ENGINEERING MATHEMATICS

UNIT-3 : *Statistical Techniques – I*

Lec-5

Today's Target

- **KURTOSIS**
- ***Problems based on Moments, Skewness and Kurtosis***
- PYQ
- DPP

Lec-1

Measure of central Tendency

1. Arithmetic Mean OR Mean
   - (i) Direct Method
   - (ii) Assume Mean Method
   - (iii) Step Deviation Method
2. Median
3. Mode
4. Geometric Mean
5. Harmonic Mean

Note : 3(Median) = Mode + 2 (Mean)

Lec-2

Moment For frequency distribution

A

Moment About Mean (μ_r)	Moment About Number (μ'_r)	Moment About Origin (ν_r)
$\mu_0 = 1$	$\mu'_0 = 1$	$\nu_0 = 1$
$\mu_1 = 0$	$\mu'_1 = \bar{x} - A$	$\nu_1 = \bar{x}$
$\mu_2 = \frac{\sum f(x-\bar{x})^2}{\sum f}$	$\mu'_2 = \frac{\sum f(x-A)^2}{\sum f}$	$\nu_2 = \frac{\sum fx^2}{\sum f}$
$\mu_3 = \frac{\sum f(x-\bar{x})^3}{\sum f}$	$\mu'_3 = \frac{\sum f(x-A)^3}{\sum f}$	$\nu_3 = \frac{\sum fx^3}{\sum f}$
$\mu_4 = \frac{\sum f(x-\bar{x})^4}{\sum f}$	$\mu'_4 = \frac{\sum f(x-A)^4}{\sum f}$	$\nu_4 = \frac{\sum fx^4}{\sum f}$

Moment Individual series

Moment About Mean (μ_r)	Moment About Number (μ'_r)	Moment About Origin (ν_r)
$\mu_0 = 1$	$\mu'_0 = 1$	$\nu_0 = 1$
$\mu_1 = 0$	$\mu'_1 = \bar{x} - A$	$\nu_1 = \bar{x}$
$\mu_2 = \frac{\sum(x-\bar{x})^2}{n}$	$\mu'_2 = \frac{\sum(x-A)^2}{n}$	$\nu_2 = \frac{\sum x^2}{n}$
$\mu_3 = \frac{\sum(x-\bar{x})^3}{n}$	$\mu'_3 = \frac{\sum(x-A)^3}{n}$	$\nu_3 = \frac{\sum x^3}{n}$
$\mu_4 = \frac{\sum f(x-\bar{x})^4}{n}$	$\mu'_4 = \frac{\sum(x-A)^4}{n}$	$\nu_4 = \frac{\sum x^4}{n}$

Lec-3

Relation between μ_r and μ'_r

$$\mu_1 = 0$$

$$\mu_2 = \mu'_2 - \mu'^2_1$$

$$\mu_3 = \mu'_3 - 3\mu'_2\mu'_1 + 2\mu'^3_1$$

$$\mu_4 = \mu'_4 - 4\mu'_3\mu'_1 + 6\mu'_2\mu'^2_1 - 3\mu'^4_1$$

Relation between μ_r and ν_r

$$\nu_1 = \bar{x}$$

$$\nu_2 = \mu_2 + \bar{x}^2$$

$$\nu_3 = \mu_3 + 3\mu_2\,\bar{x} + \bar{x}^3$$

$$\nu_4 = \mu_4 + 4\mu_3\,\bar{x} + 6\mu_2\bar{x}^2 + \bar{x}^4$$

KARL PEARSON'S β and γ Coefficients

(i) β − Coefficients

$$\beta_1 = \frac{{\mu_3}^2}{{\mu_2}^3}$$

$$\beta_2 = \frac{\mu_4}{{\mu_2}^2}$$

(ii) γ Coefficients

$$\gamma_1 = +\sqrt{\beta_1}$$

$$\gamma_2 = \beta_2 - 3$$

Lec-4

SKEWNESS

Karl Pearson's Method

Karl Pearson's coefficient of skewness (SK_p)

