UNIT - 4

Statistical Technique - II

(Probability)

Probability

Probability is a concept which numerically measures the degree of uncertainity and certainity of the occurrence of events.

$$0 \leq P \leq 1$$

$$\text{Probability} = \frac{\text{Favorable Cases}}{\text{Total no. of mutually exclusive and equally likely cases}}$$

Random Variable

It is a real valued function whose domain is the sample space of a random experiment.

It is denoted by 'X'

X = Number of things

$X : S \rightarrow R$

(S: Domain, R: Codomain)

$X(s) = x$

Where x is any real number.

Types of Random Variable

(1) Discrete random Variable (random value finite)

(2) Continous random variable. (random value infinite)

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1. **Discrete Random Variable:**

A random variable which can assume only a finite no. of values is called a discrete random variable

Example:- The number of tails in 3 tosses of a coin is a discrete random variable (X)

$S = \{HHH, HHT, HTH, THH, TTT, TTH, THT, HTT\}$

X = Number of tails

$X(HHH) = 0$

$X(HHT, HTH, THH) = 1$

$X(TTH, THT, HTT) = 2$

$X(TTT) = 3$

Hence $X = 0, 1, 2, 3$

Probabilities

$P(X=0) = \frac{1}{8}$

$P(X=1) = \frac{3}{8}$

$P(X=2) = \frac{3}{8}$

$P(X=3) = \frac{1}{8}$

$P(X=0) + P(X=1) + P(X=2) + P(X=3) = \frac{1}{8} + \frac{3}{8} + \frac{3}{8} + \frac{1}{8}$

$= \frac{1+3+3+1}{8}$

$= \frac{8}{8} = 1$

Probability Distribution

X	0	1	2	3
P(x)	1/8	3/8	3/8	1/8

Probability Mass Function

Let $x_1, x_2, x_3 \ldots$ be the values of a discrete random variable X and let $p_1, p_2, p_3, \ldots$ be the corresponding probabilities.

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Engg. Mathematics – IV **By Gulshan Sir**

A function $p(x)$ defined by

$$p(X=x_i) = p(x) = \begin{cases} p(x_i) \text{ or } p_i & i = 1,2,3,4\cdots \\ 0 & \text{otherwise} \end{cases}$$

is called the probability mass function of the discrete random variable X where

(i) $p(x_i) \geq 0$

(ii) $\Sigma p(x_i) = 1$

Probability distribution of a discrete random variable

X	x_1	x_2	x_3	$\cdots$ x_n
$p(x)$	p_1	p_2	p_3	$\cdots$ p_n

2/3/4 ... Balls (withdrawn)

- Successively
  - With replacement (Independent events)
  - Without Replacement (Dependent events)
- Simultaneously (Dependent)

$$nC_r = \frac{n!}{r!(n-r)!}$$

1 A coin is tossed three times if X is a random variable giving the number of heads that appear, make a table showing the probability distribution of X

$S = \{HHH, HHT, HTH, THH, TTT, TTH, THT, HTT\}$

X = Number of heads

$X(TTT) = 0$

$X(HTT, TTH, THT) = 1$

$X(HHT, HTH, THH) = 2$

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$X(HHH) = 3$

Hence

$X = 0, 1, 2, 3$

Probabilities corresponding to random variables

$P(X=0) = \frac{1}{8}$

$P(X=1) = \frac{3}{8}$

$P(X=2) = \frac{3}{8}$

$P(X=3) = \frac{1}{8}$

Probability Distribution

X	0	1	2	3
P(x)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

2. A random variable X has the following probability distribution

Value of X, X: 0 1 2 3 4 5 6 7

P(x): 0 k 2k 2k 3k k^2 $2k^2$ $7k^2+k$

Find (i) k (ii) $P(X<6)$ (iii) $P(X \geq 6)$ (iv) $P(3<X \leq 6)$

(v) Minimum value of x for which $P(X \leq x) > \frac{1}{2}$

(i) We know that

$\Sigma P(x_i) = 1$

$0+k+2k+2k+3k+k^2+2k^2+7k^2+k = 1$

$9k+10k^2 = 1$

$10k^2+9k-1 = 0$

$10k^2+10k-k-1 = 0$

$10k(k+1)-(k+1) = 0$

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$(K+1)(10K-1)=0$

$K = -1, \frac{1}{10}$

(Neglet)

