ENGINEERING MATHEMATICS

UNIT-4 : *Statistical Techniques – II*

Lec-09

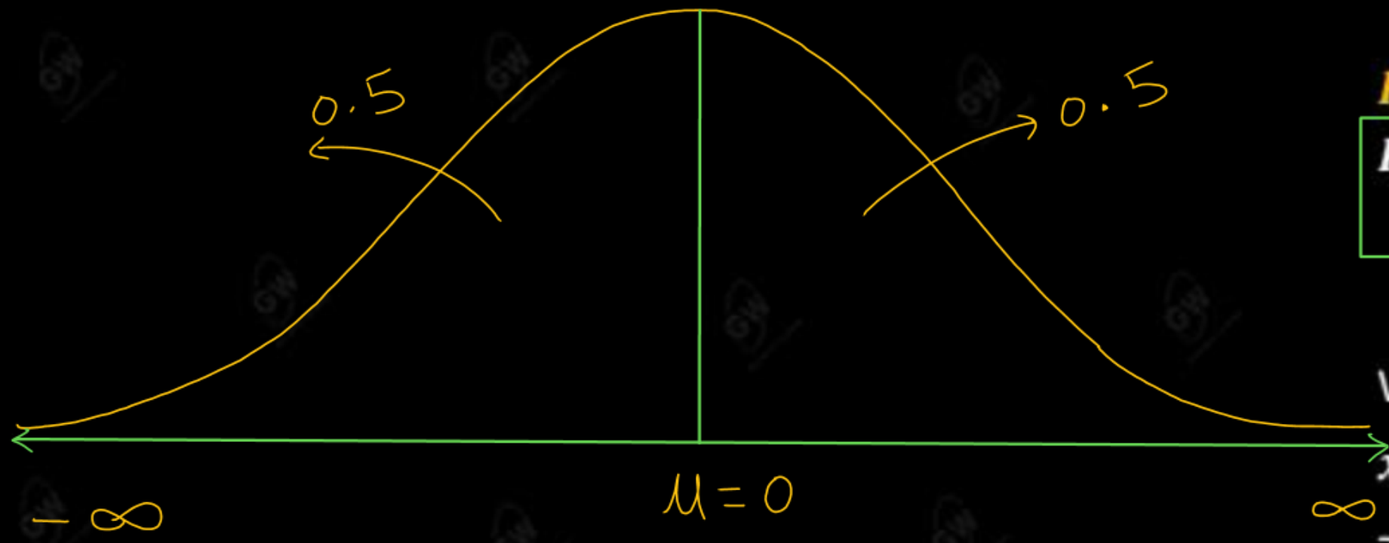
Today's Target

- $\boldsymbol{Normal\ Probability\ Distribution(Part-II)}$
- PYQs
- DPPs

Standard Normal Curve

$-\infty < z < 0$

$0 < z < \infty$



$-\infty < z < \infty$

$$z = \frac{x - \mu}{\sigma},$$

Total Area under curve = 1

From Normal table

Probability

$P(z_1 < z < z_2)$ = Area under standard normal Curve between $z = z_1$ and $z = z_2$

Where

x = Normal random variable

Z = Standard normal random variable

μ = Mean

σ = Standard deviation

Q.1 *A sample of 100 dry battery cells tested to find the length of life produced the following results:* $\bar{x} = 12$ *hours,* $\sigma = 3$ *hours. Assuming the data to be normally distributed, what % of battery cells are expected to have life* (i) *more than 15 hours* (ii) *less than 6 hours* (iii) *between 10 and 14 hours?* (*A.K.T.U. 2018, 2021*)

