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DPP-11 Unit-2 Maths-IV

Topic - Complex Fourier Transform

Q-1: Find the Fourier Transform of $F(x) = \begin{cases} x^2, & \text{if } |x| < a \\ 0 & \text{if } F(x) > a \end{cases}$

Sol: $F(x) = \begin{cases} x^2 & -a < x < a \\ 0 & x > a \text{ or } x < -a \end{cases}$

By Fourier Transform

$$f(p) = \int_{-\infty}^{\infty} F(x)\, e^{ipx}\, dx$$

$$f(p) = \int_{-\infty}^{-a} F(x) e^{ipx} dx + \int_{-a}^{a} F(x) e^{ipx} dx + \int_{a}^{\infty} F(x) e^{ipx} dx$$

$$f(p) = 0 + \int_{-a}^{a} x^2 e^{ipx} dx + 0$$

$$f(p) = \int_{-a}^{a} \underset{I}{x^2} \underset{II}{e^{ipx}}\, dx$$

$$f(p) = \left[x^2 \left(\frac{e^{ipx}}{ip} \right) - \int \underset{I}{2x} \times \frac{\underset{II}{e^{ipx}}}{ip} \right]$$

$$f(p) = \left[x^2 \left(\frac{e^{ipx}}{p} \right) - 2 \left[x \times \frac{e^{ipx}}{i^2 p^2} - \left(\frac{e^{ipx}}{i^3 p^3} \right) \right] \right]_{-a}^{a}$$

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$$f(p) = \left[x^2 \left(\frac{e^{ipx}}{ip}\right) - 2x \left(\frac{e^{ipx}}{i^2p^2}\right) + 2 \left(\frac{e^{ipx}}{i^3p^3}\right) \right]_{-a}^{a}$$

$$f(p) = \left[a^2 \left(\frac{e^{ipa}}{ip}\right) - 2a \left(\frac{e^{ipa}}{i^2p^2}\right) + 2\left(\frac{e^{ipa}}{i^3p^3}\right) - a^2 \left(\frac{e^{-ipa}}{ip}\right) - 2a \left(\frac{e^{-ipa}}{i^2p^2}\right) - 2 \left(\frac{e^{-ipa}}{i^3p^3}\right) \right]$$

$$f(p) = \frac{a^2}{ip}\left(e^{ipa} - e^{-ipa}\right) - \frac{2a}{i^2p^2}\left(e^{ipa} + e^{-ipa}\right) + \frac{2}{i^3p^3}\left(e^{ipa} - e^{-ipa}\right)$$

$$f(p) = \frac{2a^2}{p}\left(\frac{e^{ipa} - e^{-ipa}}{2i}\right) + \frac{4a}{p^2}\left(\frac{e^{ipa} + e^{-ipa}}{2}\right) - \frac{2x2}{p^3}\left(\frac{e^{ipa} - e^{-ipa}}{2i}\right)$$

$$f(p) = \frac{2a^2}{p}\sin ap + \frac{4a}{p^2}\cos ap - \frac{4}{p^3}\sin ap$$

$$\boxed{f(p) = \left(\frac{2a^2}{p} - \frac{4}{p^3}\right)\sin ap + \frac{4a}{p^2}\cos ap}$$

Q-2: Find the fourier transform of $f(x) = e^{-|x|}$. Ans

$f(x) = e^{-|x|}$

By Fourier Transform

$$f(p) = \int_{-\infty}^{\infty} f(x) e^{ipx}\, dx$$

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$$f(p) = \int_{-\infty}^{0} f(x) e^{ipx} dx + \int_{0}^{\infty} f(x) e^{ipx} dx$$

$$f(p) = \int_{-\infty}^{0} e^{x} \times e^{ipx} dx + \int_{0}^{\infty} e^{-x} \times e^{ipx} dx$$

$$f(p) = \int_{-\infty}^{0} e^{(1+ip)x} dx + \int_{0}^{\infty} e^{(ip-1)x} dx$$

$$f(p) = \left[\frac{e^{(ip+1)x}}{1+ip}\right]_{-\infty}^{0} + \left[\frac{e^{-(1-ip)x}}{ip-1}\right]_{0}^{\infty}$$

$$f(p) = \left[\frac{e^{0} - e^{-\infty}}{1+ip \quad 1+ip}\right] + \left[\frac{e^{-\infty} - e^{0}}{ip-1 \quad ip-1}\right]$$

$$f(p) = \frac{1}{1+ip} + \frac{1}{1-ip}$$

$$f(p) = \frac{2}{1^2 - i^2 p^2}$$

$$\boxed{f(p) = \frac{2}{1+p^2}}$$

Q-3: Find the Fourier transform of $F(x) = \begin{cases} 1, & |x|<1, \\ 0, & |x|>1 \end{cases}$ hence evaluate $\int_{0}^{\infty} \frac{\sin x}{x} dx$

