ENGINEERING MATHEMATICS

UNIT-5 : *Statistical Techniques – III*

(Testing a Hypothesis and Statistical Quality Control)

Lec-09

Today's Target

- $Control\ Charts\ for\ Attributes\ (p-chart\ ,\ np-chart\ and\ C-chart)$
- PYQ
- DPP

Types of control charts

There are many types of control charts. Most commonly used control charts are

(i) Control Charts for variables

They are used to measure quality characteristics

(1) Control chart for sample mean ($\bar{X}$ – chart)

(2) Control chart for sample range (R – chart)

L - 8

(ii) Control Charts for attributes

They are used to maintain and achieve an acceptable quality level

(1) Control chart for fraction defective (p – chart)

(2) Control chart for number of defective (np – chart)

(3) Control chart for number of defects (c – chart)

L - 9

Control Charts for attributes

(i) Control charts for Fraction Defective (P – chart)

Fraction Defective (p)

$$p = \frac{\text{Number of Defective item in a sample } (d)}{\text{Sample Size } (n)}$$

$$\boxed{p = \frac{d}{n}}$$

$$d = np$$

$$\text{Total number of defective in all samples} = \sum d = \sum np$$

$$\text{Total number of items in all samples} = \sum n = N$$

Mean Fraction Defective ($\bar{p}$)

$$\bar{p} = \frac{\text{Total number of defective in all samples}}{\text{Total number of items in all samples}} = \frac{\sum np}{\sum n} = \frac{\sum d}{N}$$

Control limit on P – Chart are based on Binomial Distribution

By Binomial Distribution

$$\sigma_p = \sqrt{\frac{\bar{P}(1-\bar{P})}{n}}$$

3 σ control limits for P – chart

(*i*) $CL_p = \bar{P}$

(*ii*) $UCL_p = \bar{p} + 3\sigma_P$

(*iii*) $LCL_p = \bar{p} - 3\sigma_P$

Note : *If LCL is negative then it is considered as Zero*

Q.1 *Following is the data of defective of 10 samples of size 100 each.* **(AKTU – 2021 22)**

Sample No.	1	2	3	4	5	6	7	8	9	10
Number of defective (d)	15	11	9	6	5	4	3	2	7	1

construct p – chart and state whether the process is in statistical control

Given

$K = 10$

$n = 100$

$\Sigma d = 63$

$N = 10 \times 100 = 1000$

$\bar{p} = \frac{\Sigma d}{N} = \frac{63}{1000} = 0.063$

Sample NO	d	$P = \frac{d}{100}$
1	15	0.15
2	11	0.11
3	9	0.09
4	6	0.06
5	5	0.05
6	4	0.04
7	3	0.03
8	2	0.02
9	7	0.07
10	1	0.01

$$\bar{p} = 0.063$$

$$\sigma_p = \sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$3\sigma_p = 3\sqrt{\frac{0.063(1-0.063)}{100}}$$

