Engineering Mathematics

UNIT : Partial Differential Equation

Today's Target

- Non-Homogeneous Linear Partial Differential Equation with constant coefficient

Particular Integral (PI) : Case-3 and Case-4

- PYQs
- DPP

Course Details(Paid) : All Subjects

1	Recorded Video Lectures (100 % Syllabus Coverage)
2	Pdf Notes
3	Lecture wise DPP with Answers
4	DPP Solutions
5	Unit wise set of PYQs

Paid Courses are available in Gateway Classes Application
Link in Description

Non - Homogeneous Linear Partial Differential Equation with constant coefficient
(All Partial Derivatives are not of same order)

Given PDE

$$\phi(D,\ D') = F(x,\ y)$$

CF — PI

Methods to find *PI*

CASE-3:- When $F(x,\ y) = x^m y^n$

$$PI = \frac{1}{\phi(D,D')}\, x^m y^n$$

$$PI = \left[1 + \phi(D,D')\right]^{-1} x^m y^n$$

Note :-

(i) If a separate constant term is present , take it common

(ii) If $n = m$ (Taking either D or D' as common factor)

(iii) If $n > m$ (Taking D' as a common factor)

(iv) If $m > n$ (Taking D as a common factor)

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After taking common, Apply the formulae given below

(1) $(1+x)^{-1} = 1 - x + x^2 - x^3 + x^4 + \ldots\ldots$

(2) $(1-x)^{-1} = 1 + x + x^2 + x^3 + x^4 + \ldots\ldots$

(3) $(1+x)^{-2} = 1 - 2x + 3x^2 - 4x^3 + \cdots$

(4) $(1-x)^{-2} = 1 + 2x + 3x^2 + 4x^3 + \cdots$

Q.1 Solve : $s + p - q = z + xy$.

$$s + p - q = z + xy$$

$$\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y} - z = xy$$

$$(DD' + D - D' - 1)z = xy$$

$$(DD' - D' + D - 1)z = xy$$

$$[D'(D-1) + 1(D-1)]z = xy$$

$$\underline{(D-1)(D'+1)z = xy}$$

CF corresponding to $D - 1 = e^x f_1(y)$

CF " " " $\underline{(D'+1)} = e^{-y} f_2(x)$

$$\boxed{CF = e^x f_1(y) + e^{-y} f_2(x)}$$

$$PI = \frac{1}{(D-1)(D'+1)} xy$$

$$PI = \frac{-1}{(1-D)(1+D')} xy$$

$$PI = -1\left[(1-D)^{-1}(1+D')^{-1}\right] xy$$

$$PI = -\left[(1 + D + D^2 + \cdots)(1 - D' + D'^2 - \cdots)\right] xy$$

$$PI = -\left[1 - D' + D - DD' + \cdots\right] xy$$

$$PI = -\left[xy - D'(xy) + D(xy) - DD'(xy)\right]$$

$$PI = -(xy - x + y - 1)$$

$$PI = -xy + x - y + 1$$

complete solution

$$Z = CF + PI$$

$$\boxed{Z = e^{x} f_1(y) + e^{-y} f_2(x) - xy + x - y + 1}$$

Q.2 Solve: $D(D+D'-1)(D+3D'-2)z = x^2 - 4xy + 2y^2$.

$$D(D+D'-1)(D+3D'-2)z = x^2-4xy+2y^2$$

$$CF = f_1(y) + e^{x} f_2(y-x) + e^{2x} f_3(y-3x)$$

$$PI = \frac{1}{D(D+D'-1)(D+3D'-2)}(x^2-4xy+2y^2)$$

$$PI = \frac{1}{D(-1)\left[1-(D+D')\right](-2)\left[1-\left(\frac{D}{2}+\frac{3}{2}D'\right)\right]}(x^2-4xy+2y^2)$$

$$PI = \frac{1}{2D}\left[1-(D+D')\right]^{-1}\left[1-\left(\frac{D}{2}+\frac{3}{2}D'\right)\right]^{-1}(x^2-4xy+2y^2)$$

$$PI = \frac{1}{2D}\left[1+(D+D')+(D+D')^2+\cdots\right]\left[1+\left(\frac{D}{2}+\frac{3}{2}D'\right)+\left(\frac{D}{2}+\frac{3}{2}D'\right)^2+\cdots\right](x^2-4xy+2y^2)$$

$$PI = \frac{1}{2D}\left[1+D+D'+D^2+D'^2+2DD'\right]\left[1+\frac{D}{2}+\frac{3}{2}D'+\frac{D^2}{4}+\frac{9}{4}D'^2+\frac{3}{2}DD'\right](x^2-4xy+2y^2)$$

$$PI = \frac{1}{2D}\left[1+\frac{D}{2}+\frac{3}{2}D'+\frac{D^2}{4}+\frac{9}{4}D'^2+\frac{3}{2}DD'+D+\frac{D^2}{2}+\frac{3}{2}DD'+D'+\frac{DD'}{2}+\frac{3}{2}D'^2+\right.$$
$$\left.D^2+D'^2+2DD'\right](x^2-4xy+2y^2)$$

$$PI = \frac{1}{2D}\left[1+\frac{3}{2}D+\frac{5D'}{2}+\frac{7D^2}{4}+\frac{19D'^2}{4}+\frac{11}{2}DD'\right](x^2-4xy+2y^2)$$

$$PI = \frac{1}{2D}\left[x^2-4xy+2y^2+\frac{3}{2}D(x^2-4xy+2y^2)+\frac{5}{2}D'(x^2-4xy+2y^2)+\frac{7}{4}D^2(x^2-4xy+2y^2)\right.$$
$$\left.+\frac{19}{4}D'^2(x^2-4xy+2y^2)+\frac{11}{2}DD'(x^2-4xy+2y^2)\right.$$

