ENGINEERING MATHEMATICS

UNIT-4 : *Statistical Techniques – II*

Lec-03

Today's Target

- ***Random variables (PART-3)***
- PYQ
- DPP

Types of Random variable

(1) Discrete random variable

(2) Continous random variable

Discrete random variable

A random variable which can assume only a finite number of values is called a discrete random variable.

- Probability Mass Function
- Probability Distribution
- Expectation and Variance

Continous random variable

- A random variable which can assume any real value within an interval



- The interval may be finite or infinite

Probability Density Function

Let X be a continous random variable, then in the interval $[a, b]$, the PDF is defined by

$$P(a \leq X \leq b) = \int_a^b f(x)\,dx$$

Such that

$$(i)\quad f(x) \geq 0, \qquad \forall\, x$$

$$(ii)\quad \int_{-\infty}^{\infty} f(x)\,dx = 1$$

Note: $P((a \le X \le b) = f(x) or\ p(x) = \begin{cases} 0 & x < a \\ \emptyset(x) & a \le x \le b \\ 0 & x > b \end{cases}$

$$e^{-\infty} = \frac{1}{e^{\infty}} = \frac{1}{\infty} = 0$$

Q.1 *If the function $f(x)$ is defined by $f(x) = ce^{-x}, 0 \le x \le \infty$, find the value of c which changes $f(x)$ to a probability density function.*

$f(x) = ce^{-x}, \quad 0 \le x \le \infty$

$f(x)$ is a PDF if

(i) $f(x) \ge 0, \forall x$

(ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_{-\infty}^{0} f(x)\,dx + \int_{0}^{\infty} f(x)\,dx = 1$$

$$0 + \int_{0}^{\infty} ce^{-x}\,dx = 1$$

$$\left(-ce^{-x}\right)_{0}^{\infty} = 1$$

$$-c\left[e^{-\infty} - e^{0}\right] = 1$$

$$-c[0 - 1] = 1$$

$$\boxed{c = 1}$$

$f(x) = ce^{-x}$

$e^{-x} \ge 0$

$c \ge 0$

$\Rightarrow f(x) \ge 0$

Q.2 *If $f(x)$ has probability density $cx^2, 0 < x < 1$, determine c and find the probability that $\frac{1}{3} < x < \frac{1}{2}$ i.e., $P\left(\frac{1}{3} < x < \frac{1}{2}\right)$.*

$f(x) = cx^2, \quad 0 < x < 1$

Since $f(x)$ is a PDF

$\Rightarrow$ (i) $f(x) \geq 0$

(ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

From (ii)

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_{-\infty}^{0} f(x)\,dx + \int_{0}^{1} f(x)\,dx + \int_{1}^{\infty} f(x)\,dx = 1$$

$$0 + \int_{0}^{1} cx^2\,dx + 0 = 1$$

$$c\left(\frac{x^3}{3}\right)_0^1 = 1$$

$$c\left(\frac{1}{3} - 0\right) = 1$$

$$\frac{c}{3} = 1$$

$$\boxed{c = 3}$$

$$f(x) = 3x^2$$

$$P\left(\frac{1}{3} < x < \frac{1}{2}\right) = \int_{1/3}^{1/2} f(x)\,dx$$

$$= \int_{1/3}^{1/2} 3x^2\,dx$$

$$= \cancel{3}\left(\frac{x^3}{\cancel{3}}\right)_{1/3}^{1/2}$$

$$= \frac{1}{8} - \frac{1}{27}$$

$$= \frac{27 - 8}{216}$$

$$\boxed{P\left(\frac{1}{3} < x < \frac{1}{2}\right) = \frac{19}{216}}$$

Expectation and variance

If x is a continous random variable, then the expectation of x is

$$E(x) = \int_{-\infty}^{\infty} x\, f(x)\, dx$$

Note : $E(x) = \bar{x} = \mu = \mu_1' = \int_{-\infty}^{\infty} x\, f(x)\, dx$

Expectation or mean or first moment about origin

Also

- $E(x) = \int_{-\infty}^{\infty} x\, f(x)\, dx$ **(First moment about origin)**
- $E(x^2) = \int_{-\infty}^{\infty} x^2\, f(x)\, dx$ **(Second moment about origin)**
- $E(x^r) = \int_{-\infty}^{\infty} x^r\, f(x)\, dx$ **(r^{th} moment about origin)**