(Probability can never be negative)

(ii) $P(X<6) = P(X=0)+P(X=1)+P(X=2)+P(X=3)+P(X=4)+P(X=5)$

$= 0+K+2K+2K+3K+K^2$

$= K^2+8K$

$= \left(\frac{1}{10}\right)^2 + 8\times\frac{1}{10}$

$= \frac{1}{100} + \frac{8}{10}$

$$\boxed{P(X<6) = \frac{81}{100}}$$

(iii) $P(X\geq 6) = 9K^2+K$

$= \frac{9}{100} + \frac{1}{10}$

$= \frac{9+10}{100}$

$$\boxed{P(X\geq 6) = \frac{19}{100}}$$

(iv) $P(3<X\leq 6) = P(X=4)+P(X=5)+P(X=6)$

$= 3K+K^2+2K^2$

$P(3<X\leq 6) = 3K^2+3K$

$= \frac{3}{100} + \frac{3}{10}$

$= \frac{3+30}{100}$

$$\boxed{P(3<X\leq 6) = \frac{33}{100}}$$

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(v) $P(X \leq 1) = 0 + K$

$P(X \leq 1) = K$

$P(X \leq 1) = \frac{1}{10} < \frac{1}{2}$

$P(X \leq 3) = 0 + K + 2K + 2K$

$P(X \leq 3) = 5K$

$P(X \leq 3) = \frac{5}{10} = \frac{1}{2}$

$P(X \leq 4) = 0 + K + 2K + 2K + 3K$

$P(X \leq 4) = 8K$

$\boxed{P(X \leq 4) = \frac{8}{10} > \frac{1}{2}}$

Hence Minimum value of x is 4

3 Five defective bulbs are accidentally mixed with twenty good ones. It is not possible to just look at a bulb and tell wheter or not it is defective. Find the probability distribution of the number of defective bulbs, if four bulbs are drawn at random from this lot.

Given :-

No. of Defective bulbs = 5

No. of good bulbs = 20

Total no. of bulbs = 25

Soln: X = No. of Defective bulbs

$X = 0, 1, 2, 3, 4$

Probabilities corresponding to random variable :-

$$P(X=0) = \frac{20C_4}{25C_4}$$

$$P(X=0) = \frac{20!}{4! \times 16!} \times \frac{4! \times 21!}{25!} \qquad \left(nCr = \frac{n!}{r!(n-r)!}\right)$$

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$$= \frac{20 \times 19 \times 18 \times 17 \times 16! \times 21!}{16! \times 25 \times 24 \times 23 \times 22 \times 21!}$$

$$= \frac{19 \times 3 \times 17}{5 \times 2 \times 11 \times 23}$$

$$P(x=0) = \frac{969}{2530}$$

$$P(x=1) = \frac{{}^5C_1 \times {}^{20}C_3}{{}^{25}C_4}$$

$$P(x=1) = \frac{1140}{2530}$$

$$P(x=2) = \frac{{}^5C_2 \times {}^{20}C_2}{{}^{25}C_4}$$

$$P(x=2) = \frac{380}{2530}$$

$$P(x=3) = \frac{{}^5C_3 \times {}^{20}C_1}{{}^{25}C_4}$$

$$P(x=3) = \frac{40}{2530}$$

$$P(x=4) = \frac{{}^5C_4}{{}^{25}C_4}$$

$$P(x=4) = \frac{1}{2530}$$

X	0	1	2	3	4
P(x)	$\frac{969}{2530}$	$\frac{1140}{2530}$	$\frac{380}{2530}$	$\frac{40}{2530}$	$\frac{1}{2530}$

4. A random Variable X takes values 1, 2, 3, ... with probability mass function $\frac{\lambda^x}{x!}$, $x = 1, 2, 3 \ldots \infty$. Find the value of λ.

