ENGINEERING MATHEMATICS

UNIT-4 : *Statistical Techniques – II*

Lec-01

Today's Target

- ***Random variables(PART-1)***
- PYQ
- DPP

UNIT – 4 : Statistical Technique – II (Probability)

Aktu Syllabus

- **Random variables**

(i) **Discrete random variables**

- **Probability Mass Function**
- **Expectation and Variance**

(ii) **Continous random variables**

- **Probability Density Function**
- **Expectation and Variance**

- **Probability Distribution**

(i) **Binomial** → **Based on Discrete random variable**

(ii) **Poisson** → **Based on Discrete random variable**

(iii) **Normal** → **Based on Continous random variable**

Probability

Probability is a concept which numerically meansures the degree of uncertainity and certanity of the occurrence of events.

$0 \leq P \leq 1$

$$\text{Probability} = \frac{\text{Favourable Cases}}{\text{Total number of mutually exclusive and equally likely cases}}$$

Random variable

It is a real valued function whose domain is the sample space of a random experiment

It is denoted by X

$$X = \text{Number of things}$$

$$X : S \longrightarrow R$$

S: Domain, R: codomain

$X(s) = x$

Where x is any real number

Types of Random variable

(1) Discrete random variable

(2) Continous random variable

Discrete random variable

A random variable which can assume only a finite number of values is called a discrete random variable.

Example: –The number of tails in 3 tosses of a coin is a discrete random variable (X)

$$S = \{HHH, HHT, HTH, THH, TTT, TTH, THT, HTT\}$$

X = Number of Tails

$X(HHH) = 0$

$X(HHT, HTH, THH) = 1$

$X(TTH, THT, HTT) = 2$

$X(TTT) = 3$

Hence

$X = 0, 1, 2, 3$

Probabilities

$$P(X=0) = \frac{1}{8}$$

$$P(X=1) = \frac{3}{8}$$

$$P(X=2) = \frac{3}{8}$$

$$P(X=3) = \frac{1}{8}$$

$$P(X=0) + P(X=1) + P(X=2) + P(X=3) =$$

$$\frac{1}{8} + \frac{3}{8} + \frac{3}{8} + \frac{1}{8}$$

$$= \frac{1+3+3+1}{8}$$

$$= \frac{8}{8} = 1$$

Probability Distribution

X	0	1	2	3
P(X)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

Prabability Mass Function

Let $x_1, x_2, x_3, \ldots\ldots$ be the values of a discrete random variable X and let $p_1, p_2, p_3, \ldots\ldots$ be the corresponding probabilities.

A function $p(x)$ defined by

$$p(X = x_i) = p(x) = \begin{cases} p(x_i) \text{ or } p_i & i = 1, 2, 3, 4 \ldots\ldots \\ 0, & \text{otherwise} \end{cases}$$

is called the probability mass function of the discrete random variable X where

(i) $p(x_i) \geq 0$

(ii) $\sum p(x_i) = 1$

V. Imp

Probability distribution of a discrete random variable

X	x_1	x_2	x_3	……	x_n
$p(x)$	p_1	p_2	p_3	……	p_n

2/3/4….Balls (withdrawn)

- **Successively**
  - **With replacement (Independent events)**
  - **Without Replacement (Dependent events)**
- **Simultaneously** (Dependent)

$$nC_r = \frac{n!}{r!(n-r)!}$$

Q. 1 *A coin is tossed three times. if X is a random variable giving the number of ~~tails~~ Heads that appear, make a table showing the probability distribution of X.*

$S = \{HHH, HHT, HTH, THH, TTT, TTH, THT, HTT\}$

X = Number of Heads

$X(TTT) = 0$

$X(TTH, THT, HTT) = 1$

$X(HHT, HTH, THH) = 2$

$X(HHH) = 3$

Hence

$X = 0, 1, 2, 3$

Probabilities corresponding to random variables

$P(X=0) = \frac{1}{8}$

$$P(X=1) = \frac{3}{8}$$

$$P(X=2) = \frac{3}{8}$$

$$P(X=3) = \frac{1}{8}$$

X	0	1	2	3
P(X)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

Q. 2 *A random variable X has the following probability distribution:*

Value of X, x:	0	1	2	3	4	5	6	7
$p(x)$:	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

Find (i) k (ii) $P(X<6)$ (iii) $P(X\geq 6)$ (iv) $P(3<X\leq 6)$

(v) Minimum value of n for which $P(X\leq n)>\frac{1}{2}$

Solution

(i) We know that

$$\sum P(x_i)=1$$

$$0+k+2k+2k+3k+k^2+2k^2+7k^2+k=1$$

$$9k+10k^2=1$$

$$10k^2 + 9k - 1 = 0$$

$$10k^2 + 10k - k - 1 = 0$$

$$10k(k+1) - (k+1) = 0$$

$$(k+1)(10k-1) = 0$$

$k = -1$ (Neglect)