$$3\sigma_p = 0.073$$

3σ control limits

$$CL_p = \bar{p}$$

$$CL_p = 0.063$$

$$UCL_p = \bar{p} + 3\sigma_p$$

$$= 0.063 + 0.073$$

$$UCL_p = 0.136$$

$$LCL_p = \bar{p} - 3\sigma_p$$

$$= 0.063 - 0.073$$

$$= -0.01$$

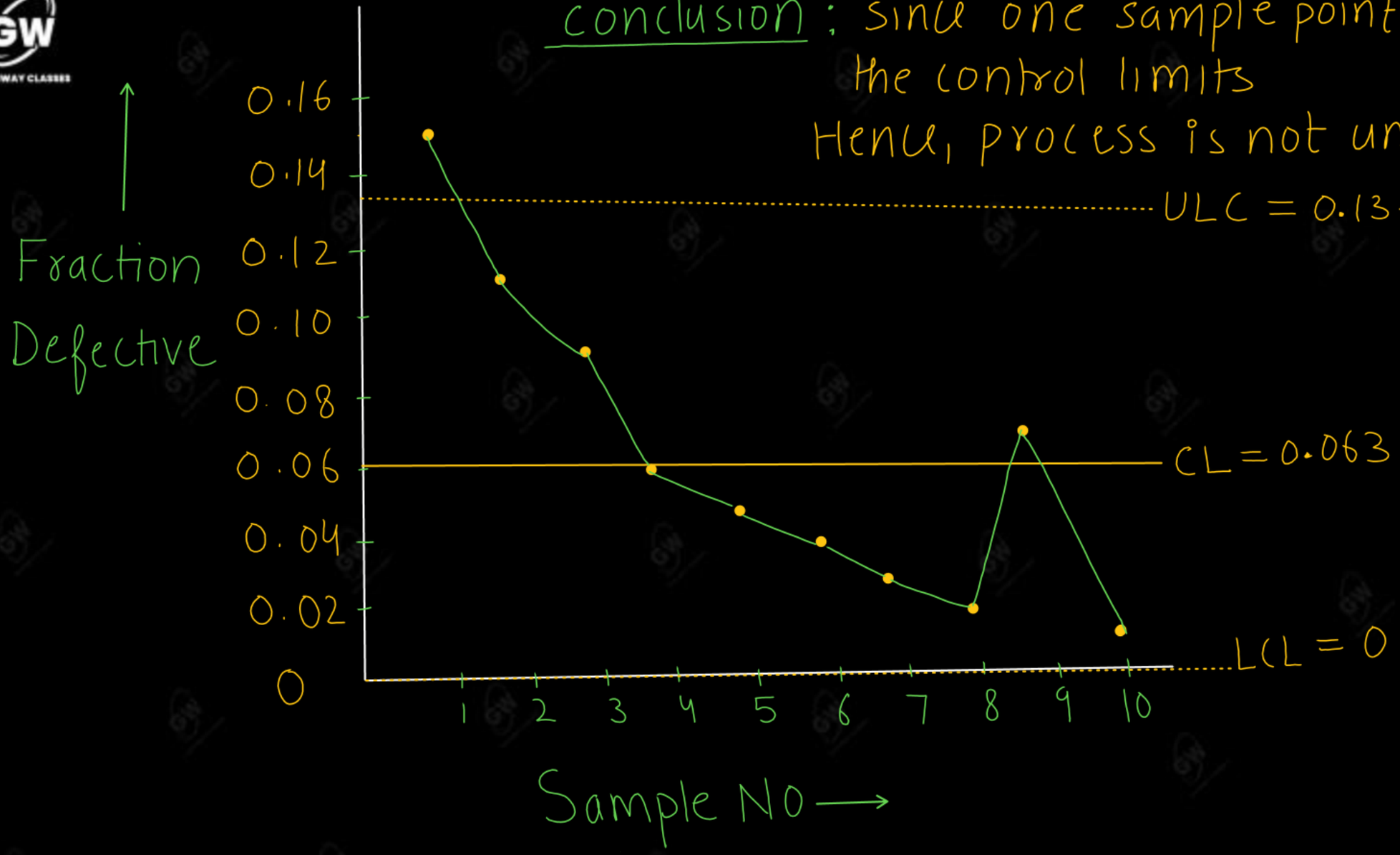
$$LCL_p \simeq 0$$

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conclusion : since one sample point lie outside the control limits

Hence, process is not under statistical control



(ii) Control chart for number of defectives ($np-chart$)

Fraction Defective (p)

$$p = \frac{\text{Number of Defective item in a sample } (d)}{\text{Sample Size } (n)}$$

$$p = \frac{d}{n}$$

$$d = np$$

Total number of defective in all samples $= \sum d = \sum np$

Total number of items in all samples $= \sum n = N$

Mean Fraction Defective ($\bar{p}$)

$$\bar{p} = \frac{\text{Total number of defective in all samples}}{\text{Total number of items in all samples}} = \frac{\sum np}{\sum n} = \frac{\sum d}{N}$$

