ENGINEERING MATHEMATICS

UNIT-4 : *Statistical Techniques – II*

Lec-05

Today's Target

- $\boldsymbol{Binomial\ probability\ distribution(Part-II)}$
- PYQ
- DPP

Binomial probability Distribution

Let a random variable X denote the number of successes in n independent trials in an experiment.

Let p be the probability of success and q be the probablity of failure in a single trial so that

$$p + q = 1$$

r successes can be obtained in n trials in n_{C_r} ways.

Probability of r successes in n trials

$$P(X = r) = n_{C_r}\, P(S)\, P(S) \dots\dots\dots P(S) \quad P(F)\, P(F) \dots\dots\dots P(F)$$

$$P(X = r) = n_{C_r}\ \mathrm{p}\ \mathrm{p}\ \mathrm{p}\ \dots\dots\dots\dots \mathrm{p}\ \ \mathrm{q}\ \mathrm{q}\ \mathrm{q}\ \dots\dots\dots\dots \mathrm{q}$$

$$\boxed{P(X = r) = n_{C_r}\, p^r\, q^{n-r}}$$

➤ *If n independent trials constitute one experiment and this experiment is repeated N times.*

➤ *The frequuency of success is*

$$N \; n_{c_r} \; p^r \; q^{n-r}$$

And Binomial Distrivution is $N(q+p)^n$

Mean, variance and standard deviation of Binomial distribution

(i) **Mean of Binomial Distribution,**

$$\mu = np$$

(i) **Variance of Binomial Distribution,**

$$\text{Variance} = npq$$

(i) ***Standard Deviation,***

$$\sigma = \sqrt{npq}$$

Third moment about Mean $= npq(q-p)$

Q.1 *If 10% of bolts produced by a machine are defective, determine the probability that out of 10 bolts choosen at random. (i)1 (ii)None (iii)at most 2 bolts will be defective.*

$$P(\text{defective}) = \frac{\cancel{10}}{\cancel{100}\ 10}$$

$$\boxed{P = \frac{1}{10}}$$

$$P(\text{Non Defective}) = 1 - \frac{1}{10}$$

$$\boxed{q = \frac{9}{10}}$$

$$n = 10$$

(i) $P(X = r) = {}^{n}C_{r}\, p^{r} q^{n-r}$

$$P(X = 1) = {}^{10}C_{1} \left(\frac{1}{10}\right)^{1} \times \left(\frac{9}{10}\right)^{9}$$

$$P(X = 1) = \cancel{10} \times \frac{1}{\cancel{10}} \times (0.9)^{9}$$

$$\boxed{P(X = 1) = 0.3874}$$

(ii) $P(X=r) = {}^{n}C_r\, p^r q^{n-r}$

$$P(X=0) = {}^{10}C_0 \left(\frac{1}{10}\right)^0 \times \left(\frac{9}{10}\right)^{10}$$

$$P(X=0) = 1 \times 1 \times (0.9)^{10}$$

$$\boxed{P(X=0) = 0.3487}$$

(iii) $P(r \le 2) = P(X=0) + P(X=1) + P(X=2)$ — ①

$$P(X=2) = {}^{10}C_2\, p^2 q^8$$

$$P(X=2) = \frac{10!}{8! \times 2!} \times \left(\frac{1}{10}\right)^2 \times \left(\frac{9}{10}\right)^8$$

$$= \frac{10 \times 9 \times 8!}{8! \times 2} \times \frac{1}{100} \times (0.9)^8$$

$$\boxed{P(X=2) = 0.1937}$$

Required Probability

$$P(r \le 2) = 0.3487 + 0.3874 + 0.1937$$

$$\boxed{P(r \le 2) = 0.9298}$$

Q.2 *If the probability of hitting a target is 10% and 10 shots are fired independently. What is the probability that the target will be hit at least once?* **AKTU 2019**

$$P(\text{Hitting a target}) = \frac{10}{100}$$

$$P = \frac{1}{10}$$

$$P(\text{Not Hitting target}) = 1 - \frac{1}{10}$$

$$q = \frac{9}{10}$$

$$n = 10$$

$$P(r \geq 1) = 1 - P(x = 0)$$

$$= 1 - {}^{n}C_{r}\, p^{r} q^{n-r}$$

$$= 1 - {}^{10}C_{0} \left(\frac{1}{10}\right)^{0} \times \left(\frac{9}{10}\right)^{10}$$

$$= 1 - 1 \times 1 \times (0.9)^{10}$$

$$\boxed{P(r \geq 1) = 0.6513}$$

Q.3 *In 800 families with 5 children each, how many families would be expected to have (i) 3 boys and 2 girls, (ii) 2 boys and 3 girls, (iii) no girl (iv) at the most two girls. Assume probabilities for boys and girls to be equal.* **AKTU-2017** V.imp