Given:-

$$P(x_i) = \frac{\lambda^x}{x!}$$

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$\sum P(x_i) = 1$

$\sum \frac{\lambda^x}{x!} = 1$

$\frac{\lambda}{1!} + \frac{\lambda^2}{2!} + \frac{\lambda^3}{3!} + \cdots = 1$

$1 + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} + \frac{\lambda^3}{3!} + \cdots = 1+1$

$e^\lambda = 2$

Taking log both side

$\lambda \log_e e = \log_e 2$

$$\boxed{\lambda = \log_e 2}$$

Expectation and Variance

If X is a discrete random variable with probability distribution

X:	x_1	x_2	x_3	.....	x_n
p(x):	$p(x_1)$	$p(x_2)$	$p(x_3)$	.....	$p(x_n)$

Expectation

$E(x) = x_1p(x_1) + x_2p(x_2) + x_3p(x_3) + \cdots + x_np(x_n)$

$$\boxed{E(x) = \sum xp(x)}$$

$E(x) = \bar{x} = \mu = \mu_1' = \sum xp(x)$

Expectation or Mathematical Expectation or Mean or first moment about origin

Important points

$\Sigma(x) = \sum xp(x) = \mu_1'$ (First moment about origin)

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$E(x^2) = \Sigma(x^2)p(x) = M_2'$ (Second moment about origin)

$E(x^3) = \Sigma(x^3)p(x) = M_3'$ (Third moment about origin)

$E(x^r) = \Sigma(x^r)p(x) = M_r'$ (r^{th} moment about origin)

Properties of Expectation:-

(i) $E(c) = c$

(ii) $E(ax) = a\,E(x)$

(iii) $E(x \pm y) = E(x) \pm E(y)$

(iv) $E(xy) = E(x) \cdot E(y)$

Varience

r^{th} moment about mean

$$M_r = \frac{\Sigma p(x-\bar{x})^r}{\Sigma p_i}$$

But $\Sigma p_i = 1$

$$M_r = \Sigma p(x-\bar{x})^r$$

$M_1 = \Sigma p(x-\bar{x})^1$

$M_2 = E(x-\bar{x})$

$M_1 = E(x) - E(\bar{x})$

$M_1 = E(x) - \bar{x}$

$M_1 = \bar{x} - \bar{x}$

$M_1 = 0$

$M_2 = \Sigma p(x-\bar{x})^2$

$M_2 = \Sigma p(x-x)^2$

Varience $= \Sigma p(x-M)^2$

$M_2 = \Sigma p(x-\bar{x})^2$

$M_2 = E(x-\bar{x})^2$

$M_2 = E(x^2 + \bar{x}^2 - 2x\bar{x})$

$M_2 = E(x^2) + E(\bar{x}^2) - E(2x\bar{x})$

$M_2 = E(x^2) + \bar{x}^2 - 2\bar{x}\,E(x)$

$M_2 = E(x^2) + \bar{x}^2 - 2\bar{x}\,\bar{x}$

$M_2 = E(x^2) + \bar{x}^2 - 2\bar{x}^2$

$M_2 = E(x^2) - (\bar{x})^2$

$$M_2 = E(x^2) - [E(x)]^2$$

Varience or second moment about mean.

