UNIT-3 Statistical Techniques-I

Measure of Central tendency

(An average is a measures of central tendency of data)

There are 5 types of averages in common use

1. Airthmetic Mean or Mean
2. Median
3. Mode
4. Geometric Mean
5. Harmonic Mean

Data

- Ungrouped Data
  - (i) Individual Series
  - (ii) Discrete Series
- Grouped data
  - (i) Continuous Series
  - (ii) Discontinuous Series

Mean

Methods to find Mean:-

(i) Direct Method
(ii) Shortcut Method or Assumed mean method
(iii) Step Deviation method

For individual series (Where frequency are not given)

Let $x_1, x_2, x_3 \ldots x_n$ are n observation

(i) Direct Method :-

$$\text{Mean} = \frac{\text{Sum of Observations}}{\text{Number of observations}}$$

$$\bar{x} = \frac{x_1 + x_2 + x_3 + \cdots + x_n}{n}$$

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$$\bar{x} = \frac{\Sigma x}{n}$$

Q.1. The weights (in kilogram) of 5 persons are 67, 65, 71, 57 and 45. Find the mean of their weights.

Soln:-

Given:-

$x_1 = 67$, $x_2 = 65$, $x_3 = 71$, $x_4 = 57$, $x_5 = 45$ and $n = 5$

Findout:-

mean

$$\bar{x} = \frac{\Sigma x}{n}$$

$$\bar{x} = \frac{67 + 65 + 71 + 57 + 45}{5}$$

$$\bar{x} = 61 \text{ kg}$$

For discrete Series (Where frequency is given)

Let $x_1, x_2, x_3, \cdots x_n$ are n observation with respective frequencies $f_1, f_2, f_3 \cdots f_n$

Method used to find Mean

(i) Direct Method:-

$$\bar{x} = \frac{f_1x_1 + f_2x_2 + \cdots + f_nx_n}{f_1 + f_2 + f_3 + \cdots + f_n}$$

$$\bar{x} = \frac{\Sigma fx}{\Sigma f}$$

Also $\Sigma f = N$

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(ii) Shortcut Method or Assumed mean Method :-

$$\bar{x} = A + \frac{\Sigma f d}{\Sigma f}$$

Also $\Sigma f = N$

Where

A = Assume Mean

Deviation (d) = $x - A$

(iii) Step Deviation Method :-

$$\bar{x} = A \times h \times \frac{\Sigma f u}{\Sigma f}$$

Also $\Sigma f = N$

Where

A = Assume mean

h = Common factor

$u = \frac{x - A}{h}$

Q.2 Find the mean of the following frequency distribution of marks in a test in mathematics.

Marks	(x)	10	20	30	40	50	60	70	80
No. of Students	(f)	3	6	10	12	9	6	2	2

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By direct Method :-

Marks(x)	No. of Students (f)	fx
10	3	30
20	6	120
30	10	300
40	12	480
50	9	450
60	6	360
70	2	140
80	2	160
	$\Sigma f = 50$	$\Sigma fx = 2040$

$$\bar{x} = \frac{\Sigma fx}{\Sigma f}$$

$$\bar{x} = \frac{2040}{50}$$

$$\boxed{\bar{x} = 40.8}$$

By Assumed mean method

Marks (x)	No. of Students (f)	$d = x - A$	fd
10	3	-30	-90
20	6	-20	-120
30	10	-10	-100
A → 40	12	0	0
50	9	10	90
60	6	20	120
70	2	30	60
80	2	40	80
	$\Sigma f = 50$		$\Sigma fd = 40$

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$$\bar{x} = A + \frac{\Sigma fd}{\Sigma f}$$

$$\bar{x} = 40 + \frac{40}{50}$$

$$\bar{x} = 40 + 0.8$$

$$\boxed{\bar{x} = 40.8}$$

By Step Deviation Method :-

Marks (x)	No. of Students (f)	$=x-A$	$u=\frac{x-A}{h}$	fu
10	3	-30	-3	-9
20	6	-20	-2	-12
30	10	-10	-1	-10
40	12	0	0	0
50	9	10	1	9
60	6	20	2	12
70	2	30	3	6
80	2	40	4	8
	$\Sigma f = 50$			$\Sigma fu = 4$

$$\bar{x} = A + h \times \frac{\Sigma fu}{\Sigma f}$$

$$\bar{x} = 40 + 10 \times \frac{4}{50}$$

$$\bar{x} = 40 + 0.8$$

$$\boxed{\bar{x} = 40.8}$$

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For Continuous Series (Where frequency and class interval are given)

Let $x_1, x_2, x_3, \ldots x_n$ are n observation with respective frequencies $f_1, f_2, f_3, \ldots f_n$.

Note:- x = (mid value) = $\frac{\text{Lower limit of class interval} + \text{Upper limit of class interval}}{2}$

Method used to find Mean

(i) Direct Method :-

$$\bar{x} = \frac{f_1x_1 + f_2x_2 + \cdots + f_nx_n}{f_1 + f_2 + f_3 + \cdots + f_n}$$

$$\boxed{\bar{x} = \frac{\Sigma fx}{\Sigma f}}$$

Also $\Sigma f = N$

(ii) Shortcut Method or Assumed mean Method :-

$$\boxed{\bar{x} = A + \frac{\Sigma fd}{\Sigma f}}$$

Also $\Sigma f = N$

where,

A = Assume Mean

$d = x - A$

(iii) Step Deviation Method :-

$$\boxed{\bar{x} = A + h \times \frac{\Sigma fu}{\Sigma f}}$$

Also $\Sigma f = N$

where, h = width of class interval, $u = \frac{x - A}{h}$

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Q.3 Find the mean of the following frequency distribution.

Class	5-15	15-25	25-35	35-45	45-55	55-65
Frequency	10	20	25	20	15	10

By Direct Method :-

Class (I)	Frequency (f)	Mid Value (x)	fx
5-15	10	10	100
15-25	20	20	400
25-35	25	30	750
35-45	20	40	800
45-55	15	50	750
55-65	10	60	600
	$\Sigma f = 100$		$\Sigma fx = 3400$

$$\bar{x} = \frac{\Sigma fx}{\Sigma f}$$

$$\bar{x} = \frac{3400}{100}$$

$$\boxed{\bar{x} = 34}$$

By Assume Mean Method :-

Class	Frequency (f)	Mid Value (x)	$d = x - A$	fd
5-15	10	10	-20	-200
15-25	20	20	-10	-200
25-35	25	A ↖ 30	0	0
35-45	20	40	10	200
45-55	15	50	20	300
55-65	10	60	30	300
	$\Sigma f = 100$			$\Sigma fd = 400$

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$$\bar{x} = A + \frac{\Sigma fd}{\Sigma f}$$

$$\bar{x} = 30 + \frac{400}{100}$$

$$\boxed{\bar{x} = 34}$$

By Step Deviation Method :-

Class	Frequency(f)	Mid Value (x)	$u = \frac{x-A}{h}$	fu
5-15	10	10	-2	-20
15-25	20	20	-1	-20
25-35	25	30	0	0
35-45	20	40	1	20
45-55	15	50	2	30
55-65	10	60	3	30
	$\Sigma f = 100$			$\Sigma fu = 40$

$$\bar{x} = A + h \times \frac{\Sigma fu}{\Sigma f}$$

$$\bar{x} = 30 + 10 \times \frac{40}{100}$$

$$\boxed{\bar{x} = 34}$$

Moments

Moment of distribution are arithematic mean of various powers of deviation from some given numbers.

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Moments

- Moment about mean (M_r)
  - (i) For frequency distribution
  - (ii) For individual series
- Moment about any arbitrary number (M'_r)
  - (i) For frequency distribution
  - (ii) For individual series
- Moment about origin (ϑ_r)
  - (i) For frequency distribution
  - (ii) For individual series

Moment about mean (M_r)	Moment about arbitrary number (M'_r)	Moment about origin (ϑ_r)
For frequency distribution	For frequency distribution	For frequency distribution
If $x_1, x_2, x_3 \dots x_n$ are n observations with respective frequency $f_1, f_2, f_3 \dots f_n$ respectively	If $x_1, x_2, x_3 \dots x_n$ are n observations with respective frequency $f_1, f_2, f_3 \dots f_n$ respectively	If $x_1, x_2, x_3 \dots x_n$ are n observations with respective frequency $f_1, f_2, f_3 \dots f_n$ Respectively
Then the r^{th} moment M_r about mean $\bar{x}$ is defined as	Then the r^{th} moment M'_r about any arbitrary no. is defined as	Then the r^{th} moment ϑ_r about origin is defined as
$M_r = \frac{\sum f (x-\bar{x})^r}{\sum f}$	$M'_r = \frac{\sum f (x-A)^r}{\sum f}$	$\vartheta_r = \frac{\sum f x}{\sum f}$
where $r = 0, 1, 2, 3, 4 \dots$	where $r = 0, 1, 2, 3, 4 \dots$	where $r = 0, 1, 2, 3, 4 \dots$
(i) $M_0 = 1$	$M'_0 = 1$	$\vartheta_0 = 1$

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	Moment about mean (M_x)	Moment about any arbitrary no. (M'_x)	Moment about origin (ν_x)
(ii)	$M_1 = 0$	$M'_1 = \bar{x} - A$	$\nu_1 = \bar{x}$
(iii)	$M_2 = \frac{\Sigma f (x-\bar{x})^2}{\Sigma f}$	$M'_2 = \frac{\Sigma f (x-A)^2}{\Sigma f}$	$\nu_2 = \frac{\Sigma f x^2}{\Sigma f}$
(iv)	$M_3 = \frac{\Sigma f (x-\bar{x})^3}{\Sigma f}$	$M'_3 = \frac{\Sigma f (x-A)^3}{\Sigma f}$	$\nu_3 = \frac{\Sigma f x^3}{\Sigma f}$
(v)	$M_4 = \frac{\Sigma f (x-\bar{x})^4}{\Sigma f}$	$M'_4 = \frac{\Sigma f (x-A)^4}{\Sigma f}$	$\nu_4 = \frac{\Sigma f x^4}{\Sigma f}$

Moment about mean (M_x)	Moment about any arbitrary no. (M'_x)	Moment about origin (ν_x)
For individual series	For individual series	For individual series
If $x_1, x_2, x_3 \cdots x_n$ are n observations. Then the x^{th} moment M_x about mean $\bar{x}$ is defined as	If $x_1, x_2, x_3 \cdots x_n$ are n observations. Then the x^{th} moment M'_x about any arbitrary no. is defined as	If $x_1, x_2, x_3 \cdots x_n$ are n observations. Then the x^{th} moment ν_x about origin (0,0) is defined as
Remove $\rightarrow f$	Remove $\rightarrow f$	Remove $\rightarrow f$
$\Sigma f = n$ (no. of observation)	$\Sigma f = n$	$\Sigma f = n$

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	Moment about mean (M_x)	Moment about any arbitrary no. (M'_x)	Moment about origin (ν_x)
(ii)	$M_1 = 0$	$M'_1 = \bar{x} - A$	$\nu_1 = \bar{x}$
(iii)	$M_2 = \frac{\Sigma f (x-\bar{x})^2}{\Sigma f}$	$M'_2 = \frac{\Sigma f (x-A)^2}{\Sigma f}$	$\nu_2 = \frac{\Sigma f x^2}{\Sigma f}$
(iv)	$M_3 = \frac{\Sigma f (x-\bar{x})^3}{\Sigma f}$	$M'_3 = \frac{\Sigma f (x-A)^3}{\Sigma f}$	$\nu_3 = \frac{\Sigma f x^3}{\Sigma f}$
(v)	$M_4 = \frac{\Sigma f (x-\bar{x})^4}{\Sigma f}$	$M'_4 = \frac{\Sigma f (x-A)^4}{\Sigma f}$	$\nu_4 = \frac{\Sigma f x^4}{\Sigma f}$

