ENGINEERING MATHEMATICS

UNIT-4 : *Statistical Techniques – II*

Lec-07

Today's Target

- $\boldsymbol{Poisson\ \ Probability\ Distribution(Part-II)}$
- PYQs
- DPPs

Poisson Probability Distribution

- Poisson Distribution is a limiting case of binomial distribution

$$P(X = r) = \frac{\lambda^r e^{-\lambda}}{r!} \qquad (r = 0, 1, 2, 3, \dots)$$

- Recurrence formula for the Poisson Distribution

$$P(r+1) = \frac{\lambda}{(r+1)} P(r) \qquad (r = 0, 1, 2, 3, \dots.)$$

- Mean and Variance of the Poisson Distribution

$$\text{Mean} = \lambda$$

$$\text{Variance} = \lambda$$

$$\lambda = np$$

Q.1 *It is given that 2% of the electric bulbs manufactured by a company are defective. Using Poisson distribution, find the probability that a sample of 200 bulbs will contain*

(i) no defective bulb (ii) two defective bulbs (iii) at the most three defective bulbs. (**G.B.T.U. 2011**)

Prob. of Defective bulbs

$$P = \frac{2}{100}$$

$$P = 0.02$$

$$n = 200$$

$$\lambda = 0.02 \times 200$$

$$\lambda = 4$$

By poisson distribution

$$P(X=r) = \frac{e^{-\lambda}\lambda^{r}}{r!}$$

(i) Prob. of no defective bulb ($r=0$)

$$P(X=0) = \frac{e^{-4} \times (4)^{0}}{0!} = e^{-4}$$

$$P(X=0) = 0.018315$$

(ii) Prob. of two defective ($r=2$)

$$P(X=2) = \frac{e^{-4}(4)^{2}}{2!} = \frac{16}{2} e^{-4}$$

$$= 8 e^{-4}$$

$$P(X=2) = 8e^{-4}$$

$$P(X=2) = 0.146525$$

(iii) Prob. of atmost 3 defective bulbs ($x \leq 3$)

$$P(x \leq 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$$

$$= e^{-4} + \frac{e^{-4} \times 4}{1!} + 8e^{-4} + \frac{e^{-4}(4)^3}{3!}$$

$$= e^{-4} + 4e^{-4} + 8e^{-4} + \frac{32}{3}e^{-4}$$

$$P(x \leq 3) = e^{-4}\left(13 + \frac{32}{3}\right)$$

$$= e^{-4}\left(\frac{39+32}{3}\right)$$

$$= \frac{71}{3}e^{-4}$$

$$\boxed{P(x \leq 3) = 0.43347}$$

Q.2 *A manufacturer knows that the condensors he makes contain on an average 1% of defectives. He packes them in boxes of 100. What is the probability that a box picked at random will contain 4 or more faulty condensors?*

Prob. of defective condensors

$$P = \frac{1}{100}$$

$$\boxed{P = 0.01}$$

$$n = 100$$

$$\lambda = np = 100 \times 0.01$$

$$\boxed{\lambda = 1}$$

By poisson distribution

$$P(x = r) = \frac{e^{-\lambda} \lambda^{r}}{r!}$$

Prob. of 4 or more faulty condensor

$$(r \geq 4)$$

$$P(r \geq 4) = 1 - P(r < 4)$$

$$P(x \geq 4) = 1 - [P(x=0) + P(x=1) + P(x=2) + P(x=3)]$$

$$= 1 - \left[\frac{e^{-1}(1)^0}{0!} + \frac{e^{-1} \times (1)^1}{1!} + \frac{e^{-1} \times 1^2}{2!} + \frac{e^{-1} \times 1^3}{3!}\right]$$

$$= 1 - \left[e^{-1} + e^{-1} + \frac{1}{2}e^{-1} + \frac{1}{6}e^{-1}\right]$$

$$= 1 - e^{-1}\left(2e^{-1} + \frac{e^{-1}}{2} + \frac{e^{-1}}{6}\right)$$

$$= 1 - e^{-1}\left(\frac{12e^{-1} + 3e^{-1} + e^{-1}}{6}\right)$$

$$= \left(1 - \frac{8}{3}e^{-1}\right) = 0.018988$$

$$\boxed{P(x \geq 4) = 0.018988}$$

Q.3 *In a certain factory turning out razor blades, there is a small chance of* 0.002 *for any blade to be defective. The blades are supplied in packets of* 10*. Calculate the approximate number of packets containing no defective, one defective and two defective blades in a consignment of* $10,000$ *packets.* (*Given:* $e^{-0.02} = 0.9802$) (*A.K.T.U. 2018*)