Given

$N = 100$

$\mu = 12$

$\sigma = 3$

$$z = \frac{x - \mu}{\sigma}$$

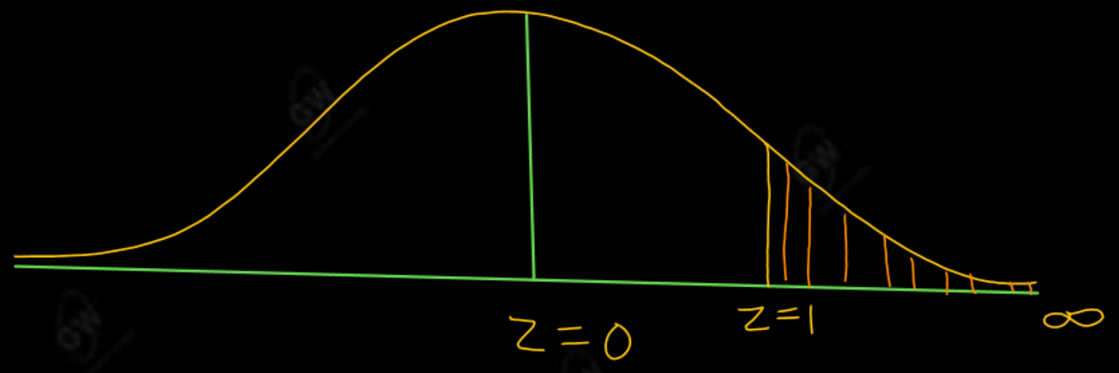
$$z = \frac{x - 12}{3} \quad \text{---} (1)$$

(i) $x = 15$

$$z = \frac{15 - 12}{3} = \frac{3}{3}$$

$z = 1$

$P(x > 15) = P(z > 1)$



$P(x > 15) = P(0 < z < \infty) - P(0 < z < 1)$

$= 0.5 - 0.3413$

$= 0.1587$

$P(x>15) = 0.1587 \times 100$

$= 15.87\%$

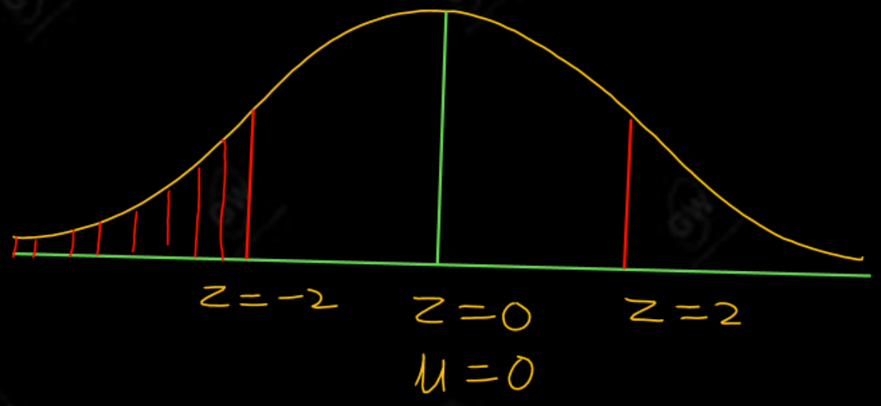
(iii) $x = 6$

$$Z = \frac{x-12}{3} = \frac{6-12}{3}$$

$Z = -2$

$P(x<6) = P(Z<-2)$

$= P(Z>2)$



$P(x<6) = P(0<z<\infty) - P(0<z<2)$

$= 0.5 - 0.4772$

$= 0.0228$

$P(x<6) = 0.0228 \times 100 = 2.28\%$

$P(x<6) = 2.28\%$

(iii) $x_1 = 10$

$x_2 = 14$

$$z_1 = \frac{x_1 - 12}{3}$$

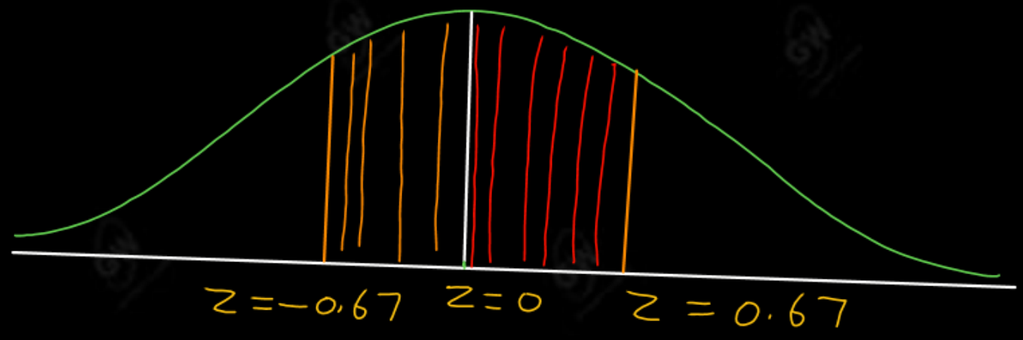
$$z_1 = \frac{10 - 12}{3}$$

$$z_1 = -0.67$$

$$z_2 = \frac{x_2 - 12}{3}$$

$$z_2 = \frac{14 - 12}{3}$$

$$z_2 = 0.67$$



$$P(10 < x < 14) = P(-0.67 < z < 0.67)$$

$$= 2P(0 < z < 0.67)$$

$$= 2 \times 0.2485 = 0.497$$

$$P(10 < x < 14) = 49.7\%$$

GW
GATEWAY CLASSES

Standard Normal C

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0754
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2485	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4255	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4930	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4999	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993

GW
GATEWAY CLASSES

Q. 2 *In a sample of 1000 cases, the mean of a certain test is 14 and S.D. is 2.5.* **(A.K.T.U. 2020)**

Assuming the distribution to be normal, find

(i) how many students score between 12 and 15?