Moment about mean (M_x) For individual series	Moment about any arbitrary no. (M'_x) For individual series	Moment about origin (ν_x) For individual series
If $x_1, x_2, x_3 \cdots x_n$ are n observations. Then the x^{th} moment M_x about mean $\bar{x}$ is defined as	If $x_1, x_2, x_3 \cdots x_n$ are n observations. Then the x^{th} moment M'_x about any arbitrary no. is defined as	If $x_1, x_2, x_3 \cdots x_n$ are n observations. Then the x^{th} moment ν_x about origin (0,0) is defined as
Remove $\rightarrow f$	Remove $\rightarrow f$	Remove $\rightarrow f$
$\Sigma f = n$ (no. of observation)	$\Sigma f = n$	$\Sigma f = n$

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Q.1 Find the first four moments for the following individual series:

x	3	6	8	10	18

Solution:-

S.NO	x	$(x-\bar{x})$	$(x-\bar{x})^2$	$(x-\bar{x})^3$	$(x-\bar{x})^4$
1	3	-6	36	-216	1296
2	6	-3	9	-27	81
3	8	-1	1	-1	1
4	10	1	1	1	1
5	18	9	81	729	6561
n = 5	$\Sigma x = 45$	$\Sigma(x-\bar{x}) = 0$	$\Sigma(x-\bar{x})^2 = 128$	$\Sigma(x-\bar{x})^3 = 486$	$\Sigma(x-\bar{x})^4 = 7940$

Mean $(\bar{x})$:- $\bar{x} = \frac{\Sigma x}{n}$

$\bar{x} = \frac{45}{5}$

$\boxed{\bar{x} = 9}$

Moments

$\boxed{M_1 = 0}$

$M_2 = \frac{\Sigma(x-\bar{x})^2}{n}$

$M_2 = \frac{128}{5}$

$\boxed{M_2 = 25.6}$

$M_3 = \frac{\Sigma(x-\bar{x})^3}{n}$

$M_3 = \frac{486}{5}$

$\boxed{M_3 = 97.2}$

$M_4 = \frac{\Sigma(x-\bar{x})^4}{n}$

$M_4 = \frac{82134}{5}$

$\boxed{M_4 = 1588}$

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Q2 The no. of flowers on sun-flower plants are given below:

No. of flowers (x)	3	6	12	16	25
No. of plants (f)	1	2	3	4	5

Calculate the first four moments about the mean

x	f	fx	$(x-\bar{x})$	$f(x-\bar{x})$	$f(x-\bar{x})^2$	$f(x-\bar{x})^3$	$f(x-\bar{x})^4$
3	1	3	-13	-13	169	-2197	28561
6	2	12	-10	-20	200	-2000	20000
12	3	36	-4	-12	48	-192	768
16	4	64	0	0	0	0	0
25	5	125	9	45	405	3645	32805
	$\Sigma f =$ 15	$\Sigma fx =$ 240		$\Sigma f(x-\bar{x}) = 0$	$\Sigma f(x-\bar{x})^2 =$ 822	$\Sigma f(x-\bar{x})^3 =$ -744	$\Sigma f(x-\bar{x})^4 =$ 82134

Mean ($\bar{x}$)

$$\bar{x} = \frac{\Sigma fx}{\Sigma f}$$

$$\bar{x} = \frac{240}{15}$$

$$\boxed{\bar{x} = 34}$$

Moments

$$M_1 = 0$$

$$M_2 = \frac{\Sigma f (x-\bar{x})^2}{\Sigma f}$$

$$M_2 = \frac{822}{15}$$

$$\boxed{M_2 = 54.8}$$

$$M_3 = \frac{\Sigma f (x-\bar{x})^3}{\Sigma f}$$

$$M_3 = \frac{-744}{15}$$

$$\boxed{M_3 = -49.6}$$

$$M_4 = \frac{\Sigma f (x-\bar{x})^4}{\Sigma f}$$

$$M_4 = \frac{82134}{15}$$

$$\boxed{M_4 = 5475.6}$$

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Q.3 Calculate M_1, M_2, M_3, M_4 for the following frequency distribution

Marks	5-15	15-25	25-35	35-45	45-55	55-65
No. of Students	10	20	25	20	15	10

Marks	frequency (f)	Mid Value (x)	fx	$(x-\bar{x})$	$f(x-\bar{x})$	$f(x-\bar{x})^2$	$f(x-\bar{x})^3$	$f(x-\bar{x})^4$
5-15	10	10	100	-24	-240	5760	-138240	3317760
15-25	20	20	400	-14	-280	3920	-54880	768320
25-35	25	30	750	-4	-100	400	-1600	6400
35-45	20	40	800	6	120	720	4320	25920
45-55	15	50	750	16	240	3840	61440	983040
55-65	10	60	600	26	260	6760	175760	4569760
	$\Sigma f = 100$	Σ	$\Sigma fx =$ 3400		$\Sigma f(x-\bar{x}) = 0$	$\Sigma f(x-\bar{x})^2 =$ 21400	$\Sigma f(x-\bar{x})^3 =$ 46800	$\Sigma f(x-\bar{x})^4 =$ 9671200

Mean $(\bar{x})$

$$\bar{x} = \frac{\Sigma f x}{\Sigma f}$$

$$\bar{x} = \frac{3400}{100}$$

$$\boxed{\bar{x} = 34}$$

Moments

$$\boxed{M_1 = 0}$$

$$M_2 = \frac{\Sigma f(x-\bar{x})^2}{\Sigma f}$$

$$M_2 = \frac{21400}{100}$$

$$\boxed{M_2 = 214}$$

$$M_3 = \frac{\Sigma f(x-\bar{x})^3}{\Sigma f}$$

$$M_3 = \frac{46800}{100}$$

$$\boxed{M_3 = 468}$$

$$M_4 = \frac{\Sigma f(x-\bar{x})^4}{\Sigma f}$$

$$M_4 = \frac{9671200}{100}$$

$$\boxed{M_4 = 96712}$$

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Moments

- Moment about mean (M_r)
- Moment about any arbitrary no. (M'_r)
- Moment about origin (ν_r)

$M_0 = 1$
$M'_0 = 1$
$\nu_0 = 1$

$M_1 = 0$
$M'_1 = \bar{x} - A$
$\nu_1 = \bar{x}$

$$\text{Varience} = \frac{\sum f(x_i - \bar{x})^r}{N} = M_2$$

$$\boxed{\text{Varience} = M_2}$$

$$\text{Varience} = (S.D)^2 = \sigma^2$$

$$\boxed{SD = \sqrt{M_2}}$$

Relation between M_r and M'_r

$$\boxed{M_1 = 0}$$

$$\boxed{M_2 = M'_2 - M'^2_1}$$

$$\boxed{M_3 = M'_3 - 3M'_2M'_1 + 2M'^3_1}$$

$$\boxed{M_4 = M'_4 - 4M'_3M'_1 + 6M'_2M'^2_1 - 3M'^4_1}$$

Relation between M_r and ν_r

$$\boxed{\nu_1 = \bar{x}}$$

$$\boxed{\nu_2 = M_2 + \bar{x}^2}$$

$$\boxed{\nu_3 = M_3 + M_2\bar{x} + \bar{x}^3}$$

$$\boxed{\nu_4 = M_4 + M_3\bar{x} + 6M_2\bar{x}^2 + \bar{x}^4}$$

KARL PEARSON'S β AND γ COEFFICIENTS

$$\beta_1 = \frac{M_3^2}{M_2^3}$$

$$\beta_2 = \frac{M_4}{M_2^2}$$

→ β-Coefficient

$$\gamma_1 = +\sqrt{\beta_1}$$

$$\gamma_2 = \beta_2 - 3$$

→ γ-coefficient.

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Q.1 The first three moments of a distribution about the value 2 of the variable are 1, 16 and −40.

(i) Show that the mean is 3, variance is 15 and $M_3 = -86$.

(ii) Find the values of the first three moments about the origin

Given:- $A = 2$

$M'_1 = 1$, $M'_2 = 16$, $M'_3 = -40$

(i) we know that

$M'_1 = \bar{x} - A$

$\bar{x} = M_1' + A$

$\bar{x} = 1 + 2$

$\boxed{\bar{x} = 3}$

Variance $= M_2$

$= M'_2 - M'^2_1$

$= 16 - 1^2$

$\boxed{\text{Variance} = 15}$

(ii) we know that

$\nu_1 = \bar{x}$

$\boxed{\nu_1 = 3}$

$\nu_2 = M_2 + \bar{x}^2$

$\nu_2 = 15 + 3^2$

$\boxed{\nu_2 = 24}$

$\nu_3 = M_3 + 3M_2\bar{x} + \bar{x}^3$

$\nu_3 = -86 + 3 \times 15 \times 3 + 3^3$

$\boxed{\nu_3 = 76}$

Q.2 For a distribution, the mean is 10, variance is 16, γ_1 is 1 and β_2 is 4. Find the first four moments about the origin

Given:-

$\bar{x} = 10$, $M_2 = 16$, $\gamma_1 = 1$, $\beta_2 = 4$

we know that

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$\gamma_1 = +\sqrt{\beta_1}$

$1 = \sqrt{\beta_1}$

$\boxed{\beta_1 = 1}$

$\beta_1 = \frac{M_3^2}{M_2^3}$

$1 = \frac{M_3^2}{(16)^3}$

$M_3^2 = 1 \times (16)^3$

$M_3 = \sqrt{16 \times 16 \times 16}$

$M_3 = 16 \times 4$

$\boxed{M_3 = 64}$

Also

$\beta_2 = \frac{M_4}{M_2^2}$

$4 = \frac{M_4}{16^2}$

$\boxed{M_4 = 256 \times 4}$

$\boxed{M_4 = 1024}$

$\boxed{M_2 = 16}$

$\boxed{M_1 = 0}$

Q.3 The first four moments of a distribution about the value '4' of the variable are -1.5, 17, -30 and 108. Find the moments about mean, about the origin, β_1 and β_2. Also find the moments.

Given :-

$A = 4, M_1' = -1.5, M_2' = 17, M_3' = -30, M_4' = 108.$

Moment about mean

$\boxed{M_1 = 0}$

$M_2 = M_2' - M_1'^2$

$M_2 = 17 - (-1.5)^2$

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$$M_2 = 17 - 2.25$$

$$\boxed{M_2 = 14.75}$$

$$M_3 = M'_3 - 3M'_2M'_1 + 2M_1^3$$

$$M_3 = -30 - 3 \times 17 \times (-1.5) + 2(-1.5)^2$$

$$\boxed{M_3 = 39.75}$$

$$M_4 = M'_4 - 4M'_3M_1 + 6M'_2M'^2_1 - 3M'^4_1$$

$$M_4 = 108 - 4(-30)(-15) + 6(17)(-1.5)^2 - 3(-1.5)^4$$

$$\boxed{M_4 = 142.3125}$$

Moments about origin

$$\nu_1 = \bar{x}$$

$$\nu_2 = M'_1 + A$$

$$\nu_1 = -1.5 + 4$$

$$\boxed{\nu_1 = 2.5}$$

$$\nu_2 = M_2 + \bar{x}^2$$

$$\nu_2 = 14.75 + (2.5)^2$$

$$\boxed{\nu_2 = 21}$$

$$\nu_3 = M_3 + 3M_2\bar{x} + \bar{x}^3$$

$$\nu_3 = 39.75 + 3 \times 14.75 \times 2.5 + (2.5)^3$$

$$\boxed{\nu_3 = 166}$$

$$\nu_4 = M_4 + 4M_3\bar{x} + 6M_2\bar{x}^2 + \bar{x}^4$$

$$\nu_4 = 142.3125 + 4 \times 39.75 \times 2.5 + 6 \times 14.75 \times (2.5)^2 + (2.5)^4$$

$$\boxed{\nu_4 = 1132}$$

Karl Pearson's coefficient :–

$$\beta_1 = \frac{M_3^2}{M_2^3} = \frac{(39.75)^2}{(14.75)^3} = \boxed{0.4923775}$$

