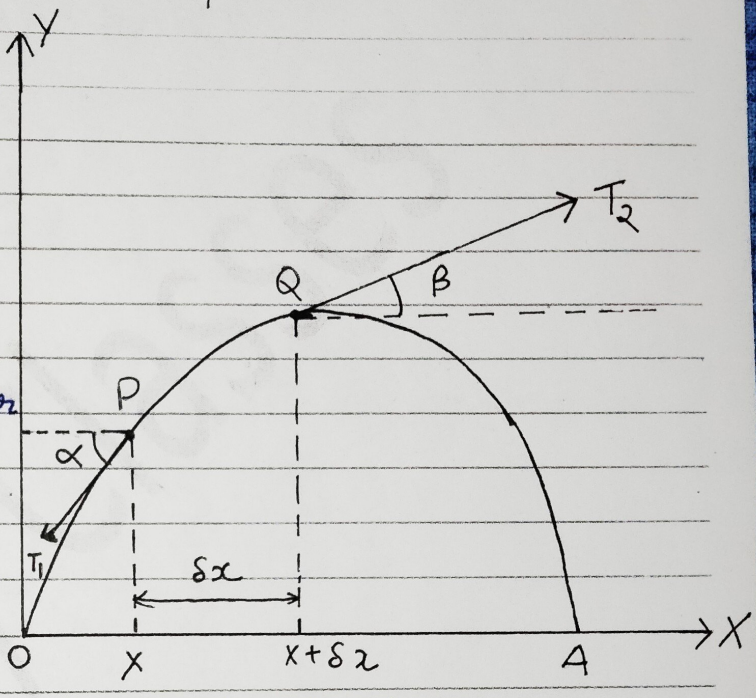
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DPP-3 Maths-IV Unit-2

Topic- Solution of one Dimensional Wave Equation

Que: Derive one dimensional wave equation.

Sol: Consider a uniform elastic string of length l stretched tightly between two points O and A and displaced slightly from its equilibrium position OA. Taking the O as the origin, OA as the x axis and a perpendicular line through O as the y-axis, we shall find the displacement y as a function of a distance x and the time t.



Let m be the mass per unit length of the string. Consider the motion of an element PQ of length δs. Since the string doesn't offer resistance to bending, the tensions T_1 and T_2 at P and Q respectively are tangential to the curve.

Since, there is no motion in the horizontal direction, we have

$$T_1 \cos\alpha = T_2 \cos\beta = T \text{ (constant)}$$

Mass of element PQ is $m\delta s$. By Newton's second law of motion, the equation of motion in the vertical direction is-

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$$m\delta s \frac{\delta^2 y}{\delta t^2} = T_2 \sin\beta - T_1 \sin\alpha$$

$$\frac{m\delta s}{T} \frac{\delta^2 y}{\delta t^2} = \frac{T_2 \sin\beta}{T_2 \cos\beta} - \frac{T_1 \sin\alpha}{T_1 \cos\alpha} \qquad \text{From eq(1)}$$

$$\frac{\delta^2 y}{\delta t^2} = \frac{T}{m\delta s} (\tan\beta - \tan\alpha)$$

$$\frac{\delta^2 y}{\delta t^2} = \frac{T}{m\delta x} \left[\left(\frac{\delta y}{\delta x}\right)_{x+\delta x} - \left(\frac{\delta y}{\delta x}\right)_x \right]$$

[Since $\delta s = \delta x$ to a first approximation and $\tan\alpha$ and $\tan\beta$ are the slopes of the curve of string at x and $x+\delta x$]

$$\frac{\delta^2 y}{\delta t^2} = \frac{T}{m} \left[\frac{\left(\frac{\delta y}{\delta x}\right)_{x+\delta x} - \left(\frac{\delta y}{\delta x}\right)_x}{\delta x} \right]$$

$$\frac{\delta^2 y}{\delta t^2} = \frac{T}{m} \frac{\delta^2 y}{\delta x^2}, \qquad \text{as } \delta x \to 0$$

$$\boxed{\frac{\delta^2 y}{\delta t^2} = c^2 \frac{\delta^2 y}{\delta x^2}} \qquad \text{where } c^2 = \frac{T}{m}$$

This is the partial differtial equation giving the transverse vibrations of the string. It is also called the one dimmensional wave equation.

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