Variance

$$\text{Var}(x) = E(x^2) - [E(x)]^2$$

Second moment about origin

Standard Deviation

$$S.D = \sqrt{Variance}$$

Q.3 *If the probability density function of a random variable is given by*

AKTU 2021

$$f(x) = \begin{cases} k(1-x^2), & 0 \le x \le 1 \\ 0, & \text{elsewhere} \end{cases}$$

Find (i) k (ii) *Mean* (iii) *Varience* (iv) *Probability between* 0.1 *and* 0.2

(v) *Probability greater than* 0.5

Since $f(x)$ is a PDF

$\therefore$ (i) $f(x) \geq 0 \quad \forall x$

(ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

From (ii)

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_{-\infty}^{0} f(x)\,dx + \int_{0}^{1} f(x)\,dx + \int_{1}^{\infty} f(x)\,dx = 1$$

$$0 + \int_{0}^{1} k(1-x^2) + 0 = 1$$

$$k\left(x - \frac{x^3}{3}\right)_0^1 = 1$$

$$k\left[1 - \frac{1}{3}\right] = 1$$

$$\frac{2}{3}k = 1$$

$$k = \frac{3}{2}$$

$$k = 1.5$$

(ii) $\text{Mean} = \int_{-\infty}^{\infty} x f(x)\,dx$

$$E(x) = \int_{-\infty}^{0} x f(x)\,dx + \int_{0}^{1} x f(x)\,dx + \int_{1}^{\infty} x f(x)\,dx$$

$$E(x) = 0 + \int_{0}^{1} x K(1-x^2)\,dx + 0$$

$$E(x) = K\int_{0}^{1} (x - x^3)\,dx$$

$$E(x) = K\left[\frac{x^2}{2} - \frac{x^4}{4}\right]_0^1$$

$$E(x) = K\left[\frac{1}{2} - \frac{1}{4}\right]$$

$$E(x) = K \times \frac{1}{4}$$

$$E(x) = \frac{3}{2} \times \frac{1}{4}$$

$$\boxed{E(x) = \frac{3}{8}}$$

(iii) $E(x^2) = \int_{-\infty}^{\infty} x^2 f(x)\,dx$

$$E(x^2) = \int_{-\infty}^{0} x^2 f(x)\,dx + \int_{0}^{1} x^2 f(x)\,dx + \int_{1}^{\infty} x^2 f(x)\,dx$$

$$E(x^2) = 0 + \int_0^1 x^2 k(1-x^2)dx + 0$$

$$E(x^2) = k\int_0^1 (x^2 - x^4)dx$$

$$E(x^2) = k\left(\frac{x^3}{3} - \frac{x^5}{5}\right)_0^1$$

$$E(x^2) = k\left[\frac{1}{3} - \frac{1}{5}\right]$$

$$E(x^2) = k\left(\frac{5-3}{15}\right)$$

$$E(x^2) = \frac{3}{2} \times \frac{2}{15} = \frac{1}{5}$$

$$\boxed{E(x^2) = \frac{1}{5}}$$

$$\text{variance} = E(x^2) - [E(x)]^2$$

$$= \frac{1}{5} - \left(\frac{3}{8}\right)^2 = \frac{1}{5} - \frac{9}{64}$$

$$= \frac{64 - 45}{320}$$

$$\boxed{\text{Var}(x) = \frac{19}{320} = 0.0594}$$

(iv) $P(0.1 < x < 0.2) = \int_{0.1}^{0.2} f(x)\,dx$

$= \int_{0.1}^{0.2} k(1-x^2)\,dx$

$= k\left(x - \frac{x^3}{3}\right)_{0.1}^{0.2}$

$= \frac{3}{2}\left[0.2 - \frac{0.008}{3} - \left(0.1 - \frac{0.001}{3}\right)\right]$