$$\beta_2 = \frac{M_4}{M_2^2} = \frac{142.3125}{14.75} = \boxed{0.65412}$$

Moment About A = 2

$$A = 2$$

$$M'_1 = \bar{x} - A$$

$$M'_1 = 2.5 - 2 = \boxed{0.5}$$

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$M_2 = M'_2 - M'^2_1$

$M'_2 = M_2 + M'^2_1$

$M'_2 = 14.75 + (0.5)^2$

$\boxed{M'_2 = 15}$

$M_3 = M'_3 - 3M'_2 M'_1 + 2M'^3_1$

$M'_3 = M_3 + 3M'_2 M'_1 + 2M'^3_1$

$M'_3 = 39.75 + 3 \times 15 \times 0.5 + 2 \times (0.5)^3$

$\boxed{M'_3 = 62}$

$M_4 = M'_4 - 4M'_3 M'_1 + 6M'_2 M'^2_1 - 3M'^4_1$

$M'_4 = M_4 + 4M'_3 M'_1 - 6M'_2 M'^2_1 + 3M'^4_1$

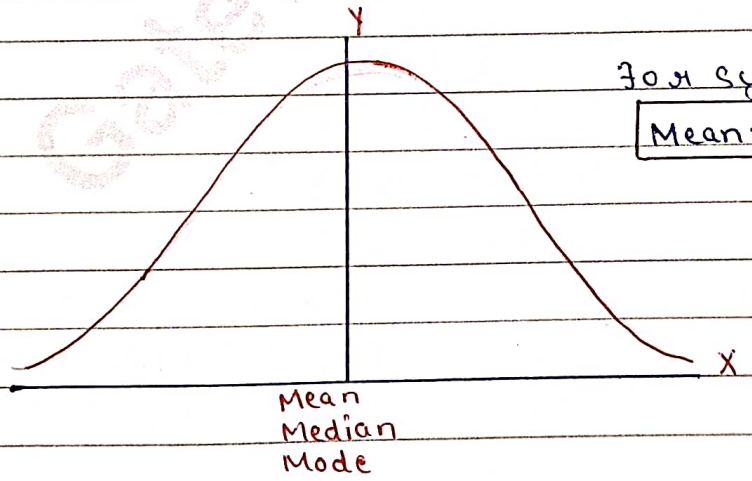
$M'_4 = 142.3125 + 4 \times 62 \times 0.5 - 6 \times 15 \times (0.5)^2 + 3 \times (0.5)^4$

$\boxed{M'_4 = 244}$

Skewness

Skewness means lack of symmetry in frequency distribution

1. Symmetrical Distribution



For Symmetrical Distribution

$\boxed{\text{Mean} = \text{Median} = \text{Mode}}$

Note:- If mean = Median = Mode, then there is no Skewness in the distribution.

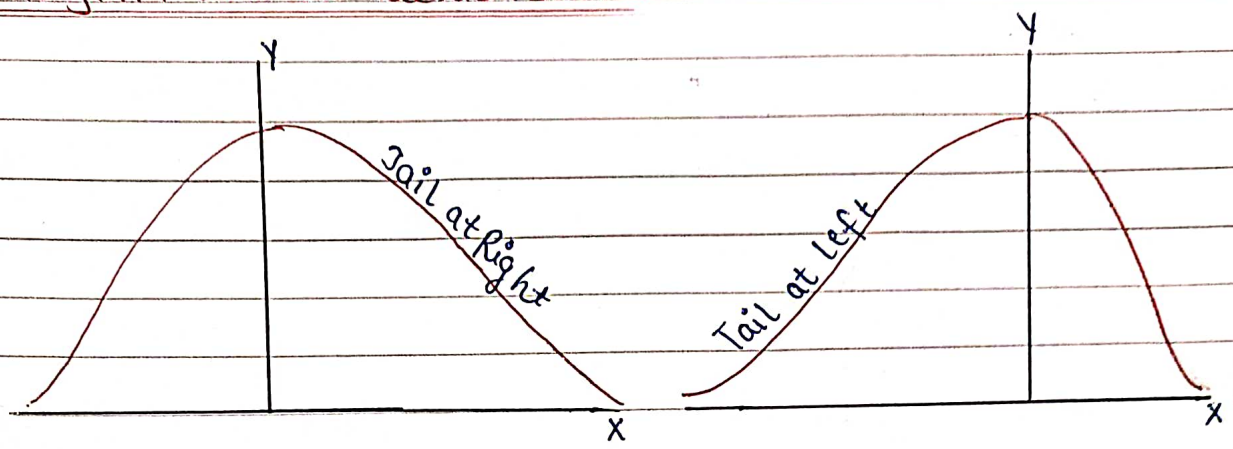
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2. Unsymmetrical Distribution



(i) Positively skewed Distribution
Mean > Mode

(ii) Negatively skewed Distribution
Mean < Mode

Methods of Measuring Skewness

(i) Karl Pearson's Method

(ii) Method of Moments

Note:- Relative measures of skewness are called coefficient of skewness.

Karl Pearson's Method :-

Karl Pearson's coefficient of skewness (Skp)

$$Skp = \frac{Mean - Mode}{S.D}$$

Mode = 3Median − 2Mean

$$Skp = \frac{3(Mean - Median)}{SD}$$

SD = Standard Deviation (σ)

Note (i) if Skp = 0
Mean = Mode
Distribution is symmetrical

(ii) if Skp > 0
Mean > Mode
Distribution is tively skewed

(iii) if Skp < 0
Mean < Mode
Distribution is -ively skewed.

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Standard Deviation

$$S.D = \sqrt{\text{Varience}}$$

$$\text{varience} = \mu_2$$

$$\boxed{S.D = \sqrt{\mu_2}}$$

Relation between μ_r and μ'_r

$$\mu_1 = 0$$

$$\mu_2 = \mu'_2 - \mu'^2_1$$

$$\mu_3 = \mu'_3 - 3\mu'_2\mu'_1 + 2\mu'^3_1$$

$$\mu_4 = \mu'_4 - 4\mu'_3\mu'_1 + 6\mu'_2\mu'^2_1 - 3\mu'^4_1$$

Q.1 For a moderately skewed data, the arithmetic mean is 100, the varience is 35 and Karl Pearson's coefficient of skewness is 0.2. Find its mode and median.

Given:-

Mean($\bar{x}$) = 100, Varience 35, Skp = 0.2

We know that

$$SD = \sqrt{\text{Variance}}$$

$$\boxed{SD = \sqrt{35}}$$

Karl pearson's coefficient of Skewness

$$Skp = \frac{\text{Mean} - \text{Mode}}{SD}$$

$$0.2 = \frac{100 - \text{Mode}}{\sqrt{35}}$$

$$\text{Mode} = 100 - 0.2 \times \sqrt{35}$$

$$\boxed{\text{Mode} = 98.816}$$

$$\text{Mode} = 3\,\text{Median} - 2\,\text{Mean}$$

$$98.816 = 3\,\text{Median} - 2 \times 100$$

$$\boxed{\text{Median} = 99.61}$$

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Q.2 The sum of 20 observations is 300 and sum of their squares is 5000. The median is 15. Find the karl Pearson's coefficient of skewness.

Given

$n = 20, \Sigma x = 300, \Sigma x^2 = 5000$, Median $= 15$

We know that

$$\text{Mean} = \frac{\Sigma x}{n}$$

$$\text{Mean} = \frac{300}{20}$$

$$\boxed{\text{Mean} = 15}$$

$$M_2 = D_2 - \bar{x}^2$$

$$M_2 = \frac{\Sigma x^2}{n} - \bar{x}^2$$

$$M_2 = \frac{5000}{20} - (15)^2$$

$$M_2 = 250 - 225$$

$$\boxed{M_2 = 25}$$

$$S.D = \sqrt{M_2} = \sqrt{25}$$

$$\boxed{S.D = 5}$$

$$Skp = \frac{3(\text{Mean} - \text{Median})}{S.D}$$

$$Skp = \frac{3(15-15)}{5}$$

$$\boxed{Skp = 0}$$

The distribution is symmetrical.

Methods of Moments :-

In this Method, Second and third central moments (M_2 & M_3) of the distribution are used.

This measure of skewness is called the moment coefficient of skewnes

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Moment coefficient of skewness (SKM) or γ_1

$$SK_M = \frac{M_3}{\sqrt{M_2^3}}$$

$$SK_M = \sqrt{\beta_1} = \gamma_1$$

Note:- We know that

$$\beta_1 = \frac{M_3^2}{M_2^3}$$

$$\sqrt{\beta_1} = \frac{M_3}{\sqrt{M_2^3}}$$

$$SK_M = \sqrt{\beta_1}$$

Q.3 The first three central moments of a distribution are 0, 15, -31. Find the moment coefficient of skewness

Given:-

$M_1 = 0$, $M_2 = 15$, $M_3 = -31$

Soln $SK_M = \frac{M_3}{\sqrt{M_2^3}}$

$$= \frac{-31}{\sqrt{(15)^3}}$$

$$SK_M = -0.53$$

The distribution is Negatively skewed.

Q.4 The first four moments of a distribution about the value 5 of the variable are 2, 20, 40 and 50. Calculate the moment coefficient of skewness.

Given $A = 5$, $M'_1 = 2$, $M'_2 = 20$, $M'_3 = 40$, $M'_4 = 50$

We know that

$$M_2 = M'_2 - M'^2_1$$

$$M_2 = 20 - (2)^2 = 16$$

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$$M_3 = M'_3 - 3M'_2M'_1 + 2M'^3_1$$

$$M_3 = 40 - 3\times 20 \times 2 + 2(2)^3$$

$$\boxed{M_3 = -64}$$

$$Sk_M = \frac{M_3}{\sqrt{M_2^3}}$$

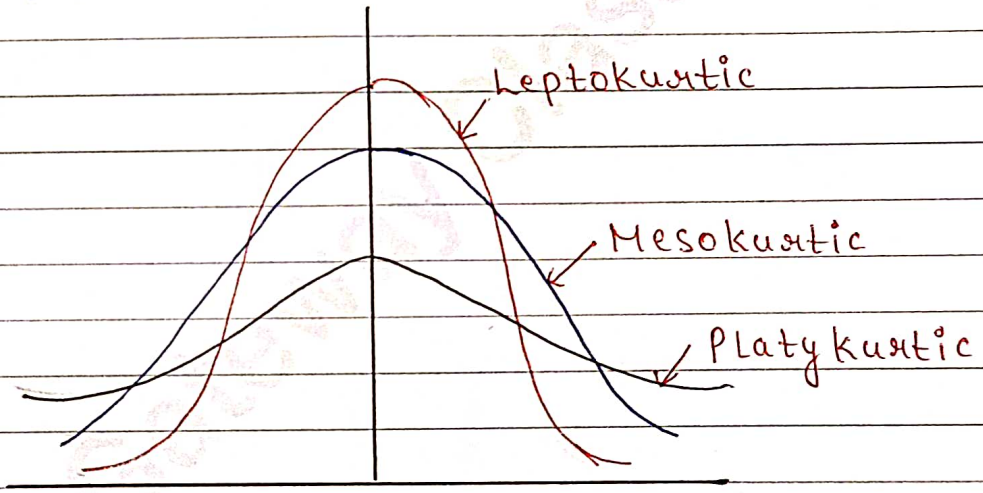
$$Sk_M = \frac{-64}{\sqrt{16^3}} = \frac{-64}{16\times 4}$$

$$\boxed{Sk_M = -1}$$

The distribution is Negatively skewed.

Kurotosis–

Measure of peakedness or flatness of a distribution.



Measure of Kurtosis:-

$$\boxed{\beta_2 = \frac{M_4}{M_2^2}}$$

Kurtosis of Distribution is also defined as

$$\boxed{\gamma_2 = \beta_2 - 3}$$

where M_2 and M_4 are the second and fourth moment about mean of the distribution.