Prob. of defective blade

$$P = 0.002$$

$$n = 10$$

$$N = 10,000$$

$$\lambda = nP = 0.002 \times 10$$

$$\boxed{\lambda = 0.02}$$

By Poisson Distribution

$$P(X = r) = \frac{e^{-\lambda} \lambda^r}{r!}$$

(i) Number of packets containing no defective blades

$$P(X = 0) = \frac{e^{-0.02} \times (0.02)^0}{0!} = e^{-0.02}$$

$P(X=0) = e^{-0.02}$

$P(X=0) = 0.9802$

Number of packets

$= 10,000 \times 0.9802$

$= 9802$

(ii) $P(X=1) = \dfrac{e^{-0.02} \times (0.02)^1}{1!}$

$P(X=1) = 0.0196$

Number of packets having 1 defective blades

$= 0.0196 \times 10000$

$= 196$

(iii) $P(X=2) = \dfrac{e^{-0.02} \times (0.02)^2}{2!}$

$= 0.000196$

Number of packets having 2 defective blades

$= 0.000196 \times 10,000 = 1.96$

$= 2$

Q.4 *The number of accidents in a year involving taxi drivers in a city follows a Poisson distribution with mean equal to 3. Out of 1000 taxi drivers, find approximately the numberof drivers with (i)no accidents (ii)more than 3 accidents in a year.* **(A.K.T.U. 2021)**

Given

$\lambda = 3$

$N = 1000$

By poisson Distribution

$$P(X=r) = \frac{e^{-\lambda}\,\lambda^{r}}{r!}$$

(i) $$P(X=0) = \frac{e^{-3}(3)^{0}}{0!} = e^{-3}$$

$$P(X=0) = e^{-3} = 0.04978$$

Numbn of drivers having no accidents

$$= 0.04978 \times 1000$$

$$= 49.7$$

$$= 50$$

(ii) $P(x>3) = 1-[P(x=0)+P(x=1)+P(x=2)+P(x=3)]$

$$= 1-\left[e^{-3}+\frac{e^{-3}\times(3)^1}{1!}+\frac{e^{-3}\times(3)^2}{2!}+\frac{e^{-3}\times(3)^3}{3!}\right]$$

$$= 1-\left[e^{-3}+3e^{-3}+\frac{9}{2}e^{-3}+\frac{9}{2}e^{-3}\right]$$

$$= 1-e^{-3}(4+4.5+4.5)$$

$$= 1-13e^{-3} = 0.3527$$

Number of drivers having more than 3 accidents

$$= 0.3527\times 1000 = 352.7$$

$$\boxed{= 353}$$

Q.5 *The following table gives the no. of days in a 50 day period during which automobile accidents occured in a city.*

No. of accidents :	0	1	2	3	4
No. of days (f) :	21	18	7	3	1

Fit a Poisson distribution to the data. (A.K.T.U. 2022)

x	f	fx	Theoritical frequencies
0	21	0	20
1	18	18	18
2	7	14	8
3	3	9	3
4	1	4	1
	$\Sigma f = 50$	$\Sigma fx = 45$	

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\lambda = \frac{45}{50}$$

$$\boxed{\lambda = 0.9}$$

$$N = 50$$

Poisson Distribution $= N \frac{e^{-\lambda} \lambda^r}{r!}$

$= \frac{50 \times e^{-0.9} (0.9)^r}{r!}$

When $r = 0$

$= 20.32 = 20$

When $r = 0$

$= 18.29 = 18$

When $r = 2$

$= 8.2 = 8$

When $r = 3$

$= 2.46$

$= 3$

When $r = 4$

$= 0.555$

$= 1$

DPP- 07

Topic : Poisson Probability Distribution (Part – II)

Q. 1 In a certain factory manufacturing razor blades, there is a small chance of 0.002 for any blade to be defective. The blades are supplied in packets of 10. Use suitable distribution to calculate the approximate number of packets containing no defective, one defective and two defective blades respectively in a consignment of 50,000 packets. (AKTU 2018)

Q. 2 The distribution of the number of road accidents per day in a city is Poisson with mean 4. Find the number of days out of 100 days when there will be (i) no accident (ii) at least 2 accidents (iii) at most 3 accidents (iv) between 2 and 5 accidents.

Q. 3 Fit a Poisson distribution to the following data and calculate theoretical frequencies.

Deaths:	0	1	2	3	4
Frequencies:	122	60	15	2	1

Q. 4 Fit a Poisson distribution to the following data given the number of yeast cells per square for 400 squares:

No. of cells per sq. :	0	1	2	3	4	5	6	7	8	9	10
No. of squares :	103	143	98	42	8	4	2	0	0	0	0

GATEWAY CLASSES

AKTU Full Courses (Paid)

Download Gateway Classes Application

From Google Play store

All Subjects

Link in Description

Thank You

GATEWAY CLASSES