(i) $SK_p = \frac{Mean - Mode}{S.D}$

Mode = 3 Median – 2 Mean

(ii) $SK_p = \frac{3(Mean - Mode)}{S.D}$

Standard Deviation

$$S.D = \sqrt{variance}$$

$$S.D = \sqrt{\mu_2}$$

V.Imp

Method of Moments

Moment coefficient of skewness (SK_M)

$$\gamma_1 = SK_M = \frac{\mu_3}{\sqrt{\mu_2^3}}$$

Note:- We know that

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3}$$

$$\sqrt{\beta_1} = \frac{\mu_3}{\sqrt{\mu_2^3}}$$

$$SK_M = \gamma_1 = \sqrt{\beta_1}$$

GW
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Note:- (i) If Skp = 0

Mean = Mode

Distribution is symmetrical

(ii) If Skp > 0

Mean > Mode

Distribution is positively skewed

(iii) If Skp < 0

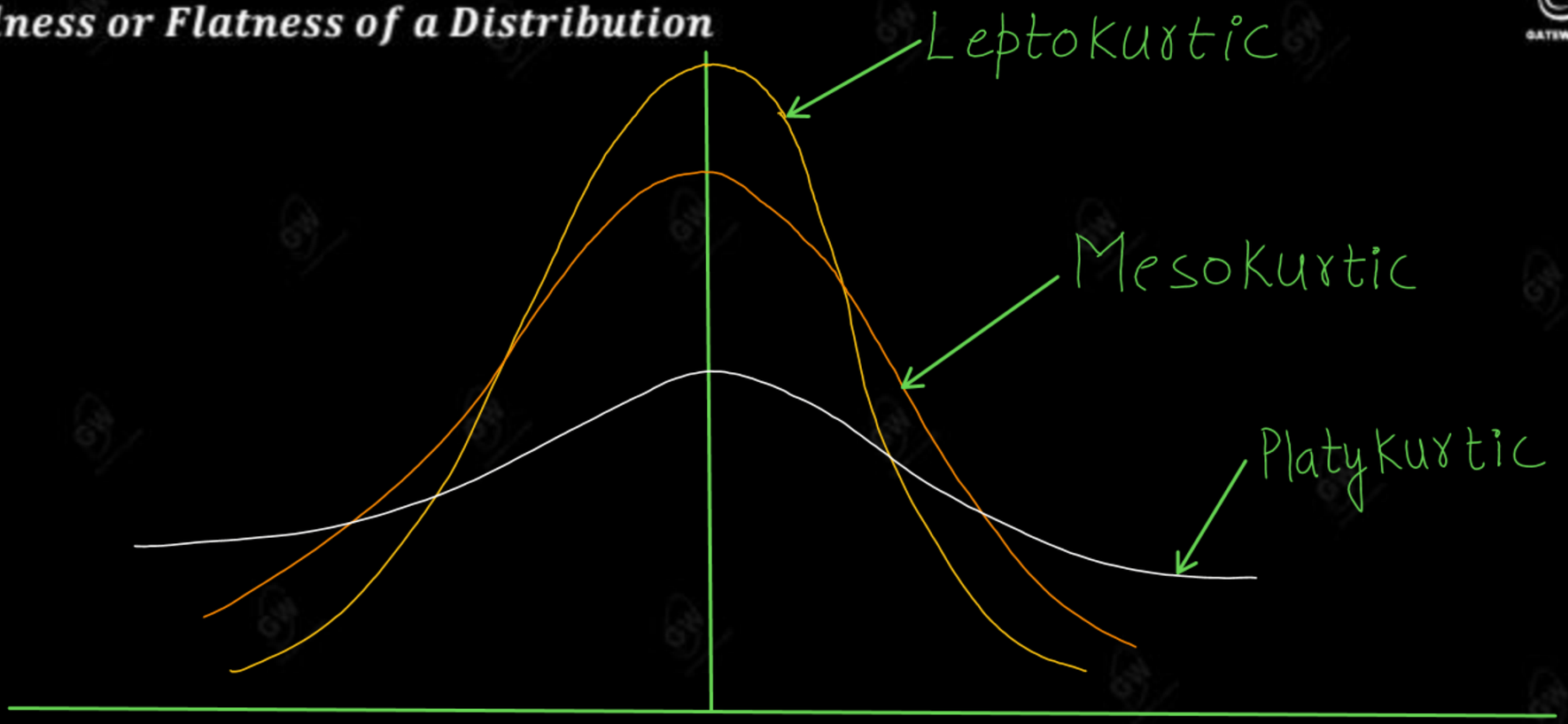
Mean < Mode

Distribution is negatively skewed

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KURTOSIS

Measure of peakedness or Flatness of a Distribution



Measure of Kurtosis

$$\beta_2 = \frac{\mu_4}{{\mu_2}^2}$$

Kurtosis of Distribution is also difined as

$$\gamma_2 = \beta_2 - 3$$

Where and μ_2 are μ_4 the second and fourth moment about mean of the distribution.