(ii) how many score above 18?

(iii) how many score below 8?

(iv) how many score 16?

Given

$\mu = 14$

$\sigma = 2.5$

$N = 1000$

$z = \frac{x - \mu}{\sigma}$

$z = \frac{x - 14}{2.5}$ ——— (1)

(i) $x_1 = 12$, $x_2 = 15$

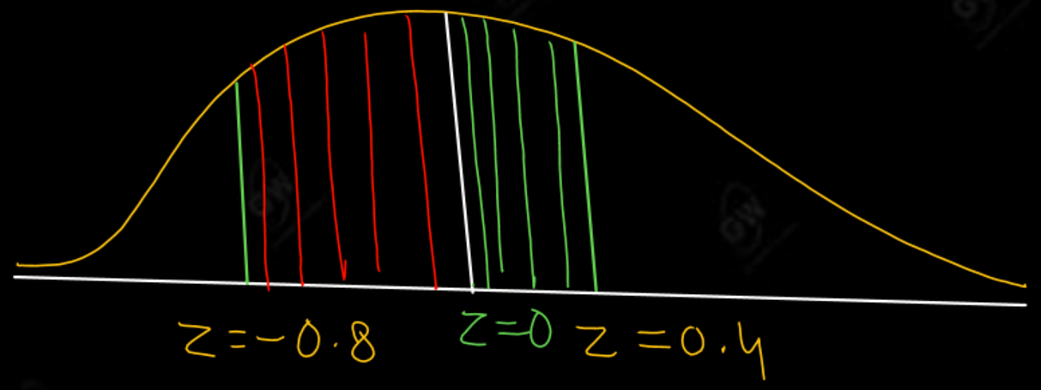
$z_1 = \frac{x_1 - 14}{2.5} = \frac{12 - 14}{2.5}$

$z_1 = -0.8$

$z_2 = \frac{x_2 - 14}{2.5} = \frac{15 - 14}{2.5}$

$z_2 = 0.4$

$P(12 < x < 15) = P(-0.8 < z < 0.4)$



$$P(12<x<15) = P(0<z<0.4) + P(0<z<0.8)$$

$$= 0.1554 + 0.2881$$

$$= 0.4435$$

Number of students $= 1000 \times 0.4435$

$= 443.5$

$= 444$

(ii) $x = 18$

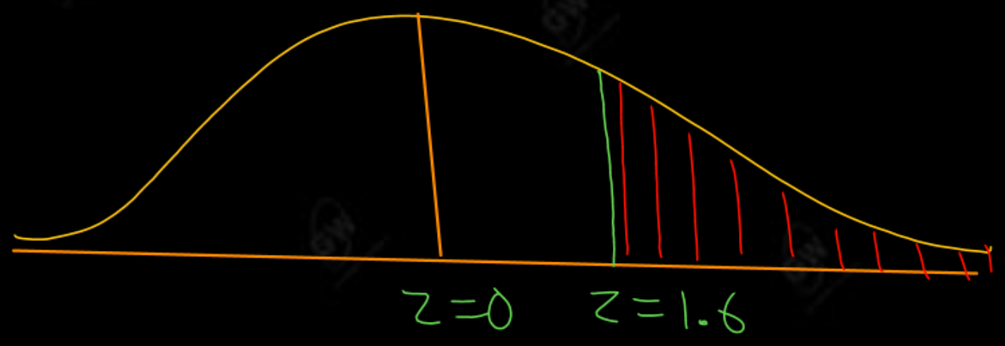
$$z = \frac{x-14}{2.5} = \frac{18-14}{2.5} = \frac{4}{2.5}$$