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Note :-

(i) The distribution is Leptokurtic if

$$\beta_2 > 3$$
$$\gamma_2 > 3$$

(ii) The distribution is Mesokurtic if

$$\beta_2 = 3$$
$$\gamma_2 = 0$$

(iii) The distribution is Platykurtic if

$$\beta_2 < 3$$
$$\gamma_2 < 0$$

Q.1 The fourth central moment is 48. What must be its standard deviation in order that the distribution be Mesokurtic.

Given $\mu_4 = 48$

For Mesokurtic distribution $\beta_2 = 3$

We know that

$$\beta_2 = \frac{\mu_4}{\mu_2^2}$$

$$3 = \frac{48}{\mu_2^2}$$

$$\mu_2^2 = \frac{48}{3} = 16$$

$$\mu_2 = \sqrt{16}$$

$$\boxed{\mu_2 = 4}$$

$$S.D = \sqrt{\mu_2}$$

$$S.D = \sqrt{4}$$

$$\boxed{S.D = 2}$$

Q.2 The first four moments of a distribution about $x = 4$ are 1, 4, 10 and 45. Obtain the various characteristics of the

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distribution on the basic of the given information. Comment upon the nature of the distribution.

Given:- $A = 4$, $M_1' = 1$, $M_2' = 4$, $M_3' = 10$, $M_4' = 45$.

Sol^n:-

Moments About Mean

$$M_2 = M_2' - M_1'^2$$
$$= 4 - 1^2$$
$$\boxed{M_2 = 3}$$

$$M_3 = M_3' - 3M_2'M_1' + 2M_1^3$$
$$M_3 = 10 - 3 \times 4 \times 1 + 2(1)^3$$
$$M_3 = 10 - 12 + 2$$
$$\boxed{M_3 = 0}$$

$$M_4 = M_4' - 4M_3'M_1' + 6M_2'M_1'^2 - 3M_1^4$$
$$M_4 = 45 - 4 \times 10 \times 1 + 6 \times 4 \times (1)^2 - 3 \times (1)^4$$
$$M_4 = 45 - 40 + 24 - 3$$
$$\boxed{M_4 = 26}$$

Skewness:-

$$SK_M = \frac{M_3}{\sqrt{M_2^3}}$$
$$SK_M = \frac{0}{\sqrt{3^3}}$$
$$\boxed{SK_M = 0}$$

The distribution is symmetrical.

Kurtosis:-

$$\beta_2 = \frac{M_4}{M_2^2} = \frac{26}{9}$$
$$\boxed{\beta_2 = 2.89}$$
$$\beta_2 < 3$$

Hence it is platykurtic

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Q.3 The first four moments of a distribution are 0, 2.5, 0.7 and 18.71 Find the coefficient of skewness and kurtosis.

Given:- $M_1 = 0$, $M_2 = 2.5$, $M_3 = 0.7$, $M_4 = 18.71$

Soln:-

Coefficient of Skewness

We know that

$$\gamma_1 = Sk_M = \frac{M_3}{\sqrt{M_2^3}}$$

$$Sk_M = \frac{0.7}{\sqrt{(2.5)^3}}$$

$$\boxed{Sk_M = 0.17708}$$

Positively skewed

Kurtosis

$$\beta_2 = \frac{M_4}{M_2^2} = \frac{18.71}{(2.5)^2}$$

$$\boxed{\beta_2 = 2.9936}$$

$\beta_2 < 3$

It is platykurtic

Q.4 The first four moments about the working mean 28.5 of a distribution are 0.294, 7.144, 42.409 and 454.98. Calculate the moments about the mean. Also evaluate β_1, β_2 and comment upon the skewness and kurtosis of the distribution.

Given:-

$A = 28.5$, $M'_1 = 0.294$, $M'_2 = 7.144$, $M'_3 = 42.409$, $M'_4 = 454.98$

Moments About Mean

$$M_2 = M'_2 - M'^2_1$$

$$M_2 = 7.144 - (0.294)^2$$

$$\boxed{M_2 = 7.0576}$$

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$$M_3 = M'_3 - 3M'_2M'_1 + 2M'^3_1$$

$$M_3 = 42.409 - 3 \times 7.144 \times 0.294 + 2(0.294)^3$$

$$\boxed{M_3 = 36.1588}$$

$$M_4 = M'_4 - 4M'_3M'_1 + 6M'_2M'^2_1 - 3M'^4_1$$

$$M_4 = 454.98 - 4 \times 42.409 \times 0.294 + 6 \times 7.144 \times (0.294)^2 - 3(0.294)^4$$

$$\boxed{M_4 = 408.7896}$$

Now

$$\beta_1 = \frac{M_3^2}{M_2^3}$$

$$\beta_1 = \frac{(36.1588)^2}{(7.0576)^3}$$

$$\boxed{\beta_1 = 3.7193}$$

$$\beta_2 = \frac{M_4}{M_2^2}$$

$$\beta_2 = \frac{408.7896}{(7.0576)^2}$$

$$\boxed{\beta_2 = 8.2070}$$

Skewness

$$Sk_M = \gamma_1 = \frac{M_3}{\sqrt{M_2^3}}$$

$$Sk_M = \frac{36.1588}{\sqrt{(7.0576)^3}}$$

$$\boxed{Sk_M = 1.9285}$$

The distribution is positively skewed

Kurtosis

$$\boxed{\beta_2 = 8.2070}$$

$$\beta_2 > 3$$

It is leptokurtic

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Q.5 The following table represents the heights of a batch of 100 students. Calculate Skewness and Kurtosis.

Height (in cm)	59	61	63	65	67	69	71	73	75
No. of Students	0	2	6	20	40	20	8	2	2

Height(x)	No. of Students (f)	$d = x - 67$	$u = \frac{x-67}{2} = h$	fu	fu^2	fu^3	fu^4
59	0	−8	−4	0	0	0	0
61	2	−6	−3	−6	18	−54	162
63	6	−4	−2	−12	24	−48	96
A → 65	20	−2	−1	−20	20	−20	20
67	40	0	0	0	0	0	0
69	20	2	1	20	20	20	20
71	8	4	2	16	32	64	128
73	2	6	3	6	18	54	162
75	2	8	4	8	32	128	512
	$\Sigma f = 100$			$\Sigma fu = 12$	$\Sigma fu^2 = 164$	$\Sigma fu^3 = 144$	$\Sigma fu^4 = 1100$

Moments About 67

$$M'_1 = \frac{\Sigma fu}{N} \times h$$

$$M'_1 = \frac{12}{100} \times 2$$

$$\boxed{M'_1 = 0.24}$$

$$M'_2 = \frac{\Sigma fu^2}{N} \times h^2$$

$$M'_2 = \frac{164}{100} \times 4$$

$$\boxed{M'_2 = 6.56}$$

$$M'_3 = \frac{\Sigma fu^3}{N} \times h^3$$

$$M'_3 = \frac{144}{100} \times 8$$

$$\boxed{M'_3 = 11.52}$$

$$M'_4 = \frac{\Sigma fu^4}{N} \times h^4$$

$$M'_4 = \frac{1100 \times 16}{100}$$

$$\boxed{M'_4 = 176}$$

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Moment about Mean

$\boxed{M_1 = 0}$

$M_2 = M'_2 - M'^2_1$

$M_2 = 6.56 - (0.24)^2$

$\boxed{M_2 = 6.5024}$

$M_3 = M'_3 - 3M'_2 M'_1 + 2M'^3_1$

$M_3 = 11.52 - 3 \times 6.56 \times 0.24 + 2(0.24)^3$

$\boxed{M_3 = 6.824}$

$M_4 = M'_4 - 4M'_3 M'_1 + 6M'_2 M'^2_1 - 3M'^4_1$

$M_4 = 176 - 4 \times 11.52 \times 0.24 + 6 \times 6.56 (0.24)^2 - 3 \times (0.24)^4$

$\boxed{M_4 = 167.19798}$

Skewness

$Sk_M = \frac{M_3}{\sqrt{M_2^3}}$

$Sk_M = \frac{6.824}{\sqrt{(6.5024)^2}}$

$\boxed{Sk_M = 0.411}$

The distribution is positively skewed.

Kurtosis:-

$\beta_2 = \frac{M_4}{M_2^2}$

$= \frac{167.19798}{(6.5024)^2}$

$\boxed{\beta_2 = 3.9544}$

$\beta_2 > 3$

It is leptokurtic

Q.6 Find the measure of skewness and kurtosis on the basis of moment for the following distribution and draw your conclusion.

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Engg. Mathematics – IV **By Gulshan Sir**

Marks	5-15	15-25	25-35	35-45	45 55
No. of Students	1	3	5	7	4

Marks	No. of Stu. (f)	Mid value (x)	$d = x - 30$	$u = \frac{x-30}{10} = h$	fu	fu^2	fu^3	fu^4
5-15	1	10	-20	-2	-2	4	-8	16
15-25	3	20	-10	-1	-3	3	-3	3
25-35	5	30	0	0	0	0	0	0
35-45	7	40	10	1	7	7	7	7
45-55	4	50	20	2	8	16	32	64
	$\Sigma f = 20$				$\Sigma fu = 10$	$\Sigma fu^2 = 30$	$\Sigma fu^3 = 28$	$\Sigma fu^4 = 90$

Moments About 30

$$M'_1 = \frac{\Sigma fu}{N} \times h$$

$$M'_1 = \frac{10}{20} \times 10$$

$$\boxed{M'_1 = 5}$$

$$M'_2 = \frac{\Sigma fu^2}{N} \times h^2$$

$$M'_2 = \frac{30}{20} \times 100$$

$$\boxed{M'_2 = 150}$$

$$M'_3 = \frac{\Sigma fu^3}{N} \times h^3$$

$$M'_3 = \frac{28}{20} \times 1000$$

$$\boxed{M'_3 = 1400}$$

$$M'_4 = \frac{\Sigma fu^4}{N} \times h^4$$

$$M'_4 = \frac{90}{20} \times 10000$$

$$\boxed{M'_4 = 45000}$$

Moment about mean

$$\boxed{M_1 = 0}$$

$$M_2 = M'_2 - M'^2_1$$

$$M_2 = 150 - (5)^2$$

$$\boxed{M_2 = 125}$$

$$M_3 = M'_3 - 3M'_2 M'_1 + 2M'^3_1$$

$$M_3 = 1400 - 3 \times 150 \times 5 + 2(5)^3$$

$$\boxed{M_3 = -600}$$

$$M_4 = M'_4 - 4M'_3 M'_1 + 6M'_2 M'^2_1 - 3M'^4_1$$

$$\boxed{M_4 = 37625}$$

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Skwness

$$Sk_M = \frac{M_3}{\sqrt{M_2^3}}$$

$$Sk_M = \frac{-600}{\sqrt{(125)^3}}$$

$$\boxed{Sk_M = -0.4293}$$

Negatively skewed Distribution

Kurtosis

$$\beta_2 = \frac{M_4}{M_2^2}$$

$$= \frac{37625}{(125)^2}$$

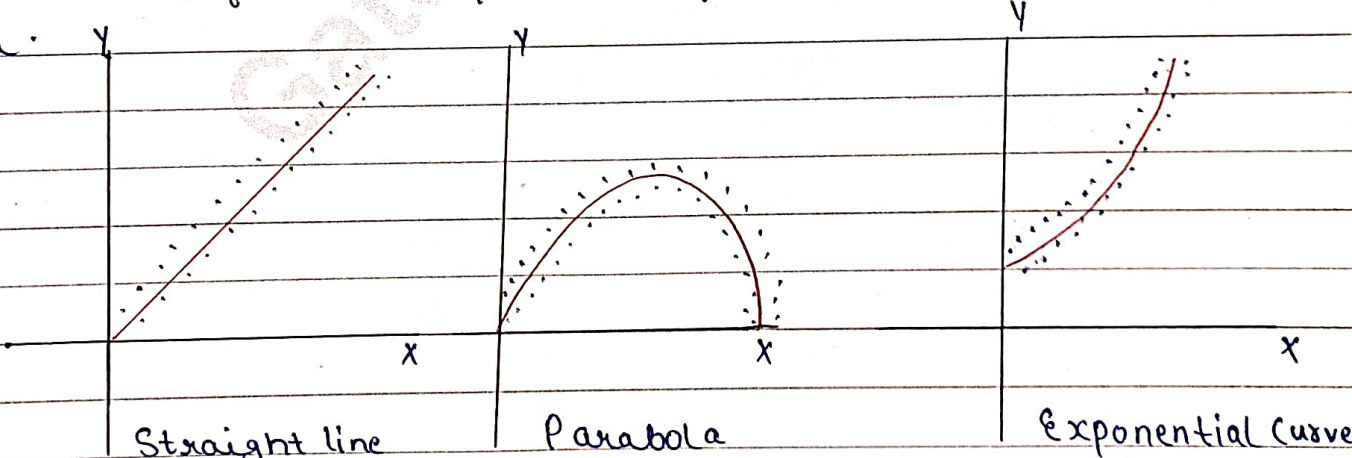
$$\boxed{\beta_2 = 2.408}$$

$\beta_2 < 3$

Platykurtic

Curve Fitting

It means to form an equation of the curve from the given data.