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Important Formulae :-

1. **Expectation** (Mean or first moment about origin)

$$E(x) = x_1p(x_1) + x_2p(x_2) + x_3p(x_3) + \cdots + x_np(x_n)$$

$$E(x) = \Sigma xp(x)$$

2. **Variance** (Second moment about mean)

$$\text{Variance} = \mu_2 = \Sigma p(x-\mu)^2$$

$$\text{Varience} = \mu_2 = E(x^2) - [E(x)]^2$$

3 **Standard Deviation**

$$S.D = \sqrt{\text{Variance}}$$

1. Find the mean, variance and standard deviation of the number of heads when three coins are tossed.

$S = \{HHH, HHT, HTH, THH, TTT, TTH, THT, HTT\}$

x = Number of heads

$X\ (TTT) = 0$

$X(TTH, THT, THH) = 1$

$X\ (HHT, HTH, THH) = 2$

$X\ (HHH) = 3$

$x = 0, 1, 2, 3$

Probability Distribution

X	0	1	2	3
$P(x)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

Probabilities

$P(x=0) = \frac{1}{8}$

$P(x=1) = 3/8$

$P(x=2) = 3/8$

$P(x=3) = \frac{1}{8}$

mean

$$E(x) = \bar{x} = 0 \times \frac{1}{8} + 1 \times \frac{3}{8} + 2 \times \frac{3}{8} + 3 \times \frac{1}{8}$$

$$\bar{x} = 0 + \frac{3}{8} + \frac{6}{8} + \frac{3}{8}$$

$$\bar{x} = \frac{3}{2}$$

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Variance

$$E(x^2) = 0^2 \times \frac{1}{8} + 1^2 \times \frac{3}{8} + 2^2 \times \frac{3}{8} + 3^2 \times \frac{1}{8}$$

$$E(x^2) = 0 + \frac{3}{8} + \frac{12}{8} + \frac{9}{8}$$

$$E(x^2) = \frac{3+12+9}{8}$$

$$E(x^2) = 3$$

$$\text{Variance} = E(x^2) - [E(x)]^2$$

$$= 3 - \left(\frac{3}{2}\right)^2$$

$$= 3 - \frac{9}{4}$$

$$\boxed{\text{Variance} = \frac{3}{4}}$$

$$\boxed{SD = \frac{\sqrt{3}}{2}}$$

2. Two cards are drawn simultaneously from a well shuffled pack of 52 cards. Find the Expectation and variance of the number of kings.

Let X = Number of kings

X = 0, 1, 2

$$P(X=0) = \frac{48C_2}{52C_2}$$

$$\boxed{P(X=0) = \frac{188}{221}}$$

$$\left(\because {}^nC_r = \frac{n!}{r!(n-r)!}\right)$$

$$P(X=1) = \frac{4C_1 \times 48C_1}{52C_2}$$

$$\boxed{P(X=1) = \frac{32}{221}}$$

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$$P(X=2) = \frac{4C_2}{52C_2}$$

$$P(X=2) = \frac{1}{221}$$

Probability Distribution

X	0	1	2
P(x)	$\frac{188}{221}$	$\frac{32}{221}$	$\frac{1}{221}$

$$E(X^2) = 0^2 \times \frac{188}{221} + 1^2 \times \frac{32}{221} + 2^2 \times \frac{1}{221}$$

$$E(X^2) = \frac{36}{221}$$

Expectation

$$E(X) = 0 \times \frac{188}{221} + 1 \times \frac{32}{221} + \frac{2}{221}$$

$$\boxed{E(x) = \frac{2}{13}}$$

Variance :-

$$Var(X) = E(X^2) - [E(X)]^2$$

$$= \frac{36}{221} - \left(\frac{2}{13}\right)^2$$

$$\boxed{Var(X) = 0.1392}$$

Standard deviation

$$SD = \sqrt{0.1392}$$

$$\boxed{SD = 0.37}$$

3. Let X Denote the sum of the numbers obtained when two fair dice are rolled. Find the expectation of X.

X = Sum of numbers on two dice

X	2	3	4	5	6	7	8	9	10	11	12
P(X)	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

