# Engineering Mathematics

## UNIT : Partial Differential Equation

**Today's Target**

- Non-Homogeneous Linear Partial Differential Equation with constant coefficient

**Particular Integral (PI) : Case-1 and Case-2**

- PYQs
- DPP

# Course Details(Paid) : All Subjects

| | |
|---|---|
| 1 | Recorded Video Lectures (100 % Syllabus Coverage) |
| 2 | Pdf Notes |
| 3 | Lecture wise DPP with Answers |
| 4 | DPP Solutions |
| 5 | Unit wise set of PYQs |

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# Non - Homogeneous Linear Partial Differential Equation with constant coefficient
## (All Partial Derivatives are not of same order)

**Given PDE**

$$\phi(D,\ D') = F(x,\ y)$$

$\phi(D,\ D')$ → CF  ;  $F(x,\ y)$ → PI

**Methods to find *PI***

**CASE-1:- When $F(x,\ y) = e^{ax+by}$**

$$PI = \frac{1}{\phi(D, D')} e^{ax+by}$$

Put $D = a$

$D' = b$

$$PI = \frac{1}{\phi(a, b)} e^{ax+by}$$

where $\phi(a, b) \neq 0$

**Q.1** Solve $(D^2 - 4DD' + 4D'^2 - D + 2D')z = e^{3x+4y}$

$$(D^2 - 4DD' + 4D'^2 - D + 2D')z = e^{3x+4y}$$

$$\left[(D-2D')^2 - (D-2D')\right]z = e^{3x+4y}$$

$$(D-2D')(D-2D'-1)z = e^{3x+4y}$$

$$(D-2D'-0)(D-2D'-1)z = e^{3x+4y}$$

$$CF = e^{0x} f_1(y+2x) + e^{-x} f_2(y+2x)$$

$$\boxed{CF = f_1(y+2x) + e^{+x} f_2(y+2x)}$$

$$PI = \frac{1}{(D^2 - 4DD' + 4D'^2 - D + 2D')} e^{3x+4y}$$

Put $D = 3$ and $D' = 4$

$$PI = \frac{1}{9 - 48 + 64 - 3 + 8} e^{3x+4y}$$

$$PI = \frac{1}{30} e^{3x+4y}$$

Complete solution

$$z = CF + PI$$

$$\boxed{z = f_1(y+2x) + e^{x} f_2(y+2x) + \frac{1}{30} e^{3x+4y}}$$

**Q.2 Solve:** $D(D-2D'-3)z = e^{x+2y}$.

$$D(D-2D'-3)z = e^{x+2y}$$

CF corresponding to $D = f_1(y)$

CF " $(D-2D'-3) = e^{3x} f_2(y+2x)$

$$CF = f_1(y) + e^{3x} f_2(y+2x)$$

$$PI = \frac{1}{D(D-2D'-3)} e^{x+2y}$$

Put $D = 1$

$D' = 2$

$$PI = \frac{1}{1(1-2\times2-3)} e^{x+2y}$$

$$PI = -\frac{1}{6} e^{x+2y}$$

complete solution

$$z = CF + PI$$

$$z = f_1(y) + f_2(y+2x) - \frac{1}{6} e^{x+2y}$$

# Non - Homogeneous Linear Partial Differential Equation with constant coefficient
## (All Partial Derivatives are not of same order)

