ENGINEERING MATHEMATICS

UNIT-5 : *Statistical Techniques – III*

(Testing a Hypothesis and Statistical Quality Control)

Lec-02

Today's Target

- $t - test\ for\ one\ sample$
- PYQ
- DPP

Parameter and Statistics: Symbols for Population and Samples

	Parameter	Statistic
S.NO.	Measures used for Population known as Parameter	Measures used for Sample known as statistic
1	Population Size (N)	Sample Size (n)
2	Population Mean (μ)	Sample Mean $(\bar{x})$
3	Population S. D (σ)	Sample S. D (s)
4	Population Variance (σ^2)	Sample Variance (s^2)
5	PopulationProportion (P)	Sample Proportion (p)

Hypothesis Testing

Test of significance

(i) t – test ✓
(ii) z – test ✓] - 7M
(iii) F – test
(iv) chi – square test ✓ 7M

t – test

OR

Students t – distribution

- It is a parametric test of hypothesis testing based on student's T distribution.
- To test the significance of the mean of a sample.
- **t – test is applicable when**

(i) sample size is small ($n \leq 30$)

(ii) Population standard deviation (σ) is not known

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$t-test$

- For one sample ✓
- For two sample — Next Lecture

$t-test$ for one sample

- To test the significant difference between the sample mean ($\bar{x}$) and population mean (μ).

Steps to solve Questions

STEP – 1 : Set Null Hypothesis (H_0)

H_0 : There is no significant difference between sample mean and population mean

$H_0 : \mu = \mu_0$ (given)

STEP – 2 : Alternative Hypothesis (H_1)

$H_1 : \mu \neq \mu_0$ (Two tailed test)

Step – 3 : Calculate |t|

(i) When S.D is not given

$$t = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}}$$

$$t = \left(\frac{\bar{x}-\mu}{s}\right)\sqrt{n}$$

$\bar{x}$ = Sample mean

μ = Population mean

n = Sample size

$$S = \text{Sample S.D} = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n-1}}$$

(ii) When S.D is given

$$t = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n-1}}}$$

Step – 4 : Write degree of freedom (n-1) and level of significance $\propto$ ($\propto = 5\%$ *OR* $\propto = 1\%$)

By default

Step – 5: From the $t-$ test table

Find the value of $t_{\propto}$ at the level of significance $\propto$ for the degree of freedom $(n-1)$

Step – 6: Conclusion

(i) If $|t| < t_{\propto}$ then H_0 is accepted

- There is no significant difference between sample mean and population mean
- Sample have been taken from correct population.

(ii) If $|t| > t_{\propto}$ then H_0 is rejected

- There is significant difference between sample mean and population mean
- Sample could not have come from this population.

Fiducial limits or confidence limits

$$\bar{x} - t_{\propto}\frac{s}{n} < \mu < \bar{x} + t_{\propto}\frac{s}{n}$$

(i) **95%** *confidence limits (level of significance is* **5%***) are* $\bar{x} \pm t_{0.05}\frac{s}{n}$

(i) **99%** *confidence limits (level of significance is* **1%***) are* $\bar{x} \pm t_{0.01}\frac{s}{n}$

Q.1 *A sample of 18 items has a mean 24 units and standard deviation 3 units. Test the hypothesis that it is a random sample from a normal population with mean 27 units.*

Given

$n = 18$

$\bar{x} = 24$

$s = 3$

$\mu = 27$

Null Hypothesis (H_0)

H_0 : ——

$H_0 : \mu = 27$

Alternative Hypothesis (H_1)

$H_1 : \mu \neq 27$ (two tailed test

Calculation of |t|

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n-1}}}$$

$$t = \frac{24 - 27}{\frac{3}{\sqrt{18-1}}}$$

$$t = -\frac{3}{3} \times \sqrt{17}$$

$$t = -\sqrt{17} = -4.123$$

$$|t| = 4.123$$

Degree of freedom = n-1

$n-1 = 18-1$

$= 17$

Level of significance

$\alpha = 5\% = \frac{5}{100}$

$\alpha = 0.05$

From the t-test table

$t_{0.05} = 2.11$

conclusion

$|t| = 4.123$ and $t_{0.05} = 2.11$

$\because |t| > t_{0.05}$

$\therefore H_0$ Hypothesis is rejected

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$t_{0.05} = 2.11$

Table 2 : SIGNIFICANT VALUES $t_\nu(\alpha)$ OF t-DISTRIBUTION (TWO TAIL AREAS) $[|t| > t_\nu(\alpha)] = \alpha$