$N = 800$

$n = 5$

$P(\text{Having a girl}) = \frac{1}{2}$

$$P = \frac{1}{2}$$

$P(\text{Having a boy}) = \frac{1}{2}$

$$q = \frac{1}{2}$$

(i) Expected number of families having 3 boys and 2 Girls

$$NP(X=2) = N\,{}^{n}C_{\gamma}\, p^{\gamma} q^{n-\gamma}$$

$$= 800 \times {}^{5}C_{2} \left(\frac{1}{2}\right)^{2} \times \left(\frac{1}{2}\right)^{3}$$

$$= 800 \times \frac{5!}{3!2!} \times \left(\frac{1}{2}\right)^{5}$$

$$= 800 \times \frac{5 \times 4 \times 3!}{3! \times 2} \times \frac{1}{32}$$

$$N(X=2) = 250$$

(ii) Number of families having 2 boys and 3 Girls

$$NP(X=3) = N\,{}^{n}C_{\gamma}\, p^{\gamma} q^{n-\gamma}$$

$$= 800 \times {}^{5}C_{3} \left(\frac{1}{2}\right)^{3} \times \left(\frac{1}{2}\right)^{2}$$

$$= 800 \times \frac{5!}{3! \times 2!} \times \left(\frac{1}{2}\right)^{5}$$

$$NP(X=3) = 250$$

(iii) Number of families having No girl

$$NP(X=0) = N\,{}^{n}C_{\gamma}\, p^{\gamma} q^{n-\gamma}$$

$$= 800 \times {}^{5}C_{0} \left(\frac{1}{2}\right)^{0} \times \left(\frac{1}{2}\right)^{5}$$

$$= \cancel{800} \times 1 \times \frac{1}{\cancel{32}}$$

$$NP(X=0) = 25$$

(iv) Number of families having atmost two Girls

$$NP(r \leq 2) = N[P(x=0) + P(x=1) + P(x=2)]$$

$$P(x=1) = {}^{5}C_1 \, p^r q^{n-r}$$

$$= 5 \times \left(\frac{1}{2}\right)^1 \times \left(\frac{1}{2}\right)^4$$

$$= 5 \times \frac{1}{25}$$

$$= \frac{5}{32}$$

$$NP(r \leq 2) = NP(x=0) + NP(x=1) + NP(x=2)$$

$$= 25 + \overset{25}{\cancel{800}} \times \frac{5}{\cancel{32}} + 250$$

$$= 25 + 125 + 250$$

$$NP(r \leq 2) = 400$$

Q. 4 *A student is given a true false examination with 8 questions. If he correct at least 7 questions, he passes the examination. Find the probability that he will pass given that he guesses all questions.*

$n = 8$

$p = \frac{1}{2}$

$q = \frac{1}{2}$

Prob. that he will pass

$P(r \geq 7) = P(x=7) + P(x=8)$

$= {}^8C_7 \left(\frac{1}{2}\right)^7 \times \left(\frac{1}{2}\right)^1 + {}^8C_8 \left(\frac{1}{2}\right)^8 \left(\frac{1}{2}\right)^0$

$P(r \geq 7) = 8 \times \left(\frac{1}{2}\right)^8 + 1 \times \left(\frac{1}{2}\right)^8$

$= \left(\frac{1}{2}\right)^8 (8+1)$

$= \frac{9}{28}$

$P(r \geq 7) = 0.035$

Q.5 *Six dies are thrown 729 times. How many times do you expect at least three dice to show a five or six?*

$N = 729$

$n = 6$

$S = \{1, 2, 3, 4, 5, 6\}$

$P(\text{having 5 or 6}) = \frac{2}{6} = 3$

$\boxed{P = \frac{1}{3}}$

$P(\text{having Not 5 or 6}) = \frac{4}{6} = \frac{2}{3}$

$\boxed{q = \frac{2}{3}}$

$$NP(x \geq 3) = N[P(x=3) + P(x=4) + P(x=5) + P(x=6)]$$

$$= 729\left[{}^6C_3\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^3 + {}^6C_4\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^2 + {}^6C_5\left(\frac{1}{3}\right)^5\left(\frac{2}{3}\right)^1 + {}^6C_6\left(\frac{1}{3}\right)^6\left(\frac{2}{3}\right)^0\right]$$

$$NP(x \geq 3) = 729\left[{}^6C_3 \times \frac{8}{36} + {}^6C_4 \times \frac{4}{36} + {}^6C_5 \times \frac{2}{36} + {}^6C_6 \times \frac{1}{36}\right]$$

$$NP(x \geq 3) = \frac{729}{3^6}\left[8\,{}^6C_3 + 4\,{}^6C_4 + 2\,{}^6C_5 + {}^6C_6\right]$$

$$= \frac{729}{729}\left[8 \times \frac{6!}{3! \times 3!} + 4 \times \frac{6!}{4! \times 2!} + 2 \times 6 + 1\right]$$

$$= \frac{8 \times 6 \times 5 \times 4 \times 3!}{3 \times 2 \times 1 \times 3!} + \frac{4 \times 6 \times 5 \times 4!}{4! \times 2 \times 1} + 12 + 1\Big]$$

$$= 160 + 60 + 12 + 1$$

$$\boxed{NP(x \geq 3) = 233}$$

Topic : Binomial probability distribution(Part – II)

Q.1 Out of 800 families with 4 children each, how many families would be expected to have (i)2 boys and 2 girls (ii)at least one boy (iii)no girl (iv)almost two girls? Assume equal probabilities for boys and girls. UPTU 2014

Q.2 A bag contains 5 white , 7 red and 8 black balls. If four balls are drawn, one by one, with replacement. what is the probability that

(i) none is white

(ii) all are white

(iii) at least on is white

(iv) only 2 are white ?

Q.3 Five cards are drawn successively with repelacement from a wellshuffled deck of 52 cards. What is the probability that

(i) All the five cards are spades

(ii) only three are spades ?

(iii) none is spade ?

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