**Given PDE**

$$\phi(D,\ D') = F(x,\ y)$$

Where

$$\phi(D, D') \neq 0$$

**Methods to find *PI***

**CASE-2:-** When $F(x,\ y) = sin(ax + by)/Cos(ax + by)$

$$PI = \frac{1}{\phi(D, D')} \sin(ax+by)/\cos(ax+by)$$

Put $D^2 = -a^2$

$DD' = -ab$

$D'^2 = -b^2$

**Q.3 Solve:** $(D - D' - 1)(D - D' - 2)z = \sin(2x + 3y)$ **(AKTU-2021-22)**

$$(D-D'-1)(D-D'-2)z = \sin(2x+3y)$$

$$CF = e^{x} f_1(y+x) + e^{2x} f_2(y+x)$$

$$PI = \frac{1}{(D-D'-1)(D-D'-2)} \sin(2x+3y)$$

$$PI = \frac{1}{D^2 - DD' - 2D - D'D + D'^2 + 2D' - D + D' + 2} \sin(2x+3y)$$

$$PI = \frac{1}{D^2 - 2DD' + D'^2 - 3D + 3D' + 2} \sin(2x+3y)$$

Put $D^2 = -4$

$DD' = -6$

$D'^2 = -9$

$$PI = \frac{1}{-4 + 12 - 9 - 3D + 3D' + 2} \sin(2x+3y)$$

$$PI = \frac{1}{-3D + 3D' + 1} \sin(2x+3y)$$

$$PI = \frac{1}{-[3D - 3D' - 1]} \sin(2x+3y)$$

$$PI = \frac{-1 \times [(3D-3D')+1]}{[(3D-3D')-1][(3D-3D')+1]} e^{2x+3y}$$

$$PI = \frac{-[(3D-3D')+1]}{(3D-3D')^2 - 1^2} e^{2x+3y}$$

$$PI = \frac{-[3D-3D'+1]}{9D^2+9D'^2-18DD'-1} e^{2x+3y}$$

Put $D^2 = -4$

$D'^2 = -9$

$DD' = -6$

$$PI = \frac{-(3D-3D'+1)}{-36-81+108-1} \sin(2x+3y)$$

$$PI \quad \frac{+(3D-3D'+1)}{+10} \sin(2x+3y)$$

$$PI = \frac{1}{10}\left[3D\sin(2x+3y) - 3D'\sin(2x+3y) + \sin(2x+3y)\right]$$

$$PI = \frac{1}{10}\left[3\times 2\cos(2x+3y) - 9\cos(2x+3y) + \sin(2x+3y)\right]$$

$$PI = \frac{1}{10}\left[\sin(2x+3y) - 3\cos(2x+3y)\right]$$

complete solution

$$\boxed{Z = CF + PI}$$

**Q.4** *Solve the following partial differential equations:*

$$(D^2 - DD' - 2D'^2 + 2D + 2D')z = \sin(2x + y)$$ *(AKTU-2016)*

$$(D^2 - DD' - 2D'^2 + 2D + 2D')z = \sin(2x+y)$$

$$(D^2 + DD' - DD' - DD' - 2D'^2 + 2D + 2D')z = \sin(2x+y)$$

$$(D^2 + DD' - 2DD' - 2D'^2 + 2D + 2D')z = \sin(2x+y)$$

$$[D(D+D') - 2D'(D+D') + 2(D+D')]z = \sin(2x+y)$$

$$(D+D')(D-2D'+2)z = \sin(2x+y)$$

$$CF = e^{0x} f_1(y-x) + e^{-2x} f_2(y+2x)$$

$$CF = f_1(y-x) + f_2(y+2x)$$

$$PI = \frac{1}{D^2 - DD' - 2D'^2 + 2D + 2D'} \sin(2x+y)$$

Put $D^2 = -4$

$D'^2 = -1$

$DD' = -2$

$$PI = \frac{1}{-4 + 2 + 2 + 2D + 2D'} \sin(2x+y)$$

$$PI = \frac{1}{2(D+D')} \sin(2x+y)$$

$$PI = \frac{D-D'}{2(D+D')(D-D')} \sin(2x+y)$$

$$PI = \frac{D-D'}{2(D^2-D'^2)} \sin(2x+y)$$

Put $D^2 = -4$

$D'^2 = -1$

$$PI = \frac{(D-D')}{2(-4+1)} \sin(2x+y)$$

$$PI = -\frac{1}{6}\left[D\sin(2x+y) - D'\sin(2x+y)\right]$$

$$PI = -\frac{1}{6}\left[2\cos(2x+y) - \cos(2x+y)\right]$$

$$PI = -\frac{1}{6}\cos(2x+y)$$

complete solution

$$Z = CF + PI$$

PDE : DPP-13

➢ Topic : Non-Homogeneous Linear Partial Differential Equation with constant coefficient

## Particular Integral (PI) : Case-1 and Case-2

Q .1 *Solve:* $(D^3 - 3DD' + D' + 4)z = e^{2x+y}$.

Q .2 *Solve* $(D^2 - D'^2 - 3D + 3D')z = e^{x-2y}$

Q .3 *Find the particular integral of* $2s + t - 3q = 5\cos(3x - 2y)$.

Q .4 *Solve the following partial differential equations:* $(D^2 - DD' + D' - 1)z = \cos(x + 2y)$

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# Thank You

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