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Note :-

(i) The distribution is Leptokurtic if

$$\beta_2 > 3$$

$$\gamma_2 > 0$$

(ii) The distribution is Mesokurtic if

$$\beta_2 = 3$$

$$\gamma_2 = 0$$

(iii) The distribution is Platykurtic if

$$\beta_2 < 3$$

$$\gamma_2 < 0$$

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Q.1 The fourth central moment is 48. What must be its standard deviation in order that the distribution be Mesokurtic. (U.P.T.U.2023)

Given

$\mu_4 = 48$

For Mesokurtic distribution

$\beta_2 = 3$

We know that

$\beta_2 = \frac{\mu_4}{\mu_2^2}$

$3 = \frac{48}{\mu_2^2}$

$\mu_2^2 = \frac{48}{3} = 16$

$\mu_2 = \sqrt{16}$

$\mu_2 = 4$

$S.D = \sqrt{\mu_2}$

$S.D = \sqrt{4}$

$S.D = 2$

Q.2 The first four moments of a distribution are 0, 2.5, 0.7 and 18.71. Find the coefficient of skewness and kurtosis. (U.P.T.U.2015)

Given

$\mu_1 = 0$

$\mu_2 = 2.5$

$\mu_3 = 0.7$

$\mu_4 = 18.71$

coefficient of skewness

$$\gamma_1 = Sk_M = \frac{\mu_3}{\sqrt{\mu_2^3}}$$

$$Sk_M = \frac{0.7}{\sqrt{(2.5)^3}}$$

$$\boxed{Sk_M = 0.17708}$$

Positively skewed

Kurtosis

$$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{18.71}{(2.5)^2}$$

$$\boxed{\beta_2 = 2.9936}$$

$\beta_2 < 3$

Platykurtic

Q.3 *The first four moments of a distribution about x = 4 are 1, 4, 10 and 45. Obtain the various characteristic of the distribution on the basic of the given information. Comment upon the nature of the distribution.*

(A.K.T.U.2016, 2018)

Given

$A = 4$

$\mu_1' = 1$

$\mu_2' = 4$

$\mu_3' = 10$

$\mu_4' = 45$

Moments About Mean

$$\mu_2 = \mu_2' - \mu_1'^2$$

$$= 4 - 1^2$$

$$\boxed{\mu_2 = 3}$$

$$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3$$

$$= 10 - 3 \times 4 \times 1 + 2(1)^3$$

$$= 10 - 12 + 2$$

$$= 0$$

$$\boxed{\mu_3 = 0}$$

$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$$

$$= 45 - 4 \times 10 \times 1 + 6 \times 4 \times (1)^2 - 3 \times (1)^4$$

$$= 45 - 40 + 24 - 3$$

$$= 26$$

$$\boxed{\mu_4 = 26}$$

Skewness

$$SK_M = \frac{\mu_3}{\sqrt{\mu_2^3}}$$

$$= \frac{0}{\sqrt{3^3}}$$

$$\boxed{SK_M = 0}$$

Symmetrical Distribution

Kurtosis

$$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{26}{9}$$

$$\boxed{\beta_2 = 2.89}$$

$$\beta_2 < 3$$

Platykurtic

Q.4 The first four moments about the working mean 28.5 of a distribution are 0.294, 7.144, 42.409 and 454.98. Calculate the moments about the mean. Also evaluate β_1, β_2 and comment upon the skewness and kurtosis of the distribution. (A.K.T.U.2016)