$z = 1.6$

$$P(x>18) = P(z>1.6)$$

$$= P(0<z<\infty) - P(0<z<1.6)$$

$$= 0.5 - 0.4452$$



$P(x > 18) = 0.0548$

Number of students $= 1000 \times 0.0548$

$= 54.8$

$\boxed{= 55}$

(iii) $x = 8$

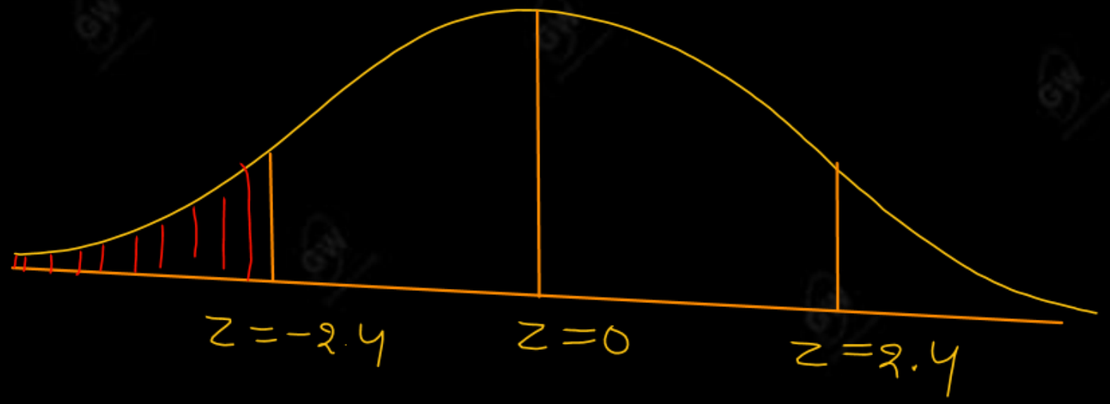
$$z = \frac{x - 14}{2.5}$$

$$z = \frac{8 - 14}{2.5}$$

$z = -2.4$

$P(x < \infty) = P(z < -2.4)$

$= P(z > 2.4)$



$P(x < 8) = P(0 < z < \infty) - P(0 < z < 2.4)$

$= 0.5 - 0.4918$

$= 0.0082$

Number of students $= 0.0082 \times 1000$

$= 8.2$

$= 8$

(IV) $x = 16$

$x_1 = 15.5, \quad x_2 = 16.5$

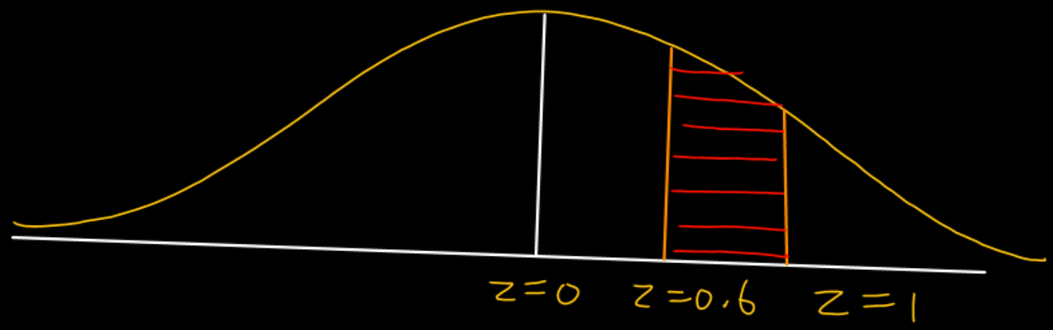
$$z_1 = \frac{x_1 - 14}{2.5} = \frac{15.5 - 14}{2.5}$$