Control limits on $np-$ Chart are based on Binomial Distribution

By binomial distribution

$$\sigma_{np} = n\sigma_p = \sqrt{\frac{n^2\bar{P}(1-\bar{P})}{n}}$$

$$\boxed{\sigma_{np} = \sqrt{n\,\bar{P}(1-\bar{P})}}$$

3 σ control limits for $np-$ chart

(*i*) $CL_{np} = n\bar{p}$

(*ii*) $UCL_{np} = n\bar{p} + 3\sigma_{np}$

(*iii*) $LCL_{np} = n\bar{p} - 3\sigma_{np}$

Note : *If LCL is negative then it is considered as Zero*

Q.2 *Distinguish between the np – chart and p – chart. Following is the data of defectives of 10 samples of size 100 each. Construct np – chart and give your comments.*

Sample no.	:	1	2	3	4	5	6	7	8	9	10
No. of difectives	:	6	9	12	5	12	8	8	16	13	7

(AKTU – 2020 21)

Given

$k = 10$

$n = 100$

$\Sigma d = 96$

$N = 1000$

$\bar{p} = \frac{\Sigma d}{N}$

$\bar{p} = \frac{96}{1000}$

$\boxed{\bar{p} = 0.096}$

$\sigma_{np} = \sqrt{n\bar{p}(1-\bar{p})}$

$3\sigma_{np} = 3\sqrt{100 \times 0.096(1-0.096)}$

$\boxed{3\sigma_{np} = 8.838}$

3σ control limits

$CL = n\bar{p} = 100 \times 0.096$

$\boxed{CL = 9.6}$

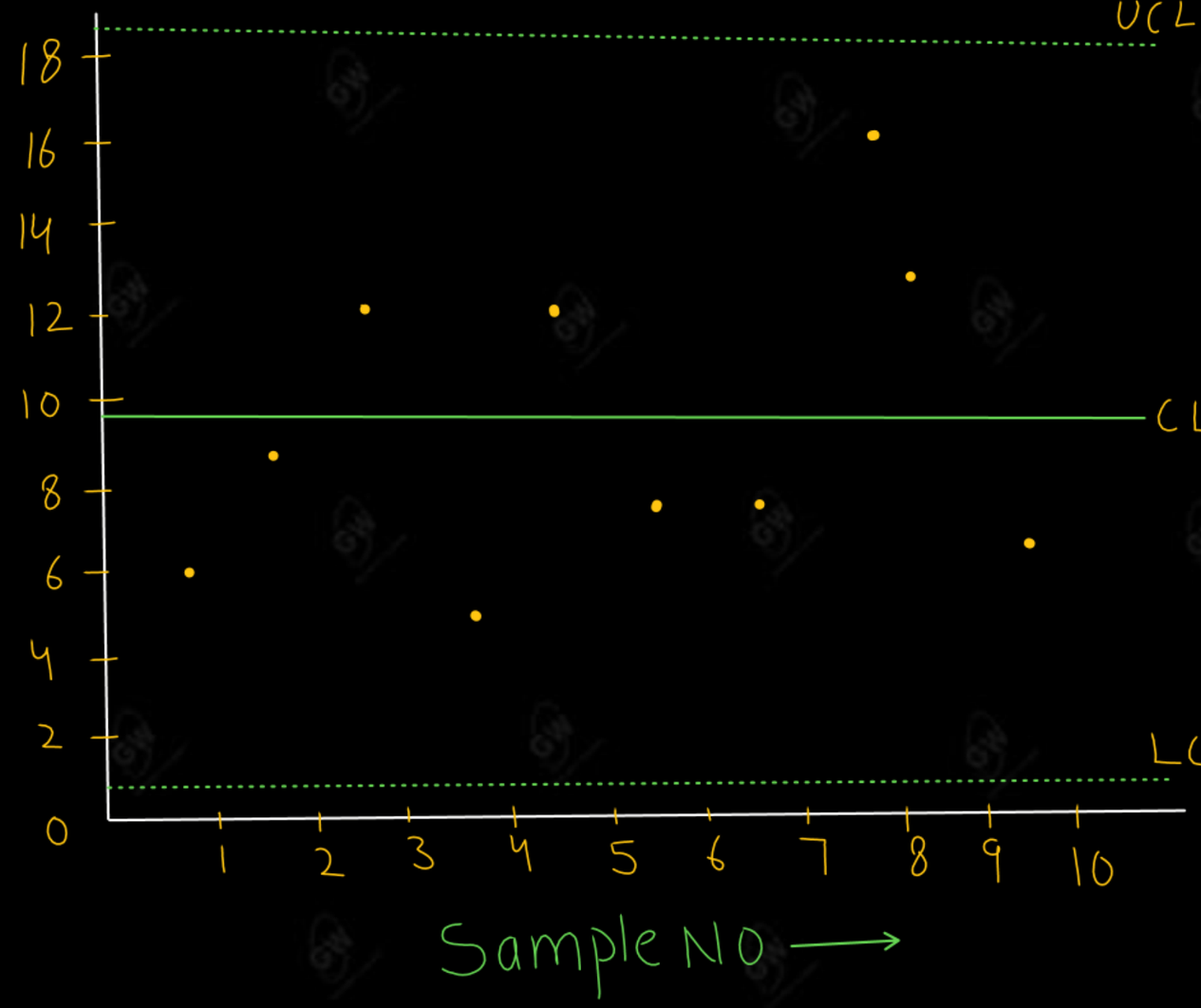