Given

$A = 28.5$

$\mu_1' = 0.294$

$\mu_2' = 7.144$

$\mu_3' = 42.409$

$\mu_4' = 454.98$

Moments About Mean

$$\mu_2 = \mu_2' - \mu_1'^2$$

$$= 7.144 - (0.294)^2$$

$$\boxed{\mu_2 = 7.0576}$$

$$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3$$

$$= 42.409 - 3 \times 7.144 \times 0.294 + 2(.294)^3$$

$$\boxed{\mu_3 = 36.1588}$$

$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$$

$$= 454.98 - 4 \times 42.409 \times 0.294 + 6 \times 7.144 \times (.294)^2 - 3(0.294)^4$$

$$\boxed{\mu_4 = 408.7896}$$

Now

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3}$$

$$\beta_1 = \frac{(36.1588)^2}{(7.0576)^3}$$

$$\boxed{\beta_1 = 3.7193}$$

$$\beta_2 = \frac{\mu_4}{\mu_2^2}$$

$$= \frac{408.7896}{(7.0576)^2}$$

$$\boxed{\beta_2 = 8.2070}$$

Skewness

$$SK_M = \gamma_1 = \frac{\mu_3}{\sqrt{\mu_2^3}}$$

$$SK_M = \frac{36.1588}{\sqrt{(7.0576)^3}}$$

$$\boxed{SK_M = 1.9285}$$

Positively skewed Distribution

KURTOSIS

$$\boxed{\beta_2 = 8.2070}$$

$\beta_2 > 3 \Rightarrow$ Leptokurtic

Q.5 *The following table represents the heights of a batch of 100 students. Calculate Skewness and kurtosis.*

(A.K.T.U.2016)

Height (in cm)	59	61	63	65	67	69	71	73	75
No. Of students	0	2	6	20	40	20	8	2	2

Height (in cm) (x)	No. of Students (f)	$d = x - 67$	$u = \frac{x-67}{2} = h$	fu	fu^2	fu^3	fu^4
59	0	−8	−4	0	0	0	0
61	2	−6	−3	−6	18	−54	162
63	6	−4	−2	−12	24	−48	96
65	20	−2	−1	−20	20	−20	20
A 67	40	0	0	0	0	0	0
69	20	2	1	20	20	20	20
71	8	4	2	16	32	64	128
73	2	6	3	6	18	54	162
75	2	8	4	8	32	128	512
	$\Sigma f = N = 100$			$\Sigma fu =$ 12	$\Sigma fu^2 =$ 164	$\Sigma fu^3 =$ 144	$\Sigma fu^4 =$ 1100

Moments About 67

$$\mu_1' = \frac{\Sigma fu}{N} \times h$$

$$\mu_1' = \frac{12}{100} \times 2$$

$$\boxed{\mu_1' = 0.24}$$

$$\mu_2' = \frac{\Sigma fu^2}{N} \times h^2$$

$$= \frac{164}{100} \times 4$$

$$\boxed{\mu_2' = 6.56}$$

$$\mu_3' = \frac{\Sigma fu^3}{N} \times h^3$$

$$\mu_3' = \frac{144}{100} \times 8$$

$$\boxed{\mu_3' = 11.52}$$

$$\mu_4' = \frac{\Sigma fu^4}{N} \times h^4$$

$$= \frac{1100}{100} \times 16$$

$$\boxed{\mu_4' = 176}$$

Moments About Mean

$$\mu_1 = 0$$

$$\mu_2 = \mu_2' - \mu_1'^2 = 6.56 - (0.24)^2$$

$$\boxed{\mu_2 = 6.5024}$$

$$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3$$

$$= 11.52 - 3 \times 6.56 \times 0.24 + 2(0.24)^3$$

$$\boxed{\mu_3 = 6.824}$$

$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$$

$$\mu_4 = 176 - 4 \times 11.52 \times 0.24 + 6 \times 6.56 (0.24)^2 - 3 \times (0.24)^4$$

$$\boxed{\mu_4 = 167.19798}$$

Skewness

$$SK_M = \frac{\mu_3}{\sqrt{\mu_2^3}}$$

$$SK_m = \frac{6.824}{\sqrt{(6.5024)^3}}$$

$$\boxed{SK_M = 0.411}$$

Positively skewed Distribution

KURTOSIS

$$\beta_2 = \frac{\mu_4}{\mu_2^2}$$

$$= \frac{167.19798}{(6.5024)^2}$$

$$\boxed{\beta_2 = 3.9544}$$

$\beta_2 > 3$

Leptokurtic

Q.6 *Find the measure of skewness and kurtosis on the basis of moments for the following distribution and draw your conclusion :* **(A.K.T.U.2018)**