d.f. (ν)	Probability (Level of Significance)					
	0.50	0.10	0.05	0.02	0.01	0.001
1	1.00	6.31	12.71	31.82	63.66	636.62
2	0.82	0.92	4.30	6.97	6.93	31.60
3	0.77	2.32	3.18	4.54	5.84	12.94
4	0.74	2.13	2.78	3.75	4.60	8.61
5	0.73	2.02	2.57	3.37	4.03	6.86
6	0.72	1.94	2.45	3.14	3.71	5.96
7	0.71	1.90	2.37	3.00	3.50	5.41
8	0.71	1.80	2.31	2.90	3.36	5.04
9	0.70	1.83	2.26	2.82	3.25	4.78
10	0.70	1.81	2.23	2.76	3.17	4.59
11	0.70	1.80	2.20	2.72	3.11	4.44
12	0.70	1.78	2.18	2.68	3.06	4.32
13	0.69	1.77	2.16	2.05	3.01	4.22
14	0.69	1.76	2.15	2.62	2.98	4.14
15	0.69	1.75	2.13	2.60	2.95	4.07
16	0.69	1.75	2.12	2.58	2.92	4.02
17	0.69	1.74	2.11	2.57	2.90	3.97
18	0.69	1.73	2.10	2.55	2.88	3.92
19	0.69	1.73	2.09	2.54	2.86	3.88
20	0.69	1.73	2.09	2.53	2.85	3.85
21	0.69	1.72	2.08	2.52	2.83	3.83
22	0.69	1.72	2.07	2.51	2.42	3.79
23	0.69	1.71	2.07	2.50	2.81	3.77
24	0.69	1.71	2.06	2.49	2.80	3.75
25	0.68	1.71	2.06	2.49	2.79	3.73
26	0.68	1.71	2.06	2.48	2.78	3.71
27	0.68	1.70	2.05	2.47	2.77	3.69
28	0.68	1.70	2.05	2.47	2.76	3.67
29	0.68	1.70	2.05	2.46	2.76	3.66
30	0.68	1.70	2.04	2.46	2.75	3.65
∞	0.67	1.65	1.96	2.33	2.58	3.29

Q. 2 *The 9 items of a sample have the following values:* x: 45, 47, 50, 52, 48, 47, 49, 53, 51.

Does the mean of these values differ significantly from the assumed mean 47.5 ?

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Given

$n = 9$

$\mu = 47.5$

x	$(x-\bar{x})$	$(x-\bar{x})^2$
45	−4.1	16.81
47	−2.1	4.41
50	0.9	0.81
52	2.9	8.41
48	−1.1	1.21
47	−2.1	4.41
49	−0.1	0.001
53	3.9	15.21
51	1.9	3.61
$\Sigma x = 442$		$\Sigma(x-\bar{x})^2 =$	54.89

$$\bar{x} = \frac{\Sigma x}{n}$$

$$\bar{x} = \frac{442}{9}$$

$$\boxed{\bar{x} = 49.1}$$

$$\Sigma(x-\bar{x})^2 = 54.89$$

$$S = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n-1}}$$

$$S = \sqrt{\frac{54.89}{8}}$$

$$\boxed{S = 2.619}$$

Null Hypothesis (H_0)

H_0 : _____

$H_0 : \mu = 47.5$

Alternative Hypothesis (H_1)

$H_1 : \mu \neq 47.5$ (Two tailed test)

calculation of $|t|$

$$t = \frac{\bar{x} - \mu}{\frac{S}{\sqrt{n}}}$$

$$= \frac{49.1 - 57.5}{\frac{2.619}{\sqrt{9}}}$$

$$t = -1.8327$$

$$\boxed{|t| = 1.8327}$$

Degree of freedom

$n-1 = 9-1$

$= 8$

Level of significance

$\alpha = 5\% = \frac{5}{100}$

$\alpha = 0.05$

From t-test table

$t_{0.05} = 2.31$

Conclusion

$|t| = 1.8327$ and $t_{0.05} = 2.31$

$\therefore |t| < t_{0.05}$

$\therefore$ H_0 Hypothesis accepted

$t_{0.05} = 2.31$

Table 2 : SIGNIFICANT VALUES $t_\nu(\alpha)$ OF t-DISTRIBUTION (TWO TAIL AREAS) $[|t| > t_\nu(\alpha)] = \alpha$

