ENGINEERING MATHEMATICS

UNIT-5 : *Statistical Techniques – III*

(Testing a Hypothesis and Statistical Quality Control)

Lec-04

Today's Target

- $Z-test$
- PYQ
- DPP

Hypothesis Testing

Test of significance

(i) t − test

(ii) z − test

(iii) F − test

(iv) chi − square test

Z–Test is applicable when

	Population S.D (σ) is Known	Population S.D (σ) is Unknown
$n \leq 30$	z − test	t − test
$n > 30$	z − test	z − test

Z – Test

(1) $Z - test\ for\ single\ mean\ (one\ sample)$

(2) $Z - test\ for\ Difference\ of\ mean\ (two\ sample)$

(3) $Z - test\ for\ single\ proportion$

(4) $Z - test\ for\ difference\ of\ proportion$

(5) $Z - test\ for\ difference\ of\ standard\ deviation$

Z – Test Table

Level of Significance			
	1%	5%	10%
Two Tailed Test	$\|Z\| = 2.58$	$\|Z\| = 1.96$	$\|Z\| = 1.645$
Right Tailed Test	$Z = 2.33$	$Z = 1.645$	$Z = 1.28$
Left Tailed Test	$Z = -2.33$	$Z = -1.645$	$Z = -1.28$

Z–test for one sample

To test the significant difference between the sample mean and population mean.

Steps to solve Questions

STEP – 1 : Set Null Hypothesis (H_0)

$H_0 : \mu = \mu_0$ (given)

STEP – 2 : Alternative Hypothesis (H_1)

$H_1 : \mu \neq \mu_0$ (Two Tail test), $\mu > \mu_0$ (Right Tail test), $\mu < \mu_0$ (Left Tail test)

Step – 3 : Calculate Z

$$Z_{cal} = \frac{\bar{x} - \mu}{\frac{S}{\sqrt{n}}} \quad OR \quad Z_{cal} = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

$\bar{x}$ = Sample mean

μ = Population mean

n = Sample size

σ = Population S.D

S = Sample S.D

Step – 4 : From the Z – test table

Find the tabulated value Z_{tab} *at the given level of significance* $\propto$

Step – 5 : Conclusion

(i) If $Z_{cal} < Z_{tab}$ *then* H_0 *is accepted*

(ii) If $Z_{cal} > Z_{tab}$ *then* H_0 *is rejected*

Confidence limits or fiducial limits

$$\bar{x} - z_{\propto}\frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + z_{\propto}\frac{\sigma}{\sqrt{n}}$$

(i) 95% *confidence limits (level of significance is* 5%*) are* $\bar{x} - 1.96\frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + 1.96\frac{\sigma}{\sqrt{n}}$

(i) 99% *confidence limits (level of significance is* 1%*) are* $\bar{x} - 2.58\frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + 2.58\frac{\sigma}{\sqrt{n}}$

Q.1 *A random sample of 900 members has a mean 3.4 cms. Can it be reasonably regarded as a sample from a large population of mean 3.2 cms and S.D. 2.3 cms?*

Given

$n = 900$

$\bar{x} = 3.4$

$\mu = 3.2$

$\sigma = 2.3$

Null Hypothesis (H_0)

Sample is drawn from large population with mean 3.2

$H_0: \mu = 3.2$

Alternative Hypothesis (H_1)

$H_1: \mu \neq 3.2$ (two tailed test)

Calculation of $|Z|$

$$Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

$$Z = \frac{3.4 - 3.2}{\frac{2.3}{\sqrt{900}}}$$

$$Z = \frac{0.2 \times 30}{2.3}$$

$$Z_{cal} = 2.61$$

From Z-test table

At the level of significance 5%

$$Z_{tab} = 1.96$$

conclusion

$Z_{cal} = 2.61$

$Z_{tab} = 1.96$

$\therefore |Z_{cal}| > |Z_{tab}|$

$\therefore H_0$ is rejected

Hence

H_1 is accepted

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$Z-Test\ for\ two\ sample$

$To\ test\ the\ significant\ difference\ between\ mean\ of\ two\ large\ independent\ samples.$

Step to sovle Questions

STEP – 1: $Set\ Null\ Hypothesis\ (H_0)$

$$H_0: \mu_1 = \mu_2$$

STEP – 2: $Alternative\ Hypothesis(\ H_1)$

$$H_1 :\ \mu \neq \mu_0(Two\ Tail\ test),\ \mu > \mu_0(Right\ Tail\ test),\ \mu < \mu_0(Left\ Tail\ test)$$

