ENGINEERING MATHEMATICS

UNIT-5 : *Statistical Techniques – III*

(Testing a Hypothesis and Statistical Quality Control)

Lec-03

Today's Target

- $t - test\ for\ two\ sample$
- PYQ
- DPP

t – test for two sample

- To test the significant difference between mean of two independent samples.
- Let two independent samples are sample – 1 and sample – 2

Sample – 1	Sample – 2
(i) $x_1, x_2, x_3 \dots \dots x_n$	(i) $y_1, y_2, y_3 \dots \dots y_n$
(ii) Mean $\bar{x} = \frac{\Sigma x}{n_1}$	(ii) Mean $\bar{y} = \frac{\Sigma y}{n_2}$
(iii) $S_1 = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n_1-1}}$	(iii) $S_2 = \sqrt{\frac{\Sigma(y-\bar{y})^2}{n_2-1}}$
(iv) Size $n_1 \leq 30$	(iv) Size $n_2 \leq 30$
(v) Population mean μ_1	(v) Population mean μ_2

Step to sovle Questions

STEP – 1: Set Null Hypothesis (H_0)

H_0: The means of the two Population are same

$H_0: \mu_1 = \mu_2$

STEP – 2: Alternative Hypothesis(H_1)

$H_1: \mu_1 \neq \mu_2$

STEP – 3: Calculate $|t|$

$$t = \frac{\bar{x} - \bar{y}}{s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

where $\bar{x}$ = Mean of Sample-1

$\bar{y}$ = Mean of Sample-2

n_1 = Size of Sample-1

n_1 = Size of sample-2

$S_1 = S.D\ of\ sample - 1$

$S_2 = S.D\ of\ sample - 2$

(i) If S.D S_1 and S_2 are given

$$S = \sqrt{\frac{n_1 S_1^2 + n_2 S_2^2}{n_1 + n_2 - 2}}$$

(ii) If S.D S_1 and S_2 are not given

$$S = \sqrt{\frac{\Sigma(x-\bar{x})^2 + \Sigma(y-\bar{y})^2}{n_1 + n_2 - 2}}$$

STEP – 4: Write degree of freedom $(n_1 + n_2 - 2)$ and level of significance $\propto$ $(\propto = 5\%\ OR \propto = 1\%)$

By Default

STEP – 5 : From the t – test table

Find the value of $t_\propto$ at level of significance for degree of freedom $(n_1 + n_2 - 2)$

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STEP – 6: Conclusion

(i) If $|t| < t_{\alpha}$ then H_0 is accepted

There is no significant difference between ~~between~~ their means

(ii) If $|t| > t_{\alpha}$ then H_0 is rejected

There is ~~no~~ significant difference between ~~between~~ their means

Q.1 *Samples of sizes 10 and 14 were taken from two normal populations with S.D. 3.5 and 5.2 The sample means were found to be 20.3 and 18.6. Test whether the means of the two populations are the same at 5% level.*

Given

$n_1 = 10$

$n_2 = 14$

$s_1 = 3.5$

$s_2 = 5.2$

$\bar{x} = 20.3$

$\bar{y} = 18.6$

We know that

$$s = \sqrt{\frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}}$$

$$s = \sqrt{\frac{10 \times (3.5)^2 + 14 \times (5.2)^2}{10 + 14 - 2}}$$

$$\boxed{s = 4.779}$$

Null Hypothesis (H_0)

H_0 : _____

$H_0 : \mu_1 = \mu_2$

Alternative Hypothesis (H_1)

$H_1 : \mu_1 \neq \mu_2$

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calculation of |t|

$$t = \frac{\bar{x} - \bar{y}}{s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$$= \frac{20.3 - 18.6}{4.772\sqrt{\frac{1}{10} + \frac{1}{14}}}$$

$$t = 0.8604$$

$$\boxed{|t| = 0.8604}$$

Degree of freedom $= n_1 + n_2 - 2$

$= 10 + 14 - 2$

$= 22$

Level of significanu (α)

$$\alpha = 5\% = \frac{5}{100} = 0.05$$

$$\alpha = 0.05$$

From t-test table

$$\boxed{t_{0.05} = 2.07}$$

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conclusion

$|t| = 0.8604$ and $t_{0.05} = 2.07$

$\therefore |t| < t_{0.05}$

$\therefore$ H_0 Hypothesis is accepted

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$t_{0.05} = 2.07$

Table 2 : SIGNIFICANT VALUES $t_\nu(\alpha)$ OF t-DISTRIBUTION (TWO TAIL AREAS) $[|t| > t_\nu(\alpha)] = \alpha$

