GATEWAY CLASSES
Empowering learners, transforming futures

DPP-5 Maths-IV Unit-3

Topic - Kurtosis

Q-1: The first four moments of a distribution about the value '4' of the variable are -1.5, 17, -30 and 108. State whether the distribution is leptokurtic or platykurtic.

Ans- $A = 4,\ \mu_1' = -1.5,\ \mu_2' = 17,\ \mu_3' = -30,\ \mu_4' = 108$

$$\mu_2 = \mu_2' - \mu_1'^2$$

$$\mu_2 = 17 - (-1.5)^2$$

$$\mu_2 = 14.75$$

$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$$
$$= 108 - 4(-30)(-1.5) + 6(17)(-1.5)^2 - 3(-1.5)^4$$
$$\mu_4 = 142.3125$$

$$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{142.31}{(14.75)^2} = 0.6541$$

Since $\beta_2 < 3$, distribution is playtykurtic.

Q-2: The first four moments of a distribution are 0, 2.5, 0.7 and 18.75. Test the kurtosis of the distribution.

Download Our App from Google PlayStore
Helpline Number : 7455961284
PDF Notes PYQs Solution Recorded Lectures

$\mu_2 = 2.5, \quad \mu_3 = 0.7, \quad \mu_4 = 18.75$

$$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{18.75}{2.5 \times 2.5}$$

$$\beta_2 = \frac{18.75}{6.25}$$

$$\boxed{\beta_2 = 3}$$

Since $\beta_2 = 3$, distribution is Mesokurtic.

Q-3- The first four moments of a distribution about the value 5 of the variable are 2, 20, 40 and 50. Calculate the moments about the mean and comment upon the skewness and kurtosis of the distribution.

Ans- $A = 5, \quad \mu_1' = 2, \quad \mu_2' = 20, \quad \mu_3' = 40, \quad \mu_4' = 50$

$$\boxed{\mu_1 = 0}$$

$$\mu_2 = \mu_2' - \mu_1'^2$$

$$\mu_2 = 20 - 4$$

$$\boxed{\mu_2 = 16}$$

$$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3$$

$$= 40 - 3 \times 20 \times 2 + 2 \times 8$$

$$\boxed{\mu_3 = -64}$$

Gatway Classes

$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$$

$$\mu_4 = 50 - 4 \times 40 \times 2 + 6 \times 20 \times 4 - 3 \times 16$$

$$\boxed{\mu_4 = 162}$$

Coefficient of skewness $= \frac{\mu_3}{\sqrt{\mu_2^3}} = \frac{-64}{\sqrt{4096}}$

$$= \frac{-64}{64}$$

$$= -1 \quad \text{(Negatively skewed)}$$

$$\beta_2 = \frac{\mu_4}{\mu_2^2}$$

$$\beta_2 = \frac{162}{256}$$

$$\boxed{\beta_2 = 0.63}$$

Since $\beta_2 < 3$, it is playtykurtic.

Q-4- Find the measures of skewness and kurtosis on the basis of moments for the following distribution.

x	1	3	5	7	9
f	1	4	6	4	1

Gateway Classes

Download Our App from Google PlayStore
Helpline Number : 7455961284
PDF Notes
PYQs Solution
Recorded Lectures

GATEWAY CLASSES
Empowering learners, transforming futures

x	f	$u = \frac{x-5}{2}$	fu	fu^2	fu^3	fu^4
1	1	-2	-2	4	-8	16
3	4	-1	-4	4	-4	4
A = 5	6	0	0	0	0	0
7	4	1	4	4	4	4
9	1	2	2	4	8	16
	$\Sigma f = 16$		$\Sigma fu = 0$	$\Sigma fu^2 = 16$	$\Sigma fu^3 = 0$	$\Sigma fu^4 = 40$

Moments about 5 -

$$\mu_1' = \frac{\Sigma fu}{N} \times h$$

$$\mu_1' = 0$$

$$\mu_2' = \frac{\Sigma fu^2}{N} \times h^2$$

$$= \frac{16}{5} \times 2 \times 2$$

$$\mu_2' = 6.4 \times 2 = 12.8$$

$$\mu_3' = \frac{\Sigma fu^3}{N} \times h^3$$

$$\boxed{\mu_3' = 0}$$

Gateway Classes

Download Our App from Google PlayStore
Helpline Number : 7455961284
PDF Notes
PYQs Solution
Recorded Lectures