$UCL = n\bar{p} + 3\sigma_{np}$

$= 9.6 + 8.838$

$\boxed{UCL = 18.438}$

$LCL = n\bar{p} - 3\sigma_{np} = 9.6 - 838$

$\boxed{LCL = 0.762}$



conclusion : since all points lie within the control limits Hence Process is under statistical control

(iii) Control chart for number of defects ($C-$ chart)

$C-$ Chart is used when number of defects per unit are counted

Number of defects per unit $= C$

Mean of C

$$\bar{C} = \frac{\text{Number of defective in all samples}}{\text{Total number of samples}} = \frac{\Sigma C}{K}$$

Control limit on $C-$ Chart are based on poisson Distribution

Standard deviation

$$\sigma_c = \sqrt{\bar{c}}$$

3σ control limits for $C-$ chart

(i) $CL = \bar{C}$

(ii) $UCL_c = \bar{C} + 3\sigma_c$

(iii) $LCL_c = \bar{C} - 3\sigma_c$

Note : If LCL is negative then it is considered as Zero

Q.3 *The number of defects observed in 10 samples of sheet of equal dimension are given below*

Sample No.	1	2	3	4	5	6	7	8	9	10
Number of defects	2	5	3	5	4	2	3	6	7	4

Draw a control chart for the number of defects and check whether the process is under control or not