Expectation

$$E(X) = \frac{2}{36} + \frac{6}{36} + \frac{12}{36} + \frac{20}{36} + \frac{30}{36} + \frac{42}{36} + \frac{40}{36} + \frac{36}{36} + \frac{33}{36} + \frac{13}{36}$$

$$\boxed{E(X) = 7}$$

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4. A random variable X has the following distribution

X :	-2	-1	0	1	2	3
P(X):	0.1	k	0.2	2k	0.3	k

Determine (i) k (ii) Mean (iii) Variance

(i) We know that

$\Sigma p(x) = 1$

$0.1 + k + 0.2 + 2k + 0.3 + k = 1$

$4k + 0.6 = 1$

$4k = 0.4$

$\boxed{k = 0.1}$

Mean

$\bar{x} = -0.2 - k + 0 + 2k + 0.6 + 3k$

$\bar{x} = -0.2 - 0.1 + 0.2 + 0.6 + 0.3$

$\boxed{\bar{x} = 0.8}$

Variance

$E(X^2) = (-2)^2 \times 0.1 + 1^2 \times k + 0 + 1^2 \times 2k + 2^2 \times 0.3 + 3^2 \times k$

$E(X^2) = 0.4 + k + 2k + 1.2 + 9k$

$= 0.4 + 0.1 + 0.2 + 1.2 + 0.9$

$E(X^2) = 2.8$

$\text{Variance} = E(X^2) - [E(X)]^2$

$= 2.8 - (0.8)^2$

$= 2.8 - 0.64$

$\boxed{\text{Var}(X) = 2.16}$

Discrete Random Variable

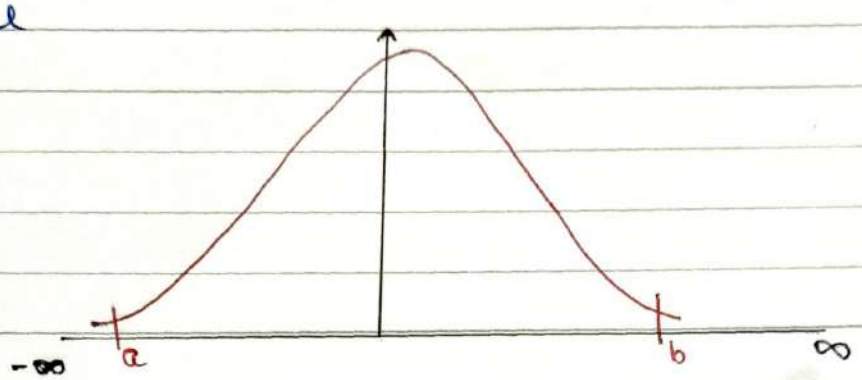
A random variable which can assume only a finite no. of values is called a discrete random variable.

* Probability Mass function
* Probability Distribution
* Expectation and Variance

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Continuous random Variable

A random variable which can assume any real value within an interval



The interval may be finite or infinite.

Probability Density function

Let X be a continous random variable, then in the interval [a,b], the PDF is defined by

$$P(a \le x \le b) = \int_a^b f(x)\,dx$$

such that

$$f(x) \ge 0 \quad \forall x$$
$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

Note $$P(a \le x \le b) = f(x) \text{ or } f(x) = \begin{cases} 0 & x < a \\ \phi(x) & a \le x \le b \\ 0 & x > b \end{cases}$$

$$e^{-\infty} = \frac{1}{e^{\infty}} = \frac{1}{\infty} = 0$$

1. If the function $f(x)$ is defined by $f(x) = ce^{-x}, 0 \le x \le \infty$, find the value of c which changes $f(x)$ to a probability density function.