Straight line $y = a + bx$

Parabola $y = a + bx + cx^2$

Exponential Curve $y = ae^{bx}$

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Curve fitting by the method of least Square

1. Fitting of straight line
2. Fitting of second degree parabola.
3. Fitting of exponential curve.

1. Fitting of straight line

Equation of straight line

$$y = a + bx \quad \text{①}$$

Normal equations

$$\Sigma y = na + b\Sigma x \quad \text{②}$$

$$\Sigma xy = a\Sigma x + b\Sigma x^2 \quad \text{③}$$

1. Fit a straight line to the following data by least Square method:

x:	0	1	2	3	4
y:	1	1.8	3.3	4.5	6.3

Equation of line

$$y = a + bx \quad \text{①}$$

Normal equations

$$\Sigma y = na + b\Sigma x \quad \text{②}$$

$$\Sigma xy = a\Sigma x + b\Sigma x^2 \quad \text{③}$$

$$n = 5$$

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x	y	xy	x^2
0	1	0	0
1	1.8	1.8	1
2	3.3	6.6	4
3	4.5	13.5	9
4	6.3	25.2	16
$\Sigma x=10$	$\Sigma y=16.9$	$\Sigma xy=47.1$	$\Sigma x^2=30$

Put Σx, Σy and n in eqⁿ (2)

$$16.9 = 5a + 10b$$

$$5a + 10b = 16.9 \quad (4)$$

Put Σxy, Σx and Σx^2 in eqⁿ (3)

$$47.1 = 10a + 30b$$

$$10a + 30b = 47.1 \quad (5)$$

Solve eqⁿ (4) and (5)

$$a = 0.72$$

$$b = 1.33$$

Put a and b in (1)

$$\boxed{y = 0.72 + 1.33x}$$

Required Straight line

2. What straight line best fits the following data?

x :	1	2	3	4
y :	3	7	13	21

Equation of line

$$y = a + bx \quad (1)$$

Normal equations

$$\Sigma y = na + b\Sigma x \quad (2)$$

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$\Sigma xy = a\Sigma x + b\Sigma x^2$ ③

$n = 4$

x	y	xy	x^2
1	3	3	1
2	7	14	4
3	13	39	9
4	21	84	16
$\Sigma x = 10$	$\Sigma y = 44$	$\Sigma xy = 140$	$\Sigma x^2 = 30$

Put Σx, Σy and n in eqⁿ ②

$44 = 4a + 10b$

$4a + 10b = 44$ ④

Put Σxy, Σx and Σx^2 in eqⁿ ③

$10a + 30b = 140$ ⑤

Solve eqⁿ ④ and ⑤

$a = -4$

$b = 6$

Put a and b in eqⁿ ①

$$\boxed{y = -4 + 6x}$$

Required straight line

Fitting of second degree Parabola

Equation of parabola

$y = a + bx + cx^2$ ①

Normal equations

$\Sigma y = na + b\Sigma x + c\Sigma x^2$ ②

$\Sigma xy = a\Sigma x + b\Sigma x^2 + c\Sigma x^3$ ③

$\Sigma x^2 y = a\Sigma x^2 + b\Sigma x^3 + c\Sigma x^4$ ④

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1 Fit a second degree parabola in the following data?

x	0	1	2	3	4
y	1	4	10	17	30

Equation of parabola

$$y = a + bx + cx^2 \quad (1)$$

Normal equations

$$\Sigma y = na + b\Sigma x + c\Sigma x^2 \quad (2)$$

$$\Sigma xy = a\Sigma x + b\Sigma x^2 + c\Sigma x^3 \quad (3)$$

$$\Sigma x^2 y = a\Sigma x^2 + b\Sigma x^3 + c\Sigma x^4 \quad (4)$$

$n = 5$

x	y	x^2	x^3	x^4	xy	x^2y
0	1	0	0	0	0	0
1	4	1	1	1	4	4
2	10	4	8	16	20	40
3	17	9	27	81	51	153
4	30	16	64	256	120	480
$\Sigma x=10$	$\Sigma y=62$	$\Sigma x^2=30$	$\Sigma x^3=100$	$\Sigma x^4=354$	$\Sigma xy=195$	$\Sigma x^2y=677$

Put these values in eqn (2), (3) and (4)

$$62 = 5a + 10b + 30c \quad (5)$$

$$195 = 10a + 30b + 100c \quad (6)$$

$$677 = 30a + 100b + 354c \quad (7)$$

by Solving eqn (5), (6) and (7)

$a = 1.2$

$b = 1.1$

$c = 1.5$

Put a, b, & c in eqn (1)

$$\boxed{y = 1.2 + 1.1x + 1.5x^2}$$

The required parabola eqn

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Fitting of an Exponential Curve

Equation of exponential curve

$$y = ae^{bx}$$

Taking log both side

$$\log_{10} y = \log_{10}(ae^{bx})$$
$$\log_{10} y = \log_{10} a + \log_{10} e^{bx}$$
$$\log_{10} y = \log_{10} a + bx \log_{10} e$$

$$\log mn = \log m + \log n$$
$$\log m^n = n \log m$$

$$\log_{10} y = \log_{10} a + (b \log_{10} e)\, x$$

Put

$$\log_{10} y = Y$$
$$\log_{10} a = A$$
$$\log_{10} e = B$$

→ By this substitution we get,

$$Y = A + Bx$$

Q2. Find the curve of best fit of the type $y = ae^{bx}$ to the following data by the method of least square:

x	1	5	7	9	12
y	10	15	12	15	21

Equation of curve

$$y = ae^{bx} \quad \text{①}$$

Taking log both side

$$\log_{10} y = \log_{10}(ae^{bx})$$
$$\log_{10} y = \log_{10} a + \log_{10} e^{bx}$$
$$\log_{10} y = \log_{10} a + (b \log_{10} e)$$

Put

$$\log_{10} y = Y,\ \log_{10} a = A,\ \log_{10} e = B$$

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$Y = A + Bx$ ②

Normal equations

$\Sigma Y = nA + B\Sigma x$ ③

$\Sigma xY = A\Sigma x + B\Sigma x^2$ ④

$n = 5$

x	y	$Y = \log_{10} y$	x^2	xY
1	10	1	1	1
5	15	1.1760	25	5.8805
7	12	1.0791	49	7.5544
9	15	1.1761	81	10.5849
12	21	1.3222	144	15.8664
$\Sigma x = 34$		$\Sigma Y = 5.7536$	$\Sigma x^2 = 300$	$\Sigma xY = 40.8862$

Put these values in eqⁿ ③ & ④

$\Sigma Y = nA + B\Sigma x$

$5.7536 = 5A + 34B$ ⑤

$\Sigma xY = A\Sigma x + B\Sigma x^2$

$40.8862 = 34A + 300B$ ⑥

By solving eqⁿ ⑤ & ⑥

$\boxed{A = 0.9766}$

$\boxed{B = 0.0256}$

Now

$\log_{10} a = A$

$\log_{10} a = 0.9766$

$a = \text{Antilog}_{10}(0.9766)$

$\boxed{a = 9.47545}$

$b \log_{10} e = B$

$b = \dfrac{B}{\log_{10} e} = \dfrac{0.0256}{0.4342} = \boxed{0.059}$

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Put a and b in eq^n ①

$$y = 9.4754\, e^{0.059x}$$

The required exponential curve

4. Fitting of the Curve

$$y = ax + bx^2$$

Normal equations

$$\Sigma xy = a\Sigma x^2 + b\Sigma x^3$$

$$\Sigma x^2 y = a\Sigma x^3 + b\Sigma x^4$$

1. By the method of least squares, find the curve $y = ax + bx^2$ that best fits the following data

x	1	2	3	4	5
y	1.8	5.1	8.9	14.1	19.8

Equation of curve

$$y = ax + bx^2 \quad ①$$

Normal equations

$$\Sigma xy = a\Sigma x^2 + b\Sigma x^3 \quad ②$$

$$\Sigma x^2 y = a\Sigma x^3 + b\Sigma x^4 \quad ③$$

x	y	x^2	x^3	x^4	xy	x^2y
1	1.8	1	1	1	1.8	1.8
2	5.1	4	8	16	10.2	20.4
3	8.9	9	27	81	26.7	80.1
4	14.1	16	64	256	56.4	225.6
5	19.8	25	125	625	99	495
		$\Sigma x^2 = 55$	$\Sigma x^3 = 225$	$\Sigma x^4 = 979$	$\Sigma xy = 194.1$	$\Sigma x^2 y = 822.9$

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Put these values in eq^n ② and ③

$$55a + 225b = 194.1 \quad ④$$

$$225a + 979b = 822.9 \quad ⑤$$

Solve eq^n ④ and ⑤

$$a = 1.51$$

$$b = 0.49$$

Put a and b in eq^n ①

$$\boxed{y = 1.51x + 0.49x^2}$$

Required parabola.