Marks	5-15	15-25	25-35	35-45	45-55
No. of Students	1	3	5	7	4

MARKS	No. of Students (f)	Mid value (x)	$d = x - 30$	$u = \frac{x-30}{10} = h$	fu	fu^2	fu^3	fu^4
5-15	1	10	−20	−2	−2	4	−8	16
15-25	3	20	−10	−1	−3	3	−3	3
25-35	5	30 (A)	0	0	0	0	0	0
35-45	7	40	10	1	7	7	7	7
45-55	4	50	20	2	8	16	32	64
	$\Sigma f = N = 20$				$\Sigma fu = 10$	$\Sigma fu^2 = 30$	$\Sigma fu^3 = 28$	$\Sigma fu^4 = 90$

Moments About 30

$$\mu_1' = \frac{\Sigma fu}{N} \times h$$

$$= \frac{10}{20} \times 10$$

$$\boxed{\mu_1' = 5}$$

$$\mu_2' = \frac{\Sigma fu^2}{N} \times h^2$$

$$= \frac{30}{\cancel{20}} \times \overset{5}{\cancel{100}}$$

$$\boxed{\mu_2' = 150}$$

$$\mu_3' = \frac{\Sigma fu^3}{N} \times h^3$$

$$= \frac{28}{\cancel{20}} \times \overset{50}{\cancel{1000}}$$

$$\boxed{\mu_3' = 1400}$$

$$\mu_4' = \frac{\Sigma fu^4}{N} \times h^4$$

$$\mu_4' = \frac{90}{\cancel{2\phi}} \overset{45}{\cancel{\times}} \cancel{1000\phi}$$

$$\boxed{\mu_4' = 45000}$$

Moments About Mean

$$\mu_1 = 0$$

$$\mu_2 = \mu_2' - \mu_1'^2$$

$$= 150 - (5)^2$$

$$\boxed{\mu_2 = 125}$$

$$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3$$

$$= 1400 - 3 \times 150 \times 5 + 2(5)^3$$

$$\boxed{\mu_3 = -600}$$

$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$$

$$= 45000 - 4 \times 1400 \times 5 + 6 \times 150 \times (5)^2 - 3 \times (5)^4$$

$$\boxed{\mu_4 = 37625}$$

Skewness

$$SK_M = \frac{\mu_3}{\sqrt{\mu_2^3}}$$

$$= \frac{-600}{\sqrt{(125)^3}}$$

$$\boxed{SK_M = -0.4293}$$

Negatively skewed Distribution

KURTOSIS

$$\beta_2 = \frac{\mu_4}{\mu_2^2}$$

$$= \frac{37625}{(125)^2}$$

$$\boxed{\beta_2 = 2.408}$$

$\beta_2 < 3$

Platykurtic

DPP- 4

Topic : *Problems based on Moments, Skewness and Kurtosis*

Q.1 *The first four moments of a distribution about the value '4' of the variable are -1.5, 17, -30 and 108. State whether the distribution is leptokurtic or platykurtic.* *(U.P.T.U.2014)*

Q.2 *The first four moments of a distribution are 0, 2.5, 0.7 and 18.75. Test the kurtosis of the distribution.* *(M.T.U.2013)*

Q.3 *The first four moments of a distribution about the value 5 of the variable are 2, 20, 40 and 50. Calculate the moments about the mean and comment upon the skewness and kurtosis of the distribution.* *(G.B.T.U.2011)*

Q.4 *Find the measure of skewness and kurtosis on the basis of moments for the following distribution :*

x	1	3	5	7	9
f	1	4	6	4	1

Q.5 *Calculate the first four moments about the mean for the following frequency distribution and hence find the coefficient of skewness and kurtosis and comment upon the nature of the distribution.* *(U.P.T.U.2014)*

Class – Interval	5-10	10-15	15-20	20-25	25-30	30-35	35-40
Frequency	6	8	17	21	15	11	2

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