d.f. (ν)	Probability (Level of Significance)					
	0.50	0.10	0.05	0.02	0.01	0.001
1	1.00	6.31	12.71	31.82	63.66	636.62
2	0.82	0.92	4.30	6.97	6.93	31.60
3	0.77	2.32	3.18	4.54	5.84	12.94
4	0.74	2.13	2.78	3.75	4.60	8.61
5	0.73	2.02	2.57	3.37	4.03	6.86
6	0.72	1.94	2.45	3.14	3.71	5.96
7	0.71	1.90	2.37	3.00	3.50	5.41
8	0.71	1.80	2.31	2.90	3.36	5.04
9	0.70	1.83	2.26	2.82	3.25	4.78
10	0.70	1.81	2.23	2.76	3.17	4.59
11	0.70	1.80	2.20	2.72	3.11	4.44
12	0.70	1.78	2.18	2.68	3.06	4.32
13	0.69	1.77	2.16	2.05	3.01	4.22
14	0.69	1.76	2.15	2.62	2.98	4.14
15	0.69	1.75	2.13	2.60	2.95	4.07
16	0.69	1.75	2.12	2.58	2.92	4.02
17	0.69	1.74	2.11	2.57	2.90	3.97
18	0.69	1.73	2.10	2.55	2.88	3.92
19	0.69	1.73	2.09	2.54	2.86	3.88
20	0.69	1.73	2.09	2.53	2.85	3.85
21	0.69	1.72	2.08	2.52	2.83	3.83
22	0.69	1.72	2.07	2.51	2.42	3.79
23	0.69	1.71	2.07	2.50	2.81	3.77
24	0.69	1.71	2.06	2.49	2.80	3.75
25	0.68	1.71	2.06	2.49	2.79	3.73
26	0.68	1.71	2.06	2.48	2.78	3.71
27	0.68	1.70	2.05	2.47	2.77	3.69
28	0.68	1.70	2.05	2.47	2.76	3.67
29	0.68	1.70	2.05	2.46	2.76	3.66
30	0.68	1.70	2.04	2.46	2.75	3.65
∞	0.67	1.65	1.96	2.33	2.58	3.29

Q.2 *The height of 6 randomly chosen sailors in inches are 63, 65, 68, 69, 71 and 72. Those of 9 randomly chosen soldiers are 61, 62, 65, 66, 69, 70, 71, 72 and 73. Test whether the sailors are on the average taller than soldiers.*

Given

Sample-1 → Height of Sailors

Sample-2 → Height of Soldiers

$n_1 = 6$

$n_2 = 9$

x	$x-\bar{x}$	$(x-\bar{x})^2$	y	$y-\bar{y}$	$(y-\bar{y})^2$
63	−5	25	61	−6.67	44.4889
65	−3	9	62	−5.67	32.1489
68	0	0	65	−2.67	7.1289
69	1	1	66	−1.67	2.7889
71	3	9	69	1.33	1.7689
72	4	16	70	2.33	5.4289
			71	3.33	11.0889
			72	4.33	18.7489
			73	5.33	28.4089
$\Sigma x =$ 408		$\Sigma(x-\bar{x})^2 =$ 60	$\Sigma y =$ 609		$\Sigma(y-\bar{y})^2 =$ 151.9995

Mean

$$\bar{x} = \frac{\Sigma x}{n_1} = \frac{408}{6}$$

$$\boxed{\bar{x} = 68}$$

$$\bar{y} = \frac{\Sigma y}{n_2}$$

$$\bar{y} = \frac{609}{9}$$

$$\boxed{\bar{y} = 67.67}$$

We know that

$$s = \sqrt{\frac{\Sigma(x-\bar{x})^2 + \Sigma(y-\bar{y})^2}{n_1 + n_2 - 2}}$$

$$s = \sqrt{\frac{60 + 151.9995}{6 + 9 - 2}}$$

$$\boxed{s = 4.038}$$

Null Hypothesis (H_0)

H_0 : _____

$H_0 : \mu_1 = \mu_2$

Calculation of $|t|$

$$t = \frac{\bar{x} - \bar{y}}{s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$$t = \frac{68 - 67.67}{4.038\sqrt{\frac{1}{6} + \frac{1}{9}}}$$

$$\boxed{|t| = 0.150\overline{5}}$$

Degree of freedom $= n_1 + n_2 - 2$

$= 6 + 9 - 2$

$= (13)$

Level of significance α

$\alpha = 5\% = \frac{5}{100} = 0.05$

$\alpha = 0.05$

From the t-test table

$$t_{0.05} = 2.16$$

conclusion

$|t| = 0.1505$ and $t_{0.05} = 2.16$

$\because |t| < t_{0.05}$

$\therefore$ H_0 Hypothesis is accepted

Means of the two population are same

Hence,

Average Height of Sailors is not taller than soldiers

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$t_{0.05} = 2.16$

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Q.1 **Two samples of sodium vapour bulbs were tested for length of life and the following results were got:**

	Size	Sample Mean	Sample S.D.
Type I	8	1234 hrs	36 hrs
Type II	7	1036 hrs	40 hrs

Is the difference in the means significant to generalise that Type I is superior to Type II regarding length of life?

Q.2 **The marks obtained by a group of 9 regular course students and another group of 11 part time course students in a test are given below:**

Regular:	56	62	63	54	60	51	67	69	58		
Part Time:	62	70	71	62	60	56	75	64	72	68	66

Examine whether the marks obtained by regular students and part time students differ significantly at 5% and 1% level of significance.

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