{(a+b)}{6}x^3\,dx$$

$$\boxed{PI = \frac{x^3y}{6}-\frac{(a+b)x^4}{24}}$$

Complete Solution $\Rightarrow$ $Z = CF + PI$

$$\boxed{Z = f_1(y-ax)+f_2(y-bx)+\frac{x^3y}{6}-\frac{(a+b)x^4}{24}}$$

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Q-2 : Solve : $(D^3+2D^2D'-DD'^2-2D'^3)z = (y+2)e^x$

$(D^3+2D^2D'-DD'^2-2D'^3)z = (y+2)e^x$

Put $D = m$, $D' = 1$

$(m^3+2m^2-m-2)z = (y+2)e^x$

Auxiliary Equation $\Rightarrow$ $m^3+2m^2-m-2 = 0$

$(m-1)(m+1)(m+2) = 0$

$m = 1, -1, -2$

$$\boxed{C.F. = f_1(y+x) + f_2(y-x) + f_3(y-2x)}$$

$$P.I = \frac{1}{D^3+2D^2D'-DD'^2-2D'^3}(y+2)e^x$$

$$P.I = \frac{1}{(D-D')(D+D')(D+2D')}(y+2)e^x$$

$$P.I = \frac{1}{(D-D')(D+2D')}\left[\frac{1}{D+D'}(y+2)e^x\right]$$

Put $y = c+x$

$$P.I = \frac{1}{(D-D')(D+2D')}\int \underset{①}{(c+x+2)}\underset{②}{e^x}\,dx$$

$$P.I = \frac{1}{(D-D')(D+2D')}\left[(c+x+2)e^x - 1 \times e^x\right]$$

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$$PI = \frac{1}{(D-D')(D+2D')}\left[(c+x+2-1)e^x\right]$$

$$PI = \frac{1}{(D-D')(D+2D')}\left[(c+x+1)e^x\right]$$

$$PI = \frac{1}{(D-D')(D+2D')}(y+1)e^x$$

$$PI = \frac{1}{D+2D'}\left[\frac{1}{D-D'}(y+1)e^x\right]$$

Put $y = c-x$

$$PI = \frac{1}{D+2D'}\int \underset{I}{(c-x+1)}\underset{II}{e^x}\,dx$$

$$PI = \frac{1}{D+2D'}\left[(c-x+1)e^x - (-1)e^x\right]$$

$$PI = \frac{1}{D+2D'}\left[(c-x+1+1)e^x\right]$$

$$PI = \frac{1}{D+2D'}(c-x+2)e^x$$

$$PI = \frac{1}{D+2D'}(y+2)e^x$$

Put $y = c+2x$

$$PI = \int \underset{①}{(c+2x+2)}\underset{②}{e^x}\,dx$$

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$PI = (C+2x+2)e^x - 2e^x$

$PI = (C+2x+2-2)e^x$

$PI = (C+2x)e^x$

$$\boxed{PI = ye^x}$$

Complete Solution $\Rightarrow$ $z = CF + PI$

$$\boxed{z = f_1(y+x) + f_2(y-x) + f_3(y-2x) + ye^x}$$

Q-3: Solve $(D^2+DD'-6D'^2)z = y\sin x$

$(D^2+DD'-6D'^2)z = y\sin x$

Put $D = m$ $D' = 1$

$(m^2+m-6)z = y\sin x$

Auxiliary Equation $\Rightarrow$ $m^2+m-6=0$

$m^2+3m-2m-6=0$

$m(m+3)-2(m+3)=0$

$(m-2)(m+3)=0$

$m = 2, -3$

$$\boxed{C.F. = f_1(y+2x) + f_2(y-3x)}$$

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$$PI = \frac{1}{(D+3D')(D-2D')} y \sin x$$

$$PI = \frac{1}{D+3D'} \left[ \frac{1}{D-2D'} y \sin x \right]$$

$$PI = \frac{1}{D+3D'} \int \underset{I}{(c-2x)} \underset{II}{\sin x} \, dx$$

$$PI = \frac{1}{D+3D'} \left[ (c-2x)(-\cos x) - \int (-2)(-\cos x) \right]$$

$$PI = \frac{1}{D+3D'} \left[ -(c-2x)\cos x - \int 2\cos x \right]$$

$$PI = \frac{1}{D+3D'} \left[ -(c-2x)\cos x - 2\sin x \right]$$

$$PI = \frac{1}{D+3D'} \left[ -y\cos x - 2\sin x \right]$$

$$PI = \int \left[ -(c+3x)\cos x - 2\sin x \right] dx$$

$$PI = -\int \underset{I}{(c+3x)} \underset{II}{\cos x} \, dx - 2\int \sin x \, dx$$

$$PI = -\left[ (c+3x)\sin x - \int 3\sin x \right] - 2(-\cos x)$$

$$PI = -\left[ (c+3x)\sin x - 3(-\cos x) \right] + 2\cos x$$

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$$PI = -\left[(c+3x)\sin x + 3\cos x\right] + 2\cos x$$

$$PI = -(c+3x)\sin x - 3\cos x + 2\cos x$$

$$PI = -(c+3x)\sin x - \cos x$$

$$\boxed{PI = -y\sin x - \cos x}$$

Complete solution $\Rightarrow$ $z = CF + PI$

$$\boxed{z = f_1(y+2x) + f_2(y-3x) - y\sin x - \cos x}$$

Ans

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