$$z_1 = 0.6$$

$$z_2 = \frac{x_2 - 14}{2.5} = \frac{16.5 - 14}{2.5}$$

$z_2 = 1$

$P(15.5 < x < 16.5) = P(0.6 < z < 1)$



$P(15.5 < x < 16.5) = P(0 < z < 1) - P(0 < z < 0.6)$

$= 0.3413 - 0.2257$

$= 0.1156$

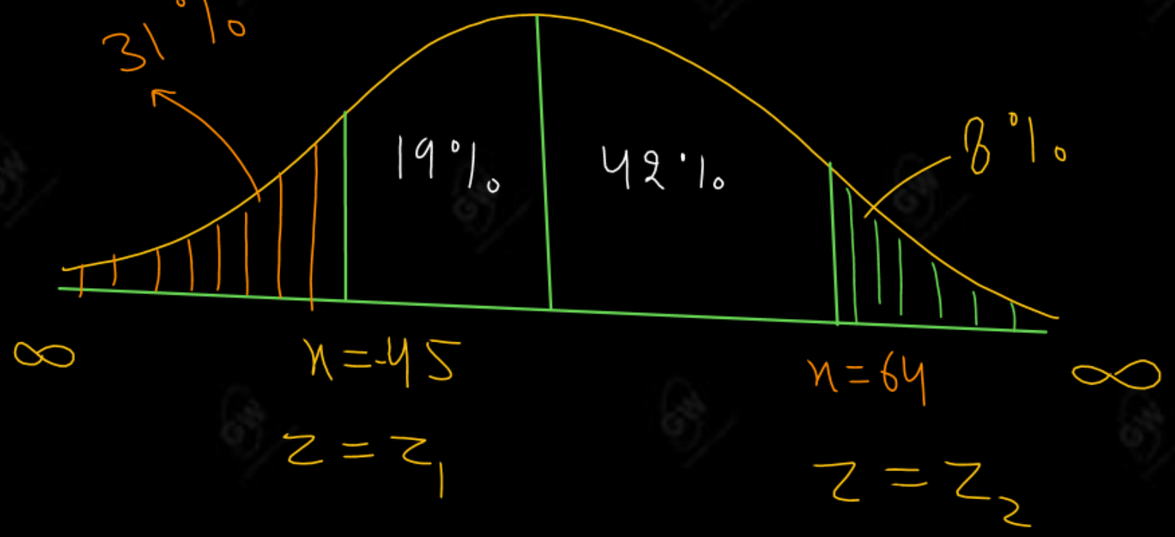
Number of students $= 0.1156 \times 1000$

$= 116$

Q.3 *In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and standard deviation of the distribution. It is given that that if*

$f(t) = \frac{1}{\sqrt{2\pi}} \int_0^t e^{-\frac{1}{2}x^2} dx$ then f(0.5) = 0.19 and f(1.4) = 0.42. **(A.K.T.U. 2020)**

Let mean = μ, Standard deviation = σ



When $x_1 = -45$, $P(-45 < x < 0) = \frac{19}{100}$

$= 0.19$

$z_1 = -0.5$

When $x_2 = 64$, $P(0 < x < 64)$

$= \frac{42}{100} = 0.42$

$z_2 = 1.4$

$$z = \frac{x - \mu}{\sigma}$$

$$z_1 = \frac{x_1 - \mu}{\sigma}$$

$$-0.5 = \frac{45 - \mu}{\sigma}$$

$$-0.5\sigma = 45 - \mu$$

$$-0.5\sigma + \mu = 45 \quad \text{— (1)}$$

$$z_2 = \frac{x_2 - \mu}{\sigma}$$

$$1.4 = \frac{64 - \mu}{\sigma}$$

$$1.4\sigma = 64 - \mu$$

$$1.4\sigma + \mu = 64 \quad \text{— (2)}$$

Subtract (1) and (2)

$$-0.5\sigma + \mu = 45$$

$$\underline{1.4\sigma + \mu = 64}$$

$$-1.9\sigma = -19$$

$$\sigma = 10$$

$$\mu = 50$$

GW
GATEWAY CLASSES

GW
GATEWAY CLASSES

Standard Normal C

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0754
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2485	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4255	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
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1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4930	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4999	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993

DPP- 09

Topic : *Normal Probability Distribution (Part – II)*

Q. 1 In a test on 2000 electric bulbs, it was found that the life of a particular make, was normally distributed with an average life of 2040 hours and S. D. of 60 hours, estimate the number of bulbs likely to burn for

(A. K. T. U. 2021)

(i) more than 2150 hours
(it) less than 1950 hours
(iii) more than 1920 hours but less than 2160 hours.

Ans. 1 (*i*) 67 (*ii*) 134 (*iii*) 1909

Q. 2 The marks X obtained in Mathematics by 1000 students are normally distributed with meas 78% and standard deviation 11% Determine:

Ans. 2(*i*) 138 (*ii*) 63.92% (*iii*) *Between* 60 *and* 96

(i) how many students got marks above 90%?
(ii) What was the highest marks obtained by the lowest 10% of students?
(iii) Within what limits did the middle 90% of the students lie?

Q. 3 In a normal distribution exactly normal, 7% of the items are under 35 and 89% areunder 63.

What are the mean and standard deviation of the distribution?

Ans. 3 50.3 *and* 10.33

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