5. Fitting of the curve $\boxed{y = \frac{C_0}{x} + C_1\sqrt{x}}$

Normal equations

$$\Sigma \frac{y}{x} = C_0 \Sigma \frac{1}{x^2} + C_1 \Sigma \frac{1}{\sqrt{x}}$$

$$\Sigma y\sqrt{x} = C_0 \Sigma \frac{1}{\sqrt{x}} + C_1 \Sigma x$$

2. Use the method of least squares to fit the curve $y = \frac{C_0}{x} + C_1\sqrt{x}$ to the following table of values.

x:	0.1	0.2	0.4	0.5	1	2
y:	21	11	7	6	5	6

Equation of curve

$$y = \frac{C_0}{x} + C_1\sqrt{x} \quad ①$$

Normal equations

$$\Sigma \frac{y}{x} = C_0 \Sigma \frac{1}{x^2} + C_1 \Sigma \frac{1}{\sqrt{x}} \quad ②$$

$$\Sigma y\sqrt{x} = C_0 \Sigma \frac{1}{\sqrt{x}} + C_1 \Sigma x \quad ③$$

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x	y	$\frac{y}{x}$	$y\sqrt{x}$	$\frac{1}{\sqrt{x}}$	$\frac{1}{x^2}$
0.1	21	210	6.64078	3.16228	100
0.2	11	55	4.91935	2.23607	25
0.4	7	17.5	4.42719	1.58114	6.25
0.5	6	12	4.24264	1.41421	4
1	5	5	5	1	1
2	6	3	8.48528	0.70711	0.25
$\Sigma x = 4.2$		$\Sigma y/x = 302.5$	$\Sigma y\sqrt{x} = 33.71524$	$\Sigma 1/\sqrt{x} = 10.10081$	$\Sigma 1/x^2 = 136.5$

Put these values in eqn ② and ③

$$136.5\,C_0 + 10.10081\,C_1 = 302.5 \quad ④$$

$$10.10081\,C_0 + 4.2\,C_1 = 33.71524 \quad ⑤$$

Solve eqn ④ and ⑤

$C_0 = 1.973267$

$C_1 = 3.281819$

Put C_0 and C_1 in eqn ①

$$\boxed{y = \frac{1.9733}{x} + 3.2818\sqrt{x}}$$

6. Fitting of the curve $\boxed{y = ax + \frac{b}{x}}$

Normal equations

$$\Sigma xy = a\Sigma x^2 + nb$$

$$\Sigma \frac{y}{x} = na + b\Sigma \frac{1}{x^2}$$

3. Fit a relation $y = ax + \frac{b}{x}$ which satisfies the following data, using method of least squares

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x	1	2	3	4	5	6	7	8
y	5.4	6.2	8.2	10.3	12.6	14.8	17.2	19.5

Equation of the curve

$$y = ax + \frac{b}{x} \quad (1)$$

Normal equations

$$\Sigma xy = a\Sigma x^2 + nb \quad (2)$$

$$\Sigma \frac{y}{x} = na + b\Sigma \frac{1}{x^2} \quad (3)$$

$n = 8$

x	y	xy	x^2	y/x	$1/x^2$
1	5.4	5.4	1	5.4	1
2	6.2	12.4	4	3.1	0.25
3	8.2	24.6	9	2.74	0.1111
4	10.3	41.2	16	2.575	0.0625
5	12.6	63	25	2.52	0.04
6	14.8	88.8	36	2.47	0.0278
7	17.2	120.4	49	2.46	0.0204
8	19.5	156	64	2.44	0.0156
		$\Sigma xy = 511.8$	$\Sigma x^2 = 204$	$\Sigma y/x = 23.705$	$\Sigma 1/x^2 = 1.5274$

Put these values in eq^n (2) and (3)

$$204a + 8b = 511.8 \quad (4)$$

$$8a + 1.5274 = 23.705 \quad (5)$$

Solve eq^n (4) and (5)

$a = 2.39139$

$b = 2.99455$ Put 'a' and 'b' in eq^n (1)

$$\boxed{y = 2.39139x + 2.99455/x}$$

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7. Fitting of the curve $y = ab^x$

$Y = A + Bx$

Normal equations

$\Sigma Y = nA + B\Sigma x$

$\Sigma xY = A\Sigma x + B\Sigma x^2$

4. Obtain a relation of the form $y = ab^x$ for the following data by the method of least squares.

x:	2	3	4	5	6
y:	8.3	15.4	33.1	65.2	127.4

Equation of curve

$y = ab^x$ ①

Taking log both side

$\log_{10} y = \log_{10} ab^x$

$\log_{10} y = \log_{10} a + \log_{10} b^x$

$\log_{10} y = \log_{10} a + (\log_{10} b)x$

Put

$\log_{10} y = Y$ ②

$\log_{10} a = A$ ③

$\log_{10} b = B$ ④

$\boxed{Y = A + Bx}$

Normal equation

$\Sigma Y = nA + B\Sigma x$ ⑤

$\Sigma xY = A\Sigma x + B\Sigma x^2$ ⑥

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x	y	$Y=\log_{10} y$	x^2	xY
2	8.3	0.9191	4	1.8382
3	15.4	1.1872	9	3.5616
4	33.1	1.5198	16	6.0792
5	65.2	1.8142	25	9.0710
6	127.4	2.1052	36	12.6312
$\Sigma x=20$		$\Sigma Y=7.5455$	$\Sigma x^2=90$	$\Sigma xY=33.1812$

Put these values in eq^n ⑤ and ⑥

$$5A + 20B = 7.5455$$

$$20A + 90B = 33.1812$$

$$A = 0.3094$$

$$B = 0.2999$$

Put A in eq^n ③

$$\log_{10} a = 0.3094$$

$$a = \text{antilog}\ (0.3094)$$

$$\boxed{a = 2.0389}$$

Put B in eq^n ④

$$\log_{10} b = 0.2999$$

$$b = \text{Antilog}\ (0.2999)$$

$$\boxed{b = 1.9948}$$

Put 'a' and 'b' in eq^n ①

$$\boxed{y = 2.0389\,(1.9948)^x}$$

8. Fitting of the curve $\boxed{pv^n = c}$

$$Y = A + BX$$

Normal equations

$$\Sigma Y = nA + B\Sigma X$$

$$\Sigma XY = A\Sigma X + B\Sigma X^2$$

$$pv^n = c$$

$$p = c / v^n$$

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$$p = cv^{-n}$$

Taking log both side

$$\log p = \log cv^{-n}$$
$$\log p = \log c + \log v^{-n}$$
$$\log p = \log c - n \log v$$

Put

$$\log p = Y$$
$$\log c = A$$
$$\log v = X$$
$$-n = B$$
$$\boxed{Y = A + BX}$$

5. The pressure of the gas corresponding to various volumes V is measured given by the following data:

V (cm^3)	50	60	70	90	100
P (kg/cm^2)	64.7	51.3	40.5	25.9	78

Fit the data to the equation $PV^n = C$

Equation of the curve

$$PV^n = C \quad (1)$$
$$P = \frac{C}{V^n}$$
$$P = CV^{-n}$$

Taking log both side

$$\log_{10} P = \log_{10} CV^{-n}$$
$$\log_{10} P = \log_{10} C - n \log_{10} V$$

Put

$$\log_{10} P = Y \quad (2)$$
$$\log_{10} C = A \quad (3)$$
$$\log_{10} V = X \quad (4)$$
$$-n = B \quad (5)$$

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$$Y = A + BX \quad (5)$$

Normal equations

$$\Sigma Y = nA + B\Sigma X \quad (6)$$

$$\Sigma XY = A\Sigma X + B\Sigma X^2 \quad (7)$$

V	P	$X = \log_{10} V$	$Y = \log_{10} P$	XY	X^2
50	64.7	1.69897	1.81090	3.07666	2.88650
60	51.3	1.77815	1.71012	3.04085	3.16182
70	40.5	1.84510	1.60746	2.96592	3.40439
80	25.9	1.95424	1.41330	2.76193	3.81905
100	78	2	1.89209	3.78418	4
		$\Sigma X = 9.27646$	$\Sigma Y = 8.43387$	$\Sigma XY = 15.62954$	$\Sigma X^2 = 17.27176$

Put these values in eqn (6) and (7)

$$5A + 9.27646B = 8.43387 \quad (8)$$

$$9.27646A + 17.27176 = 15.62954 \quad (9)$$

Solve eqn (8) and (9)

$$A = 2.2248$$

$$B = -0.2899$$

Put A in eqn (3)

$$\log_{10} C = 2.2248$$

$$C = \text{Antilog}(2.2248)$$

$$\boxed{C = 167.8031}$$

Put B in eqn (5)

$$-n = -0.2899$$

$$\boxed{n = 0.2899}$$

Put C and n in eqn (1)

$$\boxed{PV^{0.2899} = 167.8031}$$

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9 Fitting of the curve $\boxed{xy = b + ax}$

$$y = a + \frac{b}{x}$$

Normal equations

$$\Sigma y = na + b\Sigma \frac{1}{x}$$

$$\Sigma \frac{y}{x} = a\Sigma \frac{1}{x} + b\Sigma \frac{1}{x^2}$$

10. Fitting of the curve $\boxed{y = ax^2 + \frac{b}{x}}$

Normal equations

$$\Sigma x^2 y = a\Sigma x^4 + b\Sigma x$$

$$\Sigma \frac{y}{x} = a\Sigma x + b\Sigma \frac{1}{x^2}$$

11 Fitting of the curve $\boxed{y = a + \frac{b}{x} + \frac{c}{x^2}}$

Normal equations

$$\Sigma y = na + b\Sigma \frac{1}{x} + c\Sigma \frac{1}{x^2}$$

$$\Sigma \frac{y}{x} = a\Sigma \frac{1}{x} + b\Sigma \frac{1}{x^2} + c\Sigma \frac{1}{x^3}$$

$$\Sigma \frac{y}{x^2} = a\Sigma \frac{1}{x^2} + b\Sigma \frac{1}{x^3} + c\Sigma \frac{1}{x^4}$$

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Correlation

* Correlation is a statistical tool which deal with two or more variables (two or more series)
* The degree of relationship between the variables under consideration is measured through the correlation analysis

Types of correlation:-

(i) **Positive correlation** :- If two variables move in same direction then correlation is said to be positive correlation

(i) $x\uparrow$ $y\uparrow$ (ii) $x\downarrow$ $y\downarrow$

(ii) **Negative correlation**:- If two variables move in opposite direction then corelation is said to be negative correlation

(i) $x\uparrow$ $y\downarrow$ (ii) $x\downarrow$ $y\uparrow$

Method to measure Correlation:-

Co-efficient of corelation (r_{xy})

OR

Karl pearson's Co-efficient of corelation

OR

Product moment Co-efficient of corelation

$$r_{xy} = \frac{Cov(x,y)}{\sigma_x \sigma_y}$$

$$Cov(x,y) = \frac{\sum (x-\bar{x})(y-\bar{y})}{n}$$

$$\sigma_x = \sqrt{\frac{\sum (x-\bar{x})^2}{n}}$$

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$$\sigma_y = \sqrt{\frac{\Sigma (y-\bar{y})^2}{n}}$$

$$r_{xy} = \frac{\Sigma (x-\bar{x})(y-\bar{y})}{\sqrt{\Sigma (x-\bar{x})^2}\sqrt{\Sigma (y-\bar{y})^2}}$$ → Direct Method

where

$\bar{x}$ = Mean of x

$\bar{y}$ = Mean of y

$(x-\bar{x})$ = Deviation from mean

$(y-\bar{y})$ = Deviation from mean

Another form

$$r_{xy} = \frac{n\Sigma xy - \Sigma x \Sigma y}{\sqrt{n\Sigma x^2 - (\Sigma x)^2}\sqrt{n\Sigma y^2 - (\Sigma y)^2}}$$ → Direct Method

Shortcut Method

$$dx = x - A$$

$$dy = y - A$$

$$r_{xy} = \frac{n\Sigma dx\,dy - \Sigma dx \Sigma dy}{\sqrt{n\Sigma dx^2 - (\Sigma dx)^2}\sqrt{n\Sigma dy^2 - (\Sigma dy)^2}}$$

Step deviation Method

$$u = \frac{x-A}{n}$$

$$v = \frac{y-A}{n}$$

$$r_{xy} = \frac{n\Sigma uv - \Sigma u \Sigma v}{\sqrt{n\Sigma u^2 - (\Sigma u)^2}\sqrt{n\Sigma v^2 - (\Sigma v)^2}}$$

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Important points

$-1 \leq r \leq 1$

Positive correlation $(0 < r < 1)$

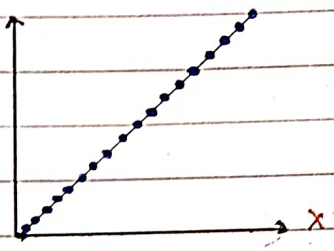
Negative correlation $(-1 < r < 0)$

Perfect +ive correlation $(r = +1)$

Perfect -ive correlation $(r = -1)$

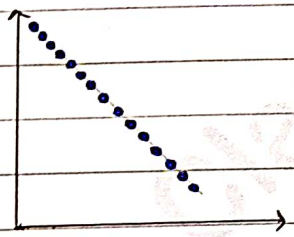
No correlation $(r = 0)$

Graphs of correlation (scattered or dotted diagrams)