d.f. (ν)	Probability (Level of Significance)					
	0.50	0.10	0.05	0.02	0.01	0.001
1	1.00	6.31	12.71	31.82	63.66	636.62
2	0.82	0.92	4.30	6.97	6.93	31.60
3	0.77	2.32	3.18	4.54	5.84	12.94
4	0.74	2.13	2.78	3.75	4.60	8.61
5	0.73	2.02	2.57	3.37	4.03	6.86
6	0.72	1.94	2.45	3.14	3.71	5.96
7	0.71	1.90	2.37	3.00	3.50	5.41
8	0.71	1.80	2.31	2.90	3.36	5.04
9	0.70	1.83	2.26	2.82	3.25	4.78
10	0.70	1.81	2.23	2.76	3.17	4.59
11	0.70	1.80	2.20	2.72	3.11	4.44
12	0.70	1.78	2.18	2.68	3.06	4.32
13	0.69	1.77	2.16	2.05	3.01	4.22
14	0.69	1.76	2.15	2.62	2.98	4.14
15	0.69	1.75	2.13	2.60	2.95	4.07
16	0.69	1.75	2.12	2.58	2.92	4.02
17	0.69	1.74	2.11	2.57	2.90	3.97
18	0.69	1.73	2.10	2.55	2.88	3.92
19	0.69	1.73	2.09	2.54	2.86	3.88
20	0.69	1.73	2.09	2.53	2.85	3.85
21	0.69	1.72	2.08	2.52	2.83	3.83
22	0.69	1.72	2.07	2.51	2.42	3.79
23	0.69	1.71	2.07	2.50	2.81	3.77
24	0.69	1.71	2.06	2.49	2.80	3.75
25	0.68	1.71	2.06	2.49	2.79	3.73
26	0.68	1.71	2.06	2.48	2.78	3.71
27	0.68	1.70	2.05	2.47	2.77	3.69
28	0.68	1.70	2.05	2.47	2.76	3.67
29	0.68	1.70	2.05	2.46	2.76	3.66
30	0.68	1.70	2.04	2.46	2.75	3.65
∞	0.67	1.65	1.96	2.33	2.58	3.29

Q.3 *The lifetime of electric bulbs for a random sample of 10 from a large consignment gave the following data:*

Item	1	2	3	4	5	6	7	8	9	10
Life in 000hrs.	4.2	4.6	3.9	4.1	5.2	3.8	3.9	4.3	4.4	5.6

Can we accept the hypothesis that the average lifetime of bulb is 4000 hrs? a

Given

$n = 10$

$\mu = 4000$

$\mu = 4$

x	$(x-\bar{x})$	$(x-\bar{x})^2$
4.2	−0.2	0.04
4.6	0.2	0.04
3.9	−0.5	0.25
4.1	−0.3	0.09
5.2	0.8	0.64
3.8	−0.6	0.36
3.9	−0.5	0.25
4.3	−0.1	0.01
4.4	0	0
5.6	1.2	1.44
$\Sigma x = 44$		$\Sigma(x-\bar{x})^2 = 3.12$

$$\bar{x} = \frac{\Sigma x}{n} = \frac{44}{10}$$

$$\bar{x} = 4.4$$

$$s = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n-1}}$$

$$= \sqrt{\frac{3.12}{9}}$$

$s = 0.589$

Null Hypothesis (H_0)

H_0 : ———

$H_0 : \mu = 4000$

Alternative Hypothesis (H_1)

$H_1 : \mu \neq 4000$ (Two tailed test)

calculation of $|t|$

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{4.4 - 4}{\frac{0.589}{10}}$$

$$t = 2.123$$

$$|t| = 2.123$$

Remaning step → HW

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$t_{0.05} = 2.26$

Table 2 : SIGNIFICANT VALUES $t_\nu(\alpha)$ OF t-DISTRIBUTION (TWO TAIL AREAS) $[|t| > t_\nu(\alpha)] = \alpha$

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	0.50	0.10	0.05	0.02	0.01	0.001
1	1.00	6.31	12.71	31.82	63.66	636.62
2	0.82	0.92	4.30	6.97	6.93	31.60
3	0.77	2.32	3.18	4.54	5.84	12.94
4	0.74	2.13	2.78	3.75	4.60	8.61
5	0.73	2.02	2.57	3.37	4.03	6.86
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8	0.71	1.80	2.31	2.90	3.36	5.04
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17	0.69	1.74	2.11	2.57	2.90	3.97
18	0.69	1.73	2.10	2.55	2.88	3.92
19	0.69	1.73	2.09	2.54	2.86	3.88
20	0.69	1.73	2.09	2.53	2.85	3.85
21	0.69	1.72	2.08	2.52	2.83	3.83
22	0.69	1.72	2.07	2.51	2.42	3.79
23	0.69	1.71	2.07	2.50	2.81	3.77
24	0.69	1.71	2.06	2.49	2.80	3.75
25	0.68	1.71	2.06	2.49	2.79	3.73
26	0.68	1.71	2.06	2.48	2.78	3.71
27	0.68	1.70	2.05	2.47	2.77	3.69
28	0.68	1.70	2.05	2.47	2.76	3.67
29	0.68	1.70	2.05	2.46	2.76	3.66
30	0.68	1.70	2.04	2.46	2.75	3.65
∞	0.67	1.65	1.96	2.33	2.58	3.29