$$PI = \frac{1}{2D}\left[x^2 - 4xy + 2y^2 + \frac{3}{2}(2x-4y) + \frac{5}{2}(-4x+4y) + \frac{7}{4}(2) + \frac{19}{4}(4) + \frac{11}{2}(-4)\right]$$

$$PI = \frac{1}{2D}\left[x^2 - 4xy + 2y^2 + 3x - 6y - 10x + 10y + \frac{7}{2} + 19 - 22\right]$$

$$PI = \frac{1}{2D}\left[x^2 - 4xy + 2y^2 - 7x + 4y + \frac{1}{2}\right]$$

$$PI = \frac{1}{2}\left[\frac{x^3}{3} - 2x^2y + 2xy^2 - \frac{7x^2}{2} + 4xy + \frac{1}{2}x\right]$$

complete solution

$$z = CF + PI$$

Non - Homogeneous Linear Partial Differential Equation with constant coefficient
(All Partial Derivatives are not of same order)

Given PDE

$$\phi(D,\ D') = F(x,\ y)$$

Methods to find *PI*

CASE-4:- When $F(x,\ y) = e^{ax+by}$ V

$$PI = \frac{1}{\phi(D, D')} e^{(ax+by)} V(x, y)$$

$$PI = e^{ax+by} \times \frac{1}{\phi(D+a, D'+b)} V(x, y)$$

Q.3 Solve : $(D - 3D' - 2)^2 z = 2e^{2x} \tan(y + 3x)$.

$$(D-3D'-2)^2 z = 2e^{2x}\tan(y+3x)$$

$$CF = e^{2x} f_1(y+3x) + x e^{2x} f_2(y+3x)$$

$$PI = \frac{1}{(D-3D'-2)^2} 2e^{2x}\tan(y+3x)$$

$$PI = 2 \times \frac{1}{(D-3D'-2)^2} e^{2x+0y}\tan(y+3x)$$

$$PI = 2e^{2x+0y} \times \frac{1}{(D+2-3D'-2)^2}\tan(y+3x)$$

$$PI = 2e^{2x} \times \frac{1}{(D-3D')^2}\tan(y+3x)$$

Put $D \longrightarrow 3,\ D' \longrightarrow 1$

case of failure

$$PI = 2e^{2x} \times x \times \frac{1}{2(D-3D')}\tan(y+3x)$$

Again case of failure

$$PI = \frac{2x^2 e^{2x}}{2} \times \frac{1}{2}\tan(y+3x)$$

$$PI = x^2 e^{2x} \tan(y+3x)$$

complete solution

$$z = CF + PI$$

Q.4 Solve the linear partial differential equation : $(D^2 - DD' + D' - 1)z = \cos(x + 2y) + e^y$.

$$(D^2 - DD' + D' - 1)z = \cos(x+2y) + e^y$$

$$[(D^2 - 1) + D' - DD']z = \cos(x+2y) + e^y$$

$$[(D-1)(D+1) - D'(D-1)]z = \cos(x+2y) + e^y$$

$$(D-1)(D+1-D')z = \cos(x+2y) + e^y$$

$$CF = e^x f_1(y) + e^{-x} f_2(y+x)$$

$$PI = \frac{1}{(D^2 - DD' + D' - 1)} \{\cos(x+2y) + e^y\}$$

$$PI = \frac{1}{(D^2 - DD' + D' - 1)} \cos(x+2y)$$

$$+ \frac{1}{(D^2 - DD' + D' - 1)} e^y$$

$$PI = P_1 + P_2 \quad \text{——} (1)$$

$$P_1 = \frac{1}{(D^2 - DD' + D' - 1)} \cos(x+2y)$$

Put $D^2 = -1$

$DD' = -2$

$$P_1 = \frac{1}{-1+2+D'-1}\cos(n+2y)$$

$$P_1 = \frac{1}{D'}\cos(n+2y)$$

$$\boxed{P_1 = -\frac{\sin(n+2y)}{2}}$$

$$P_2 = \frac{1}{(D-1)(D-D'+1)}e^y$$

$$P_2 = \frac{1}{(D-D'+1)}\left[\frac{1}{D-1}e^{0n+y}\right]$$

$$P_2 = \frac{1}{(D-D'+1)}\left(-e^{0n+y}\right)$$

$$P_2 = -e^{0n+y}\left[\frac{1}{D-(D'+1)+1}\right]$$

$$P_2 = -e^y\left(\frac{1}{D-D'-1+1}\right)$$

$$P_2 = -e^y \times \frac{1}{D-D'}e^{0n+0y}$$

$$P_2 = -e^y \times \frac{1}{0-0} e^{ax+by}$$

case of failure

$$P_2 = -xe^y \times \frac{1}{1} e^{ax+by}$$

$$P_2 = -xe^y$$

Put P_1 and P_2 in (1)

$$P_I = -\frac{\sin(x+2y)}{2} - xe^y$$

complete solution

$$\boxed{z = CF + PI}$$

PDE : DPP-14

➢ Topic : Non-Homogeneous Linear Partial Differential Equation with constant coefficient

Particular Integral (PI) : Case-3 and Case-4

Q .1 *Solve the linear partial differential equation :* $(D^2 - D'^2 - 3D + 3D')z = xy + e^{x+2y}$. *(GBTU-2011)*

Q .2 *Solve* $(D + D' - 1)^2 z = xy$ *(GBTU-2011)*

Q .3 *Solve :* $(D - 3D' - 2)^3 z = 6e^{2x} \sin(3x + y)$.

Q .4 *Solve :* $r - 4s + 4t + p - 2q = e^{x+y}$.

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Thank You

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