STEP – 3: $Calculate\ |\ z\ |$

$$Z_{cal} = \frac{\bar{x}-\bar{y}}{\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}} \quad OR \quad Z_{cal} = \frac{\bar{x}-\bar{y}}{\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}}$$

where $\bar{x}$ = Mean of Sample-1

$\bar{y}$ = Mean of Sample-2

$$\sigma^2 = \frac{n_1 S_1^2 + n_2 S_2^2}{n_1 + n_2}$$

STEP – 4 : From the Z – test table

Find the tabulated value Z_{tab} *at the level of significance* $\propto$

STEP – 5: Conclusion

(*i*) *If* $Z_{cal} < Z_{tab}$ *then* H_0 *is accepted*

(*ii*) *If* $Z_{cal} > Z_{tab}$ *then* H_0 *is rejected*

Q.2 *The average income of persons was Rs. 210 with a S.D. of Rs. 10 in sample of 100 people of a city. For another sample of 150 persons, the average income was Rs. 220 with S.D. of Rs. 12. The S.D. of incomes of the people of the city was Rs. 11. Test whether there is any significant difference between the average incomes of the localities.*

Given

Sample-1

$n_1 = 100$

$\bar{x} = 210$

$s_1 = 10$

Sample-2

$n_2 = 150$

$\bar{y} = 220$

$s_2 = 12$

Null Hypothesis (H_0)

H_0: There is no significant difference between the average income of the localities

$H_0: \bar{x} = \bar{y}$

Alternative Hypothesis (H_1)

$H_1: \bar{x} \neq \bar{y}$

Calculation of $|Z_{cal}|$

$$Z = \frac{\bar{x} - \bar{y}}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$$z = \frac{210 - 220}{\sqrt{\frac{10^2}{100} + \frac{12^2}{150}}}$$

$$z = -7.1428$$

$$\boxed{|z_{cal}| = 7.1428}$$

From z-test table

At the level of significance 5%

$$\boxed{z_{tab} = 1.96}$$

Conclusion

$\because |z_{cal}| > |z_{tab}|$

$\therefore H_0$ is rejected

Hence

H_1 is accepted

Z–Test for one single proportion

To test the significant difference between Proportion of the sample and the population.

Let X be the number of successes in n independent trials with constant probability P of success for each trial.

Steps to solve Questions

STEP – 1 : Set Null Hypothesis (H_0)

$$H_0 : \mu = \mu_0 \ (given)$$

STEP – 2 : Alternative Hypothesis (H_1)

$$H_1 : \mu \neq \mu_0$$

Step – 3 : Calculate Z

$$Z = \frac{p - P}{\sqrt{\frac{PQ}{n}}}$$

Where

Observed Proportion of Success $p = \frac{x}{n}$

x = number of success

n = Sample Size

P = Probability of success

$Q = 1 - P$ (Probability of failure)

Step – 5: From the Z – test table

Find the tabulated value Z_{tab} *at the level of significance* $\propto$

Step – 6: Conclusion

(i) $If Zca_l < Z_{tab}$ *then* H_0 *is accepted.*

(ii) $If Zca_l > Z_{tab}$ *then* H_0 *is rejected*

95% *confidence limits (level of significance is* 5%*) are* $P \pm Z_{tab}\sqrt{\frac{PQ}{n}}$

99% *confidence limits (level of significance is* 1%*) are* $P \pm Z_{tab}\sqrt{\frac{PQ}{n}}$

Q.3 *A coin was tossed 400 times and the head turned up 216 times. Test the hypothesis that the coin is unbiased.*

Given

$n = 400$

$P = 0.5$

$Q = 1 - 0.5$

$Q = 0.5$

$x = 216$

$p = \frac{x}{n}$

$p = \frac{216}{400}$

$p = 0.54$

Null Hypothesis (H_0)

H_0: The coin is unbiased

$H_0: P = 0.5$

Alternative Hypothesis (H_1)

$H_1: P \neq 0.5$ (two tailed test)

calculation of $|Z|$

$$Z = \frac{p - P}{\sqrt{\frac{PQ}{n}}} = \frac{0.54 - 0.5}{\sqrt{\frac{0.5 \times 0.5}{400}}}$$

$Z = 1.6$

$$|Z_{cal}| = 1.6$$

From z-test table

At the level of singnificance 5%

$|z_{tab}| = 1.96$

conclusion

As $|z_{cal}| < |z_{tab}|$

$\therefore$ H_0 is accepted

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Z–test for difference of proportion

To test the significant difference between the sample proportions p_1 and p_2

Step to sovle Questions

STEP – 1: Set Null Hypothesis (H_0)

$$H_0: \mu_1 = \mu_2$$

STEP – 2: Alternative Hypothesis(H_1)

$$H_1: \mu_1 \neq \mu_2$$

STEP – 3: Calculation of $|z|$

$$Z = \frac{p_1 - p_2}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

Where $p_1 = \frac{x_1}{n_1}$

$p_2 = \frac{x_2}{n_2}$

$$Where\ P = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2}$$

$$Q = 1 - P$$

GW
GATEWAY CLASSES

GW
GATEWAY CLASSES

STEP – 5 : From the Z – test table

Find the tabulated value Z_{tab} *at the level of significance* $\propto$

STEP – 6: Conclusion

(*i*) *If* $Z_{cal} < Z_{tab}$ *then* H_0 *is accepted*

(*ii*) *If* $Z_{cal} > Z_{tab}$ *then* H_0 *is rejected*

Q.4 *Before an increase in excise duty on tea, 800 people out a sample of 100 persons were found to be tea drinkers. After an increase in the duty, 800 persons were known to be tea drinkers in a sample of 1200 people. Do you think that there has been a significant decrease in the consumption of tea after the increase in the exise duty?*