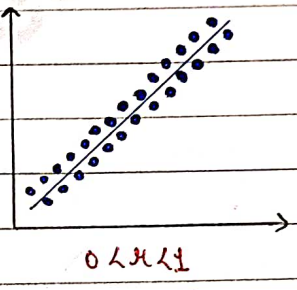
$r = +1$

Perfect +ive correlation



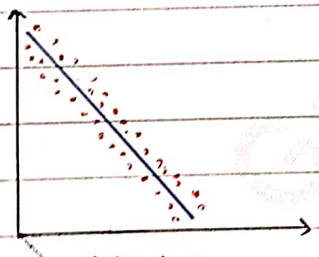
$r = -1$

Perfect -ive correlation



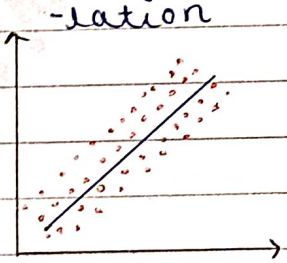
$0 < r < 1$

Strong +ive correlation



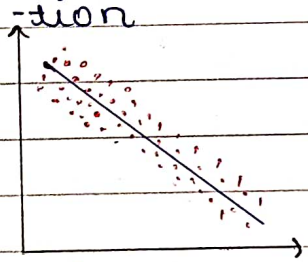
$-1 < r < 0$

Strong -ive correlation



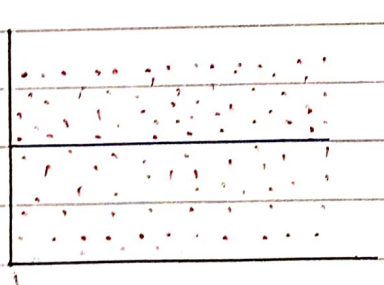
$0 < r < 1$

weak +ive correlation



$-1 < r < 0$

weak -ive correlation



$r = 0$ (No correlation)

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1 Form the data given below, find the number of items n $r_{xy} = 0.5$, $\Sigma XY = 120$, $\Sigma X^2 = 90$, $\sigma_y = 8$ where X and Y are deviations from the arithmetic mean

Given:- $r_{xy} = 0.5$, $\Sigma XY = 120$, $\Sigma X^2 = 90$, $\sigma_y = 8$

$X = x - \bar{x}$

$Y = y - \bar{y}$

$\Sigma (x-\bar{x})(y-\bar{y}) = 120$

$\Sigma (x-\bar{x})^2 = 90$

$\sigma_y = 8$

$\sqrt{\frac{\Sigma (y-\bar{y})^2}{n}} = 8$

$\frac{\Sigma (y-\bar{y})^2}{n} = 64$ (∵ By squaring on both sicle)

$\Sigma (y-\bar{y})^2 = 64n$

$r_{xy} = \frac{\Sigma (x-\bar{x})(y-\bar{y})}{\sqrt{\Sigma (x-\bar{x})^2}\sqrt{\Sigma (y-\bar{y})^2}}$

$0.5 = \frac{120}{\sqrt{90}\sqrt{64n}}$

$0.5 = \frac{120}{\sqrt{90} \times 8\sqrt{n}}$

$\sqrt{n} = \frac{120 \times 2}{8 \times \sqrt{90}}$

$\sqrt{n} = \frac{30}{\sqrt{90}}$

$n = \frac{900}{90}$ (By squaring on both side)

$\boxed{n = 10}$

2. Ten students got the following percentage of marks in principles of Economics and statistics :-

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Roll no.	1	2	3	4	5	6	7	8	9	10
Marks in Economics	78	36	98	25	75	82	90	62	65	39
Marks in statistics	84	51	91	60	68	62	86	58	53	47

Calculate the co-efficient of correlation

Mean

$$\bar{x} = \frac{\Sigma x}{n} \qquad \bar{y} = \frac{\Sigma y}{n}$$

$$\bar{x} = \frac{650}{10} \qquad \bar{y} = \frac{660}{10}$$

$$\boxed{\bar{x} = 65} \qquad \boxed{\bar{y} = 66}$$

$$r_{xy} = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\sqrt{\Sigma(x-\bar{x})^2}\sqrt{\Sigma(y-\bar{y})^2}}$$

$$r_{xy} = \frac{2734}{\sqrt{5398}\sqrt{2224}}$$

$$\boxed{r_{xy} = 0.789}$$

x	y	$x-\bar{x}$	$y-\bar{y}$	$(x-\bar{x})^2$	$(y-\bar{y})^2$	$(x-\bar{x})(y-\bar{y})$
78	84	13	18	169	324	234
36	51	-29	-15	841	225	435
98	91	33	25	1089	625	825
25	60	-40	-6	1600	36	240
75	68	10	2	100	4	20
82	62	17	-4	289	16	-68
90	86	25	20	625	400	500
62	58	-3	-8	9	64	24
65	53	0	-13	0	169	0
39	47	-26	-19	676	361	494
$\Sigma x = 650$	$\Sigma y = 660$			$\Sigma(x-\bar{x})^2 = 5398$	$\Sigma(y-\bar{y})^2 = 2224$	$\Sigma(x-\bar{x})(y-\bar{y}) = 2734$

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3. Find the coefficient of correlation between the values of x and y

x	y	x^2	y^2	xy
1	8	1	64	8
3	12	9	144	36
5	15	25	225	75
7	17	49	289	119
8	18	64	324	144
10	20	100	400	200
$\Sigma x = 34$	$\Sigma y = 90$	$\Sigma x^2 = 248$	$\Sigma y^2 = 1446$	$\Sigma xy = 582$

$$r_{xy} = \frac{n\Sigma xy - \Sigma x \Sigma y}{\sqrt{n\Sigma x^2 - (\Sigma x)^2}\ \sqrt{n\Sigma y^2 - (\Sigma y)^2}}$$

$$r_{xy} = \frac{6 \times 582 - 34 \times 90}{\sqrt{6 \times 248 - (34)^2}\ \sqrt{6 \times 1446 - (90)^2}}$$

$$\boxed{r_{xy} = 0.9879}$$

4. A computer while calculating correlation co-efficient between two variables X and Y from 25 pairs of observation obtained the following results:

$\Sigma X = 125$, $\Sigma X^2 = 650$, $n = 25$

$\Sigma Y^2 = 460$, $\Sigma XY = 508$, $\Sigma Y = 100$

It was, however, later discovered at the time of checking that he had copied down two pairs as

X	Y
6	14
8	6

while the correct values were

X	Y
8	12
6	8

Obtain the correct value of correlation co-efficient.

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$n = 25$

Corrected values

$\Sigma X = 125 - 6 - 8 + 8 + 6 = 125$

$\Sigma Y = 100 - 14 - 6 + 12 + 8 = 100$

$\Sigma X^2 = 650 - 36 - 64 + 64 + 36$

$\Sigma X^2 = 650$

$\Sigma Y^2 = 460 - 196 - 36 + 144 + 64$

$\Sigma Y^2 = 436$

$\Sigma XY = 508 - 6 \times 14 - 8 \times 6 + 8 \times 12 + 6 \times 8$

$\Sigma XY = 520$

$$r_{xy} = \frac{n\Sigma XY - \Sigma X \Sigma Y}{\sqrt{n\Sigma X^2 - (\Sigma X)^2}\ \sqrt{n\Sigma Y^2 - (\Sigma Y)^2}}$$

$$r_{xy} = \frac{25 \times 520 - 125 \times 100}{\sqrt{25 \times 650 - (125)^2}\ \sqrt{25 \times 436 - (100)^2}}$$

$$\boxed{r_{xy} = 0.67}$$

5. Find the coefficient of correlation for the following table

x:	10	14	18	22	26	30
y:	18	12	24	6	30	36

x	y	$dx = x - 22$	$dy = y - 24$	$u = \frac{x-22}{4}$	$v = \frac{y-24}{6}$	u^2	v^2	uv
10	18	-12	-6	-3	-1	9	1	3
14	12	-8	-12	-2	-2	4	4	4
18	24	-4	0	-1	0	1	0	0
22	6	0	-18	0	-3	0	9	0
26	30	4	6	1	1	1	1	1
30	36	8	12	2	2	4	4	4
$\Sigma x = 120$	$\Sigma y = 126$			$\Sigma u = -3$	$\Sigma v = -3$	$\Sigma u^2 = 19$	$\Sigma v^2 = 19$	$\Sigma uv = 12$

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$n = 6$

$$r_{xy} = \frac{n\Sigma uv - \Sigma u \Sigma v}{\sqrt{n\Sigma u^2 - (\Sigma u)^2}\ \sqrt{n\Sigma v^2 - (\Sigma v)^2}}$$

$$r_{xy} = \frac{6 \times 12 - (-3)(-3)}{\sqrt{6 \times 19 - (-3)^2}\ \sqrt{6 \times 19 - (-3)^2}}$$

$$r_{xy} = \frac{72 - 9}{\sqrt{105} \times \sqrt{105}}$$

$$r_{xy} = \frac{63}{105}$$

$$\boxed{r_{xy} = 0.6}$$

Rank Correlation

- Non Repeated Rank
- Repeated Rank

Rank correlation (Non Repeated Rank)

$$\boxed{r = 1 - \frac{6\Sigma D^2}{n(n^2-1)}}$$

where

n = no. of Observation

$D = R_1 - R_2$

r = Rank correlation coefficient

Rank Correlation (Repeated Rank)

$$\boxed{r = 1 - \frac{6[\Sigma D^2 + F]}{n(n^2-1)}}$$

where $F = \frac{m_1(m_1^2-1)}{12} + \frac{m_2(m_2^2-1)}{12} + \cdots\cdots$

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Type-1: Where Rank is given

1. Compute the rank correlation for the following data

Persons	A	B	C	D	E	F	G	H	I	J
Rank in Maths	9	10	6	5	7	2	4	8	1	3
Rank in Physics	1	2	3	4	5	6	7	8	9	10

Persons	R_1	R_2	$D = R_1 - R_2$	D_2^2
A	9	1	8	64
B	10	2	8	64
C	6	3	3	9
D	5	4	1	1
E	7	5	2	4
F	2	6	-4	16
G	4	7	-3	9
H	8	8	0	0
I	1	9	-8	64
J	3	10	-7	49
				$\Sigma D^2 = 280$

Coefficient of Rank correlation

$$r = 1 - \frac{6 \Sigma D^2}{n(n^2-1)}$$

$$r = 1 - \frac{6 \times 280}{10 \times 99}$$

$$\boxed{r = -0.6969}$$

2. Ten competitors in a beauty contest were ranked by three judges in the following orders

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Competitors	A	B	C	D	E	F	G	H	I	J
1st Judge	1	6	5	10	3	2	4	9	7	8
2nd Judge	3	5	8	4	7	10	2	1	6	9
3rd Judge	6	4	9	8	1	2	3	10	5	7

Use the method of rank correlation to determine which pair of judges has the nearest approach to common taste in beauty?

Competitors	R_1	R_2	R_3	$D_{12}=R_1-R_2$	$D_{13}=R_1-R_3$	$D_{23}=R_2-R_3$	D_{12}^2	D_{13}^2	D_{23}^2
A	1	3	6	-2	-5	-3	4	25	9
B	6	5	4	1	2	1	1	4	1
C	5	8	9	-3	-4	-1	9	16	1
D	10	4	8	6	2	-4	36	4	16
E	3	7	1	-4	2	6	16	4	36
F	2	10	2	-8	0	8	64	0	64
G	4	2	3	2	1	-1	4	1	1
H	9	1	10	8	-1	-9	64	1	81
I	7	6	5	1	2	1	1	4	1
J	8	9	7	-1	1	2	1	1	4
							ΣD_{12}^2	ΣD_{13}^2	ΣD_{23}^2
							=200	60	214

$$\rho_{12} = 1 - \frac{6\Sigma D_{12}^2}{n(n^2-1)}$$

$$\rho_{12} = 1 - \frac{6 \times 200}{10 \times 99}$$

$$\boxed{\rho_{12} = -0.212}$$

$$\rho_{13} = 1 - \frac{6\Sigma D_{13}^2}{n(n^2-1)}$$

$$\rho_{13} = 1 - 6 \times 60 / 10 \times 99$$

$$\boxed{\rho_{13} = 0.636}$$

$$\rho_{23} = 1 - \frac{6\Sigma D_{23}^2}{n(n^2-1)}$$

$$\rho_{23} = 1 - \frac{6 \times 214}{10 \times 99}$$

$$\boxed{\rho_{23} = -0.296}$$

Hence first and third judges has the nearest approach.