$$\boxed{k = \frac{1}{10}}$$

$$P(X<6) = P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4) + P(X=5)$$

$$= 0 + k + 2k + 2k + 3k + k^2$$

$$= k^2 + 8k$$

$$= \left(\frac{1}{10}\right)^2 + 8 \times \frac{1}{10} = \frac{1}{100} + \frac{8}{10}$$

$$\boxed{P(X<6) = \frac{81}{100}}$$

(iii) $P(X \geq 6) = P(X=6) + P(X=7)$

$$= 2k^2 + 7k^2 + k$$

$$P(X \geq 6) = 9k^2 + k$$

$$= \frac{9}{100} + \frac{1}{10}$$

$$= \frac{9 + 10}{100}$$

$$\boxed{P(X \geq 6) = \frac{19}{100}}$$

(iv) $P(3 < X \leq 6) =$

$= P(X=4) + P(X=5) + P(X=6)$

$= 3k + k^2 + 2k^2$

$$P(3 < X \leq 6) = 3k^2 + 3k = \frac{3}{100} + \frac{3}{10}$$

$$\boxed{P(3 < X \leq 6) = \frac{33}{100}}$$

(v) $P(X \leq 1) = 0 + k$

$P(X \leq 1) = k$

$P(X \leq 1) = \frac{1}{10} < \frac{1}{2}$

$P(X \leq 3) = 0 + k + 2k + 2k$

$P(X \leq 3) = 5k$

$P(X \leq 3) = \frac{5}{10} = \frac{1}{2}$

$P(X \leq 4) = 0 + k + 2k + 2k + 3k$

$P(X \leq 4) = 8k$

$P(X \leq 4) = \frac{8}{10} > \frac{1}{2}$

Hence Minimum value of n is 4

Q.3 *Five defective bulbs are accidentally mixed with twenty good ones. It is not possible to just look at a bulb and tell wheter or not it is defective. Find the probability distribution of the number of defective bulbs, if four bulbs are drawn at random from this lot.*

Given

Number of Defective bulbs = 5

Number of good bulbs = 20

Total number of bulbs = 25

X = Number of Defective bulbs

X = 0, 1, 2, 3, 4

Probabilities

$$P(X=0) = \frac{{}^{20}C_4}{{}^{25}C_4}$$

5 D
20 G

$$P(X=0) = \frac{20!}{4! \times 16!} \times \frac{4! \times 21!}{25!}$$

$$= \frac{20 \times 19 \times 18 \times 17 \times 16! \times 21!}{16! \times 25 \times 24 \times 23 \times 22 \times 21!}$$

5 4 2 11

$$P(X=0) = \frac{19 \times 3 \times 17}{5 \times 2 \times 11 \times 23}$$

$$P(X=0) = \frac{969}{2530}$$

$$P(X=1) = \frac{{}^{5}C_{1} \times {}^{20}C_{3}}{{}^{25}C_{4}}$$

$$P(X=1) = \frac{1140}{2530}$$

$$P(X=2) = \frac{{}^{5}C_{2} \times {}^{20}C_{2}}{{}^{25}C_{4}}$$

$$P(X=2) = \frac{380}{2530}$$

$$P(X=3) = \frac{{}^{5}C_{3} \times {}^{20}C_{1}}{{}^{25}C_{4}}$$

$$P(X=3) = \frac{40}{2530}$$

$$P(X=4) = \frac{{}^{5}C_{4}}{{}^{25}C_{4}}$$

$$P(X=4) = \frac{1}{2530}$$

X	0	1	2	3	4
P(X)	$\frac{969}{2530}$	$\frac{1140}{2530}$	$\frac{380}{2530}$	$\frac{40}{2530}$	$\frac{1}{2530}$

Q.4 *A random variable X takes values 1, 2, 3, ... with probability mass function* $\frac{\lambda^r}{r!}$, $r = 1, 2, 3 \ldots \infty$. *Find the value of* λ.

Given

$$P(x_i) = \frac{\lambda^x}{x!}$$

$$\sum P(x_i) = 1$$

$$\sum \frac{\lambda^x}{x!} = 1$$

$$\frac{\lambda}{1!} + \frac{\lambda^2}{2!} + \frac{\lambda^3}{3!} + \cdots\cdots = 1$$

$$1 + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} + \frac{\lambda^3}{3!} + \cdots = 1 + 1$$

$$e^{\lambda} = 2$$

Taking log both side

$$\lambda \log_e e = \log_e 2$$

$$\boxed{\lambda = \log_e 2}$$

Topic : *Random variables (PART-1)*

Q.1 *A box has 5 Blue and 3 Red balls. If 2 balls are to be drawn at random without replacement and X denotes the number of Blue balls, find the probability distribution for X.*

Q.2 *A random variable X has the following probability distribution:*

Value of X, x:	0	1	2	3	4	5	6	7	8
$p(x)$:	a	$3a$	$5a$	$7a$	$9a$	$11a$	$13a$	$15a$	$17a$

(*i*) *Determine the value of* a.

(*ii*) *Find* $P(X < 3), P(X \geq 3), P(2 \leq X < 5)$

(*iii*) *What is the smallest value of* x *for which* $P(X \leq x) > 0.5$?

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