Given

$k = 10$

$\Sigma c = 41$

$\bar{c} = \frac{\Sigma c}{k}$

$\bar{c} = \frac{41}{10}$

$\boxed{\bar{c} = 4.1}$

$\sigma_c = \sqrt{\bar{c}}$

$3\sigma_c = 3\sqrt{4.1}$

$\boxed{3\sigma_c = 6.075}$

3σ control limits for c-chart

$CL = \bar{c}$

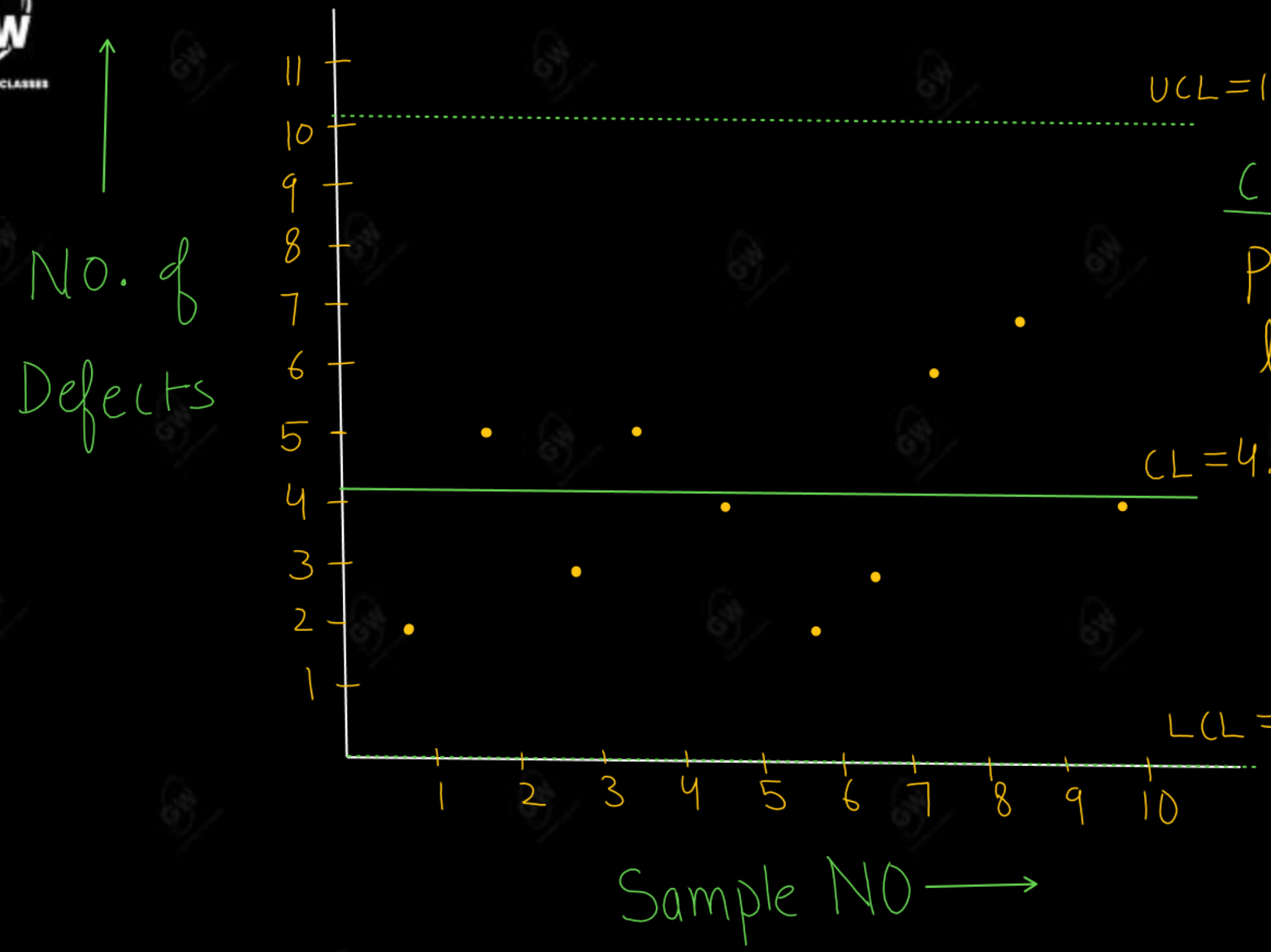
$\boxed{CL = 4.1}$

$UCL = \bar{c} + 3\sigma_c = 4.1 + 6.075$

$\boxed{UCL = 10.175}$

$LCL = \bar{c} - 3\sigma_c = 4.1 - 6.075 = -1.975$

$\boxed{LCL \simeq 0}$



conclusion: All sample points lie within control limits

Hence, process is under statistical control

Topic : *Control Charts for Attributes ($p-chart$, $np-chart$ and $C-chart$)*

Q.1 In a factory producing spark plug, the number of defectives found in inspection of 20 lots of 100 each is given below.

Lot No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
No. of defectives	5	10	12	8	6	4	6	3	3	5	4	7	8	3	3	4	5	8	6	10

Construct $p-chart$ and satte whether the process is in statistical control. (AKTU – 2011)

Q.2 Discuss how control charts can be used in quality control of industrial products. The averge percentage of defectives in 27 samples of size 1500 each was found to be 13.7%. construct $P-chart$ for this situation. Explain how the control chart can be used to control quality. (AKTU – 2014)

Q.3 An inspection of 10 samples of size 400 each from 10 lots revealed the following number of defective units. 17,15,14,26,9,4,19,12,9,15. Calculate the control limits for the number of defective units and whether the process is under control or not. (AKTU – 2021 22)

Q.4 In a blade manufacturing factory, 1000 blades are examined daily. Draw the np chart for the following table and examine whether the process is under control ? (AKTU – 2010)

Date	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
No. of defective blades	9	10	12	8	7	15	10	12	10	8	7	13	14	15	16

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