Given

$n_1 = 1000$

$x_1 = 800$

$n_2 = 1200$

$x_2 = 800$

$$p_1 = \frac{x_1}{n_1} = \frac{800}{1000}$$

$$p_1 = 0.8$$

$$p_2 = \frac{x_2}{n_2} = \frac{800}{1200}$$

$$p_2 = 2/3$$

$$p_2 = 0.67$$

$$P = \frac{p_1 n_1 + p_2 n_2}{n_1 + n_2} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{800 + 800}{1000 + 1200}$$

$$P = \frac{8}{11}$$

$$Q = 1 - P = 1 - \frac{8}{11}$$

$$Q = \frac{3}{11}$$

GW GATEWAY CLASSES

Null Hypothesis (H_0)

H_0: There is no significant decrease in the consumption of tea after increase in the excise duty

$H_0: P_1 = P_2$

Alternative Hypothesis (H_1)

$H_1: P_1 > P_2$ (one tailed test)

calculation of $|Z|$

$$Z = \frac{P_1 - P_2}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$Z = \frac{0.8 - 0.67}{\sqrt{\frac{8}{11} \times \frac{3}{11}\left(\frac{1}{1000} + \frac{1}{1200}\right)}}$$

$$Z = 6.817$$

$$\boxed{|Z_{cal}| = 6.817}$$

From t-test table

At the level of significance 5%

$|Z_{tab}| = 1.645$

conclusion

As $|Z_{cal}| > |Z_{tab}|$

$Z-$ test for difference of standard deviations

To test the significant difference between the standard deviation of two independent samples.

Step to sovle Questions

STEP – 1: Set Null Hypothesis (H_0)

$$H_0: \mu_1 = \mu_2$$

STEP – 2: Alternative Hypothesis(H_1)

$$H_1: \mu_1 \neq \mu_2$$

STEP – 3: Calculation of $|z|$

$$Z = \frac{s_1 - s_2}{\sqrt{\frac{{\sigma_1}^2}{2n_1} + \frac{{\sigma_2}^2}{2n_2}}} \qquad OR \qquad Z = \frac{s_1 - s_2}{\sqrt{\frac{{s_1}^2}{2n_1} + \frac{{s_2}^2}{2n_2}}}$$

$STEP-5$: *From the* $Z-$ *test table*

Find the tabulated value Z_{tab} *at the level of significance* $\propto$

$STEP-6$: *Conclusion*

(i) *If* $Z_{cal} < Z_{tab}$ *then* H_0 *is accepted*

(ii) *If* $Z_{cal} > Z_{tab}$ *then* H_0 *is rejected*

Q.5 *Random samples drawn from two countries gave the following data relating to the heights of adult males :*

	country A	*country B*
Mean height(in inche	67.42	67.25
Standard deviation	2.58	2.50
number in samples	1000	1200

Is the difference between the standard deviation significant?

Given

$n_1 = 1000$

$n_2 = 1200$

$s_1 = 2.58$

$s_2 = 2.50$

Null Hypothesis (H_0)

H_0: There is no significant difference between the standard deviation

$H_0: s_1 \neq s_2$ (Two tailed test)

Alternative Hypothesis(H_1)

$H_1 : S_1 \neq S_2$

Calculation of z

$$z = \frac{S_1 - S_2}{\sqrt{\frac{S_1^2}{2n_1} + \frac{S_2^2}{2n_1}}}$$

$$z = \frac{2.58 - 2.50}{\sqrt{\frac{2.58^2}{2 \times 1000} + \frac{2.50^2}{2 \times 1200}}}$$

$z = 1.0387$

$\boxed{|z_{cal}| = 1.0387}$

From z-test table

At the level of significance 5%.

$|z_{tab}| = 1.96$

conclusion

As $|z_{cal}| < |z_{tab}|$

$\therefore H_0$ is accepted

DPP-4

Topic : Z - Test

Q .1 – A machine is producing bolts of which a certain fraction is defective. A random sample of 400 is taken from a large batch and is found to contain 30 defective bolts. Does this indicate that the proportion of defectives is larger than that claimed by the manufacturer where the manufacturer claims that only 5% of his product are defective. Find 95% confifence limits of the proportion of defective bolts in batch.

Q .2 – A machine produced 16 defective articles in a batch of 500. After overhauling it produced 3 defectives in a batch of 100. Has the machine improved?

Q .3 – The mean weight obtained from aa random sample of size 100 is 64 gms. The S.D. of the weight distribution of the population is 3 gms. Test the statement that the mean weight of the pupulation is 67 gms at 5% level of significance. Also set up 99% confidence limits of the mean weight of the pupulation.

Q .4 – Intelligence tests were given to two group of boys and girls.

	Mean	S.D.	Size
Girls	75	8	60
Boys	73	10	100

Examine if the difference mean score is significant