Q.4 *A random sample of size 16 has 53 as mean. The sum of squares of the deviation from mean is 135. Can this sample be regarded as taken from the population having 56 as mean? Obtain 95% and 99% confidence limits of the mean of the population.*

Given

$n = 16$

$\bar{x} = 53$

$\Sigma (x - \bar{x})^2 = 135$

$\mu = 56$

$$s = \sqrt{\frac{\Sigma (x - \bar{x})^2}{n - 1}}$$

$$s = \sqrt{\frac{135}{16 - 1}}$$

$$s = \sqrt{9}$$

$$\boxed{s = 3}$$

Null Hypothesis (H_0)

H_0 : ———

$H_0 : \mu = 56$

Alternative Hypothesis (H_1)

$H_1 : \mu \neq 56$ (Two tailed test)

calculation of |t|

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

$$t = \frac{53 - 56}{\frac{3}{\sqrt{16}}}$$

$$t = -\frac{\cancel{3}}{\cancel{3}}\sqrt{16}$$

$$t = -4$$

$$\boxed{|t| = 4}$$

Degree of freedom

$$n - 1 = 16 - 1$$

$$= 15$$

Level of significance

$$\alpha = 5\% = \frac{5}{100}$$

$$\alpha = 0.05$$

From t-test table

$$\boxed{t_{0.05} = 2.13}$$

conclusion

$|t| = 4$ and $t_{0.05} = 2.13$

$\therefore |t| > t_{0.05}$

$\therefore H_0$ Hypothesis is rejected

When level of significance is 1%

$\alpha = 1\% = \frac{1}{100} = 0.01$

$\alpha = 0.01$

$t_{0.01} = 2.95$

confidence limit of mean at 95%

$= \bar{x} \pm t_{0.05} \frac{s}{n} = 53 \pm 2.13 \times \frac{3}{16}$

$= 51.4025, 54.5975$

confidence limit of mean at 99%

$= \bar{x} \pm t_{0.01} \frac{s}{n} = 53 \pm 2.95 \times \frac{3}{16}$

$= 50.7875, 55.2125$

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$t_{0.05} = 2.13$

Table 2 : SIGNIFICANT VALUES $t_\nu(\alpha)$ OF t-DISTRIBUTION (TWO TAIL AREAS) $[|t| > t_\nu(\alpha)] = \alpha$

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	0.50	0.10	0.05	0.02	0.01	0.001
1	1.00	6.31	12.71	31.82	63.66	636.62
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8	0.71	1.80	2.31	2.90	3.36	5.04
9	0.70	1.83	2.26	2.82	3.25	4.78
10	0.70	1.81	2.23	2.76	3.17	4.59
11	0.70	1.80	2.20	2.72	3.11	4.44
12	0.70	1.78	2.18	2.68	3.06	4.32
13	0.69	1.77	2.16	2.05	3.01	4.22
14	0.69	1.76	2.15	2.62	2.98	4.14
15	0.69	1.75	2.13	2.60	2.95	4.07
16	0.69	1.75	2.12	2.58	2.92	4.02
17	0.69	1.74	2.11	2.57	2.90	3.97
18	0.69	1.73	2.10	2.55	2.88	3.92
19	0.69	1.73	2.09	2.54	2.86	3.88
20	0.69	1.73	2.09	2.53	2.85	3.85
21	0.69	1.72	2.08	2.52	2.83	3.83
22	0.69	1.72	2.07	2.51	2.42	3.79
23	0.69	1.71	2.07	2.50	2.81	3.77
24	0.69	1.71	2.06	2.49	2.80	3.75
25	0.68	1.71	2.06	2.49	2.79	3.73
26	0.68	1.71	2.06	2.48	2.78	3.71
27	0.68	1.70	2.05	2.47	2.77	3.69
28	0.68	1.70	2.05	2.47	2.76	3.67
29	0.68	1.70	2.05	2.46	2.76	3.66
30	0.68	1.70	2.04	2.46	2.75	3.65
∞	0.67	1.65	1.96	2.33	2.58	3.29

UNIT-5 : DPP- 02

Topic : $t-test\ for\ one\ sample$

Q. 1 *A sample of 20 items has mean 42 units and S. D. 5 units. Test the hypothesis that it is a random sample from a normal population with mean 45 units.*

Q. 2 *Ten individuals are chosen at random from a normal population of students and their marks are found to be 63, 63, 66, 67, 68, 69, 70, 70, 71, 71. In the light of these data, discuss the that mean mark of the population of students is 66.*

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