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Type-2: where Rank is not given

3. Ten students got the following percentage of marks in chemistry and physics:

Students	1	2	3	4	5	6	7	8	9	10
Marks in chem.	78	36	98	25	75	82	90	62	65	39
Marks in Phys.	84	51	91	60	68	62	86	58	63	47

Calculate the rank correlation co-efficient

$$n = 10$$

$$\Sigma D^2 = 26$$

$$r = 1 - \frac{6\Sigma D^2}{n(n^2-1)}$$

$$r = 1 - \frac{6 \times 26}{10 \times 99}$$

$$\boxed{r = 0.84}$$

Students	Marks (Chem.)	R_1	Marks (Phy.)	R_2	$D = R_1 - R_2$	D^2
1	78	4	84	3	1	1
2	36	9	51	9	0	0
3	98	1	91	1	0	0
4	25	10	60	7	3	9
5	75	5	68	4	1	1
6	82	3	62	6	-3	9
7	90	2	86	2	0	0
8	62	7	58	8	-1	1
9	65	6	63	5	1	1
10	39	8	47	10	-2	4
						$\Sigma D^2 = 26$

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4. Find the rank correlation coefficient of marks of A and B from the following data:

Marks A: 15 20 27 13 45 60 20 75

Marks B: 50 30 55 30 25 10 30 70

Students	Marks (A)	R_1	Marks (B)	R_2	$D = R_1 - R_2$	D^2
1	15	7	50	3	4	16
2	20	5.5	30	5	0.5	0.25
3	27	4	55	2	2	4
4	13	8	30	5	3	9
5	45	3	25	7	-4	16
6	60	2	10	8	-6	36
7	20	5.5	30	5	0.5	0.25
8	75	1	70	1	0	0
						$\Sigma D^2 = 81.5$

Coefficient of rank correlation

$$r = 1 - \frac{6(\Sigma D^2 + F)}{n(n^2-1)} \quad \text{①}$$

$$F = \frac{m_1(m_1^2-1)}{12} + \frac{m_2(m_2^2-1)}{12}$$

$$F = \frac{2(4-1)}{12} + \frac{3(9-1)}{12}$$

$$F = \frac{6}{12} + \frac{24}{12}$$

$$F = 2.5$$

$$r = 1 - \frac{6(81.5 + 2.5)}{8(64-1)}$$

$$r = 1 - \frac{6 \times 84}{8 \times 63}$$

$$r = 1 - 1 \Rightarrow \boxed{r = 0}$$

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Regression Analysis :-

* Regression means functional relationship between two or more related variables.
* If two variables are correlated then there exists a relation between them.
* Regression analysis is a process of estimating relationship between dependent variable and independent variables

Linear Regression :-

When points in scatter diagram are concentrated around a straight line then it is linear Regression
There are two lines of Regression

(i) Line of Regression of y on x
(ii) Line of Regression of x on y

Line of Regression of y on x

$$y - \bar{y} = b_{yx}(x - \bar{x})$$

$$b_{yx} = \frac{n\Sigma xy - \Sigma x \Sigma y}{n\Sigma x^2 - (\Sigma x)^2}$$

Or

$$\boxed{b_{yx} = r\frac{\sigma_y}{\sigma_x}}$$

where
b_{yx} = Regression coefficient of y on x

Equation of a line passing through a given point (x_1, y_1) and given slope (m)

$$\boxed{y - y_1 = m(x - x_1)}$$

$$2x + 4y = 11$$
$$4y = -2x + 11$$
$$y = -\frac{1}{2}x + \frac{11}{4} \Rightarrow \boxed{b_{yx} = -\frac{1}{2}}$$

Note: Regression lines are always passes through the point $(\bar{x}, \bar{y})$

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Line of Regression of x on y

$$x - \bar{x} = b_{xy}(y - \bar{y})$$

$$b_{xy} = \frac{n\Sigma xy - \Sigma x \Sigma y}{n\Sigma y^2 - (\Sigma y)^2}$$

$$b_{xy} = r\frac{\sigma_x}{\sigma_y}$$

Regression line

$2x + 4y = 11$

$2x = -4y + 11$

$x = -2y + \frac{11}{2}$

$b_{xy} = -2$

where

b_{xy} = Regression coefficient of x on y

Note: Regression lines are always passes through the point $(\bar{x}, \bar{y})$

Relation between regression coefficient and correlation coefficient

We know that

$$b_{yx} = \frac{r\sigma_y}{\sigma_x} \quad (1)$$

$$b_{xy} = \frac{r\sigma_x}{\sigma_y} \quad (2)$$

Multiplying eqn (1) & (2)

$$b_{yx} \times b_{xy} = r\frac{\sigma_y}{\sigma_x} \times r\frac{\sigma_x}{\sigma_y}$$

$$r^2 = b_{yx} \times b_{xy}$$

$$r = \sqrt{b_{yx} \times b_{xy}}$$

r is the Geometric mean function b_{yx} and b_{xy}

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1. If the regression coefficients are 0·8 and 0·2 what would be the value of coefficient of correlation? AKtu-2021

Given $b_{yx} = 0.8$, $b_{xy} = 0.2$

We know that

$$r = \sqrt{b_{yx} \times b_{xy}}$$

$$r = \sqrt{0.8 \times 0.2}$$

$$r = \sqrt{0.16}$$

$$\boxed{r = 0.4}$$

2. Find the coefficient of correlation and obtain the equation to the lines of regression for the following data

x:	6	2	10	4	8
y:	9	11	5	8	7

x	y	x^2	y^2	xy
6	9	36	81	54
2	11	4	121	22
10	5	100	25	50
4	8	16	64	32
8	7	64	49	56
$\Sigma x = 30$	$\Sigma y = 40$	$\Sigma x^2 = 220$	$\Sigma y^2 = 340$	$\Sigma xy = 214$

Regression coefficient

$$b_{yx} = \frac{n\Sigma xy - \Sigma x \Sigma y}{n\Sigma x^2 - (\Sigma x)^2}$$

$$b_{yx} = \frac{5 \times 214 - 30 \times 40}{5 \times 220 - (30)^2}$$

$$b_{yx} = \frac{-130}{200}$$

$$\boxed{b_{yx} = -0.65}$$

$$b_{xy} = \frac{n\Sigma xy - \Sigma x \Sigma y}{n\Sigma y^2 - (\Sigma y)^2}$$

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$$b_{xy} = \frac{-130}{100}$$

$$\boxed{b_{xy} = -1.3}$$

We know that

$$r = \sqrt{b_{yx} \times b_{xy}}$$

$$r = \sqrt{0.65 \times 1.3}$$

$$\boxed{r = -0.9192}$$ → Coefficient of correlation

Mean

$$\bar{x} = \frac{\Sigma x}{n} = \frac{30}{5}$$

$$\boxed{\bar{x} = 6}$$

$$\bar{y} = \frac{\Sigma y}{n} = \frac{40}{5}$$

$$\boxed{\bar{y} = 8}$$

Line of regression of y on x

$$(y - \bar{y}) = b_{yx}(x - \bar{x})$$

$$y - 8 = -0.65(x - 6)$$

$$y - 8 = -0.65x + 3.90$$

$$\boxed{y = -0.65x + 11.9}$$

Line of regression of x on y

$$(x - \bar{x}) = b_{xy}(y - \bar{y})$$

$$x - 6 = -1.3(y - 8)$$

$$x - 6 = -1.3y + 10.4$$

$$\boxed{x = -1.3y + 16.4}$$

3. The following table gives age (x) in years of cars and annual maintenance cost (y) in hundred rupees:

x:	1	3	5	7	9
y:	15	18	21	23	22

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Estimate the maintenace cost for a 4 year old car after finding the regression equation

x	y	x^2	xy
1	15	1	15
3	18	9	54
5	21	25	105
7	23	49	161
9	22	81	198
$\Sigma x = 25$	$\Sigma y = 99$	$\Sigma x^2 = 165$	$\Sigma xy = 533$

Regression coefficient

$$byx = \frac{n\Sigma xy - \Sigma x \Sigma y}{n\Sigma x^2 - (\Sigma x)^2}$$

$$byx = \frac{5 \times 533 - 99 \times 25}{5 \times 165 - (25)^2}$$

$$\boxed{byx = 0.95}$$

Mean

$$\bar{x} = \frac{\Sigma x}{n}$$

$$\bar{x} = \frac{25}{5}$$

$$\boxed{\bar{x} = 5}$$

$$\bar{y} = \frac{\Sigma y}{n} = \frac{99}{5}$$

$$\boxed{\bar{y} = 19.8}$$

Regression line of y on x

$$(y - \bar{y}) = byx\,(x - \bar{x})$$

$$y - 19.8 = 0.95\,(x - 5)$$

$$y - 19.8 = 0.95x - 4.75$$

$$\boxed{y = 0.95x + 15.05}$$

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Maintenance cost

$x = 4$

$y = 0.95 \times 4 + 15.05$

$y = 18.85$

$y = 1885$ Rs

4 The regression equations calculated from a given set of observations for two random variables are $x = -0.4y + 6.4$ and $y = -0.6x + 4.6$

Calculated (i) $\bar{x}$ (ii) $\bar{y}$ (iii) r Aktu-2016

Regression lines

$x = -0.4y + 6.4$ (1)

$y = -0.6x + 4.6$ (2)

From (1)

$b_{xy} = -0.4$

From (2)

$b_{yx} = -0.6$

We know that

$r = \sqrt{0.6 \times 0.4}$

$r = 0.48989$ coefficient of correlation

Since regression lines always passing through $(\bar{x}, \bar{y})$

From (1)

$\bar{x} = -0.4\bar{y} + 6.4$

$\bar{x} + 0.4\bar{y} = 6.4$ (3)

From (2)

$\bar{y} = -0.6\bar{x} + 4.6$

$0.6\bar{x} + \bar{y} = 4.6$ (4)

Solve (3) and (4)

$\bar{x} = 6$

$\bar{y} = 1$

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5. In a partially destroyed laboratory record of an analysis of a correlation data, the following results only are legible:
Variance of $x = 9$
Regression equations: $8x - 10y + 66 = 0$ $40x - 18y = 214$
What were (a) the mean values of x and y (b) the standard deviation of y and the coefficient of correlation between x and y?

Given: $\sigma x^2 = 9$, $\sigma_x = 3$

Regression lines

$8x - 10y = -66$ (1)

$40x - 18y = 214$ (2)

Regression lines always passing through $(\bar{x}, \bar{y})$

From (1) and (2)

$8\bar{x} - 10\bar{y} = -66$ (3)

$40\bar{x} - 18\bar{y} = 214$ (4)

Solve (3) and (4)

$\boxed{\bar{x} = 13}$

$\boxed{\bar{y} = 17}$

From (1) and (2)

$8x - 10y = -66$

$-10y = -8x - 66$

$y = 0.8x + 6.6$

$\boxed{byx = 0.8}$

$40x - 18y = 214$

$40x = 18y + 214$

$x = \frac{18}{40} y + 214$

$x = 0.45y + 214$

$\boxed{bxy = 0.45}$

We know that

$r = \sqrt{byx \times bxy}$

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$$r = \sqrt{0.8 \times 0.45}$$

$$\boxed{r = 0.6}$$

Also

$$b_{yx} = r\frac{\sigma_y}{\sigma_x}$$

$$0.8 = 0.6\frac{\sigma_y}{3}$$

$$\sigma_y = \frac{0.8 \times 3}{0.6}$$

$$\sigma_y = \frac{8}{2}$$

$$\boxed{\sigma_y = 4}$$

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