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Sol- $F(x) = \begin{cases} 1 & -1 < x < 1 \\ 0 & x > 1 \text{ or } x < -1 \end{cases}$

By Fourier Transform

$$f(p) = \int_{-\infty}^{\infty} F(x)\, e^{ipx}\, dx$$

$$f(p) = \int_{-\infty}^{-1} F(x) e^{ipx} + \int_{-1}^{1} F(x) e^{ipx} dx + \int_{1}^{\infty} F(x) e^{ipx} dx$$

$$f(p) = \int_{-1}^{1} e^{ipx}\, dx$$

$$f(p) = \left[\frac{e^{ipx}}{ip}\right]_{-1}^{1}$$

$$f(p) = \frac{1}{ip}\left[e^{ip} - e^{-ip}\right]$$

$$f(p) = \frac{2}{p}\left[\frac{e^{ip} - e^{-ip}}{2i}\right]$$

$$\boxed{f(p) = \frac{2\sin p}{p}}$$

By Inverse Fourier transform

$$F(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(p)\, e^{-ipx}\, dp$$

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$$F(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{2}{p} \sin p \, e^{-ipx} \, dp$$

$$F(x) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{1}{p} \sin p \, (\cos px - i \sin px) \, dp$$

$$F(x) = \frac{1}{\pi} \int_{-\infty}^{\infty} \left(\underbrace{\frac{\sin p \cos px}{p}}_{\text{even}} - i \underbrace{\frac{\sin p \sin px}{p}}_{\text{odd}} \right) dp$$

By property of definite integral

$$F(x) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{\sin p \cos px}{p} \, dp$$

$$\int_{-\infty}^{\infty} \frac{\sin p \cos px}{p} \, dp = \pi F(x)$$

Put $x = 0$

$$\int_{-\infty}^{\infty} \frac{\sin p}{p} \, dp = \pi F(0)$$

$$\int_{-\infty}^{\infty} \frac{\sin p}{p} \, dp = \pi$$

By property of Definite Integral

$$2 \int_{0}^{\infty} \frac{\sin p}{p} \, dp = \pi$$

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$$\int_0^\infty \frac{\sin p}{p}\,dp = \frac{\pi}{2}$$

By property of definite integral

$$\int_0^\infty \frac{\sin x}{x}\,dx = \frac{\pi}{2}$$

Q-4:- Find the Fourier transform of Block function $f(t)$ of height 1 and duration defined by -

$$f(t) = \begin{cases} 1, & \text{for } |t| \le a/2 \\ 0, & \text{otherwise} \end{cases}$$

Sol- $f(t) = \begin{cases} 1 & -\frac{a}{2} \le t \le \frac{a}{2} \\ 0 & t > \frac{a}{2} \text{ or } t < -\frac{a}{2} \end{cases}$

By Fourier Transform

$$f(p) = \int_{-\infty}^{\infty} f(t)\, e^{ipx}\,dx$$

$$f(p) = \int_{-\infty}^{-a/2} f(t) e^{ipx}\,dx + \int_{-\frac{a}{2}}^{a/2} f(t) e^{ipx}\,dx + \int_{a/2}^{\infty} f(t) e^{ipx}\,dx$$

$$F(p) = \int_{-a/2}^{a/2} e^{ipx}\,dx$$

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$$f(p) = \left[\frac{e^{ipx}}{ip}\right]_{-a/2}^{+a/2}$$

$$f(p) = \frac{1}{ip}\left[e^{ipa/2} - e^{-ipa/2}\right]$$

$$f(p) = \frac{2}{p}\left[\frac{e^{ipa/2} - e^{-ipa/2}}{2i}\right]$$

$$\boxed{f(p) = \frac{2}{p} \times \sin\frac{ap}{2}}$$

Ans

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