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DPP-13 Maths-IV Unit-2

Q-1: Find Fourier cosine transform $F(x) = \begin{cases} x, & 0<x<1/2 \\ 1-x, & 1/2<x<1 \\ 0, & x>1 \end{cases}$

Sol- By Fourier cosine transform

$$f_c(p) = \int_0^\infty F(x)\cos px\,dx$$

$$f_c(p) = \int_0^{1/2} F(x)\cos px\,dx + \int_{1/2}^{1} F(x)\cos px\,dx + \int_1^\infty F(x)\cos px\,dx$$

$$f_c(p) = \int_0^{1/2} \underset{I}{x}\ \underset{II}{\cos px}\,dx + \int_{1/2}^{1} \underset{I}{(1-x)}\ \underset{II}{\cos px}\,dx + 0$$

$$f_c(p) = \left[x\left(\frac{\sin px}{p}\right) - 1\left(\frac{-\cos px}{p^2}\right)\right]_0^{1/2} + \left[(1-x)\left(\frac{\sin px}{p}\right) - (-1)\left(\frac{-\cos px}{p^2}\right)\right]_{1/2}^{1}$$

$$f_c(p) = \left[\frac{1}{2}\left(\frac{\sin p/2}{p}\right) + \frac{\cos p/2}{p^2} - \frac{1}{p^2}\right] + \left[-\frac{\cos p}{p^2} - \frac{1}{2}\left(\frac{\sin p/2}{p}\right) + \frac{\cos p/2}{p^2}\right]$$

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$$f_c(p) = \frac{1}{2}\left(\frac{\sin p/2}{p}\right) + \frac{\cos p/2}{p^2} - \frac{1}{p^2} - \frac{\cos p}{p^2} - \frac{1}{2}\left(\frac{\sin p/2}{p}\right) + \frac{\cos p/2}{p^2}$$

$$f_c(p) = \frac{2\cos p/2}{p^2} - \frac{\cos p}{p^2} - \frac{1}{p^2}$$

$$f_c(p) = \frac{2\cos p/2}{p^2} - \left(\frac{2\cos^2 p/2 - 1}{p^2}\right) - \frac{1}{p^2}$$

$$f_c(p) = \frac{2\cos p/2}{p^2} - \frac{2\cos^2 p/2}{p^2} + \frac{1}{p^2} - \frac{1}{p^2}$$

$$\boxed{f_c(p) = \frac{2\cos p/2\left(1 - \cos p/2\right)}{p^2}}$$

Ans

Q-2: Find the Fourier cosine transform of e^{-x^2}.

By Fourier cosine Transform,

$$f_c(p) = \int_0^\infty e^{-x^2}\cos px\,dx = I \text{ (say)} \quad -(1)$$

$$I = \int_0^\infty e^{-x^2}\cos px\,dx$$

Differentiate w.r.t 'p'

$$\frac{dI}{dp} = \int_0^\infty e^{-x^2}\frac{d}{dp}\cos px\,dx$$

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$$\frac{dI}{dp} = \int_0^\infty e^{-x^2} x(-x \sin px)\, dx$$

$$\frac{dI}{dp} = \int_0^\infty -x e^{-x^2} \sin px\, dx$$

(II) (I)

$$\frac{dI}{dp} = \left[\sin px \cdot \int -x e^{-x^2} dx\right]_0^\infty - \int_0^\infty \left[p\cos px \cdot \int -x e^{-x^2} dx\right] dx$$

$\because \int -x e^{-x^2} dx \Rightarrow$ Let $-x^2 = t$

$-2x\,dx = dt$

$-x\,dx = \frac{dt}{2}$

$$= \int e^t \frac{dt}{2} = \frac{1}{2}\int e^t dt = \frac{e^t}{2} = \frac{e^{-x^2}}{2}$$

$$\frac{dI}{dp} = \left[\sin px \cdot \frac{e^{-x^2}}{2}\right]_0^\infty - p\int_0^\infty \cos px \cdot \frac{e^{-x^2}}{2} dx$$

$$\frac{dI}{dp} = [0 - 0] - \frac{p}{2}\int_0^\infty e^{-x^2}\cos px\, dx$$

$\int_0^\infty e^{-x^2}\cos px\, dx = I$

$$\frac{dI}{dp} = -\frac{p}{2} I$$

$$\frac{dI}{I} = -\frac{p}{2} dp$$

Integerate both side

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$$\log I = -\frac{p^2}{4} + \log C$$

$$\log I - \log C = -\frac{p^2}{4}$$

$$\frac{I}{C} = e^{-p^2/4}$$

$$\boxed{I = Ce^{-p^2/4}}$$

For $p = 0$

$$\boxed{I = C}$$

$$I_{p=0} = \int_0^\infty e^{-x^2} \cos 0 \, dx$$

$$= \int_0^\infty e^{-x^2} dx$$

We know

$$\boxed{\int_0^\infty e^{-x^2} dx = \sqrt{\frac{\pi}{2}}}$$

$$I = \sqrt{\frac{\pi}{2}} = C$$

$$\boxed{I = \sqrt{\frac{\pi}{2}}\, e^{-p^2/4}}$$

Ans

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Q-3: Find Fourier cosine transform of $F(x) = 2e^{-5x} + 2e^{-2x}$

$$F(x) = 2e^{-5x} + 2e^{-2x}$$

By Fourier cosine transform

$$f_c(p) = \int_0^\infty F(x) \cos px \, dx$$

$$f_c(p) = 2\int_0^\infty e^{-5x} \cos px \, dx + 2\int_0^\infty e^{-2x} \cos px \, dx$$

$$f_c(p) = 2\left[\frac{e^{-5x}}{(-5)^2+p^2}(-5\cos px + p\sin px)\right]_0^\infty + 2\left[\frac{e^{-2x}}{(-2)^2+p^2}(-2\cos px + p\sin px)\right]_0^\infty$$

$$f_c(p) = 2\left[0 - \frac{1}{25+p^2}(-5)\right] + 2\left[0 - \frac{1}{4+p^2}(-2)\right]$$

$$f_c(p) = \frac{10}{25+p^2} + \frac{4}{4+p^2}$$

$$\boxed{f_c(p) = \frac{10}{p^2+25} + \frac{4}{p^2+4}}$$

Ans

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