GATEWAY CLASSES
Empowering learners, transforming futures

$$\mu_4' = \frac{\sum fu^4}{N} \times h^4$$

$$\mu_4' = \frac{40}{5} \times 16$$

$$\boxed{\mu_4' = 128}$$

Moments about mean

$$\boxed{\mu_1 = 0}$$

$$\mu_2 = \mu_2' - \mu_1'^2$$

$$\boxed{\mu_2 = 12.8}$$

$$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3$$

$$\boxed{\mu_3 = 0}$$

$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$$

$$\boxed{\mu_4 = 128}$$

Skewness $(SK_M) = \frac{\mu_3}{\sqrt{\mu_2^3}} = 0$

Kurtosis $\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{128}{163.84} = 0.781$

Gateway Classes

Download Our App from Google PlayStore
Helpline Number : 7455961284
PDF Notes
PYQs Solution
Recorded Lectures

GATEWAY CLASSES
Empowering learners, transforming futures

Q- Calculate the first four moment about the mean for the following frequency distribution and hence find the coefficient of skewness and kurtosis and comment nature of the distribution.

Class - Interval	5-10	10-15	15-20	20-25	25-30	30-35	35-40
Frequency	6	8	17	21	15	11	2

CLASS	Frequency	MID Value (x)	$u = \frac{x - 22.5}{5}$	fu	fu^2	fu^3	fu^4
5-10	6	7.5	-3	-18	54	-162	486
10-15	8	12.5	-2	-16	32	-64	128
15-20	17	17.5	-1	-17	17	-17	17
20-25	21	22.5	0	0	0	0	0
25-30	15	27.5	1	15	15	15	15
30-35	11	32.5	2	22	44	88	176
35-40	2	37.5	3	6	18	54	162
	$\Sigma f = 80$			$\Sigma fu = -8$	$\Sigma fu^2 = 180$	$\Sigma fu^3 = -86$	$\Sigma fu^4 = 984$

$$\mu_1' = \frac{\Sigma fu}{N} \times h = \frac{-8}{80} \times 5 = -0.5$$

Download Our App from Google PlayStore
Helpline Number : 7455961284
PDF Notes
PYQs Solution
Recorded Lectures

GATEWAY CLASSES
Empowering learners, transforming futures

$$\mu_2' = \frac{\sum fu^2}{N} \times h^2$$

$$= \frac{180}{80} \times 5 \times 5$$

$$\boxed{\mu_2' = 56.25}$$

$$\mu_3' = \frac{\sum fu^3}{N} \times h^3$$

$$= \frac{-86}{80} \times 5 \times 5 \times 5$$

$$\boxed{\mu_3' = -134.37}$$

$$\mu_4' = \frac{\sum fu^4}{N} \times h^4$$

$$= \frac{984}{80} \times 5 \times 5 \times 5 \times 5$$

$$\mu_4' = 7687.5$$

Moment about mean :-

$$\mu_1 = 0$$

$$\mu_2 = \mu_2' - \mu_1'^2$$

$$= 56.25 - 0.25$$

$$\boxed{\mu_2 = 56}$$

Gateway Classes

Download Our App from Google PlayStore
Helpline Number : 7455961284
PDF Notes
PYQs Solution
Recorded Lectures

GATEWAY CLASSES
Empowering learners, transforming futures

$$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3$$

$$= -134.37 - 3 \times 56.25 \times (-0.5) + 2 \times (-0.5)^3$$

$$= -134.37 + 84.375 - 0.25$$

$$= -50.245$$

$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$$

$$= 7687.5 - 4 \times (-134.37)(-0.5) + 6 \times 56.25 \times (-0.5)^2 - 3 \times (-0.5)^4$$

$$\boxed{\mu_4 = 7502.937}$$

Measure of kurtosis $\beta_2 = \dfrac{\mu_4}{\mu_2^2}$

$$= \frac{7502.937}{56 \times 56}$$

$$= 2.39$$

Hence $\beta_2 < 3$, it is platykurtic.

Coefficient of skewness $\Rightarrow \dfrac{\mu_3}{\sqrt{\mu_2^3}} = \dfrac{-134.37}{419.06}$

$$= -0.3206$$

(Negatively skewed)

Download Our App from Google PlayStore
Helpline Number : 7455961284
PDF Notes
PYQs Solution
Recorded Lectures