$= \frac{3}{2}\left[0.1 - \frac{0.008}{3} + \frac{0.001}{3}\right]$

$P(0.1 < x < 0.2) = 0.1466$

(v) $P(0.5 < x < 1) = \int_{0.5}^{1} f(x)\,dx$

$= k\int_{0.5}^{1} (1-x^2)\,dx$

$= \frac{3}{2}\left[x - \frac{x^3}{3}\right]_{0.5}^{1}$

$= 0.3125$

Q.4 *A continous random variable X has the probability density function*

$$f(x) = \begin{cases} kx & for\ 0 \le x < 2 \\ 2k & for\ 2 \le x < 4 \\ -kx + 6k & for\ 4 \le x \le 6 \\ 0 & elsewhere \end{cases}$$

Find (i)k (ii)Mean

Since $f(x)$ is PDF

$\therefore$ (i) $f(x) \ge 0 \ \forall x$

(ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_{-\infty}^{0} f(x)\,dx + \int_{0}^{2} f(x)\,dx + \int_{2}^{4} f(x)\,dx + \int_{4}^{6} f(x)\,dx + \int_{6}^{\infty} f(x)\,dx = 1$$

$$0 + k\int_{0}^{2} x\,dx + 2k\int_{2}^{4} dx + k\int_{4}^{6} (-x+6)\,dx + 0 = 1$$

$$k\left(\frac{x^2}{2}\right)_0^2 + 2k(x)_2^4 + k\left(-\frac{x^2}{2}+6x\right)_4^6 = 1$$

$$k(2-0) + 2k(4-2) + k\left[-18+36-(-8+24)\right] = 1$$

$$2k + 4k + k(18+8-24) = 1$$

$$2k + 4k + 2k = 1$$

$$8k = 1$$

$$\boxed{k = \frac{1}{8}}$$

(II) Mean

$$E(x) = \int_{-\infty}^{\infty} x\, f(x)\, dx$$

$$E(x) = \int_{-\infty}^{0} x f(x)\, dx + \int_{0}^{2} x f(x) dx + \int_{2}^{4} x f(x) dx + \int_{4}^{6} x f(x) dx + \int_{6}^{\infty} x f(x) dx$$

$$E(x) = k\int_0^2 x^2\,dx + 2k\int_2^4 x\,dx + k\int_4^6 (-x^2+6x)\,dx$$

$$= k\left(\frac{x^3}{3}\right)_0^2 + 2k\left(\frac{x^2}{2}\right)_2^4 + k\left(-\frac{x^3}{3}+\frac{6x^2}{2}\right)_4^6$$

$$= k\left[\frac{8}{3} - 0 + 2(8-2) + \left(-72 + 3\times 36 + \frac{64}{3} - 16\times 3\right)\right]$$

$$= \frac{1}{8}\left[\frac{8}{3} + 12 + \left(-72 + 108 + \frac{64}{3} - 48\right)\right]$$

$$\boxed{E(x) = 3}$$

Topic : *Random variables (PART-3)*

Q. 1 Find the constant k so function $f(x)$ defined as follows be a density function:

$$\begin{cases} 1/k, & a \le x \le b \\ 0, & elsewhere \end{cases}$$

Q. 2 A continous random variable X has

$$\mathrm{f}(x) = \begin{cases} \frac{1}{2}(x+1), & for\ -1 \le x \le 1 \\ 0, & elsewhere \end{cases}$$

represent the density . Find mean, variance and standard deviation.

GATEWAY CLASSES

AKTU Full Courses (Paid)

Download Gateway Classes Application

From Google Play store

All Subjects

Link in Description

Thank You

GATEWAY CLASSES