$$f(x) = ce^{-x} \qquad 0 \le x \le \infty$$

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$f(x)$ is a PDF if

(i) $f(x) \geq 0$, $\forall x$

(ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_{-\infty}^{0} f(x)\,dx + \int_{0}^{\infty} ce^{-x}\,dx = 1$$

$$(-ce^{-x})_0^{\infty} = 1$$

$$-c[e^{-\infty} - e^{0}] = 1$$

$$-c[0-1] = 1$$

$$\boxed{c = 1}$$

$$f(x) = ce^{-x}$$

$$e^{-x} \geq 0$$

$$c \geq 0$$

$\Rightarrow$ $f x \geq 0$

If $f(x)$ has probability density $cx^3, 0 < x < 1$, determine c and find the probability that $\frac{1}{3} < x < \frac{1}{2}$ i.e $P\left(\frac{1}{3} < x < \frac{1}{2}\right)$

$f(x) = cx^2$, $0 < x < 1$

Since $f(x)$ is a PDF

(i) $f(x) \geq 0$

(ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

From (ii)

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_{-\infty}^{0} f(x)\,dx + \int_{0}^{1} f(x)\,dx + \int_{1}^{\infty} f(x)\,dx = 1$$

$$0 + \int_0^1 cx^2\,dx + 0 = 1$$

$$c\left(\frac{x^3}{3}\right)_0^1 = 1$$

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$$c\left(\frac{1}{3}-0\right)=1$$

$$\frac{c}{3}=1$$

$$\boxed{c=3}$$

$$f(x)=3x^2$$

$$P\left(\frac{1}{3}<x<\frac{1}{2}\right)=\int_{1/3}^{1/2} f(x)\,dx$$

$$=\int_{1/3}^{1/2} 3x^2\,dx$$

$$=3\left(\frac{x^3}{3}\right)_{1/3}^{1/2}$$

$$=\frac{1}{8}-\frac{1}{27}$$

$$=\frac{27-8}{216}$$

$$\boxed{P\left(\frac{1}{3}<x<\frac{1}{2}\right)=\frac{19}{216}}$$

Expectation and Variance

If x is a continous random variable, then the expectation of x is

$$\boxed{E(x)=\int_{-\infty}^{\infty} x f(x)\,dx}$$

Note:- $E(x)=\bar{x}=\mu=\mu_1'=\int_{-\infty}^{\infty} x f(x)\,dx$

Expectation or mean or first moment about origin

Also

* $E(x)=\int_{-\infty}^{\infty} x f(x)\,dx$ (first moment about origin)

* $E(x^2)=\int_{-\infty}^{\infty} x^2 f(x)\,dx$ (Second moment about origin)

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* $E(x^r) = \int_{-\infty}^{\infty} x^r f(x)\,dx$ (r^{th} moment about origin)

Variance

$$\boxed{Var(x) = E(x^2) - [E(x)]^2}$$

↑ Second moment about origin

Standard Deviation:

$$\boxed{S.D = \sqrt{Variance}}$$

3. If the probability density function of a random variable is given by

$$f(x) = \begin{cases} K(1-x^3) & 0 \le x \le 1 \\ 0, & \text{elsewhere} \end{cases}$$

Find (i) K (ii) Mean (iii) Varience (iv) Probability between 0.1 and 0.2 (v) Probability greater than 0.5 (AKTU 2021)

Since $f(x)$ is a PDF

(i) $f(x) \ge 0 \quad \forall x$

(ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

From (ii)

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_{-\infty}^{0} f(x)\,dx + \int_{0}^{1} f(x)\,dx + \int_{1}^{\infty} f(x)\,dx = 1$$

$$0 + \int_0^1 K(1-x^3) + 0 = 1$$

$$K\left(x - \frac{x^3}{3}\right)_0^1 = 1$$

$$K\left[1 - \frac{1}{3}\right] = 1$$

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$\frac{2}{3}K = 1$

$K = \frac{3}{2}$

$\boxed{K = 1.5}$

(ii) $\text{Mean} = \int_{-\infty}^{\infty} x f(x)\,dx$

$E(x) = \int_{-\infty}^{0} x f(x)\,dx + \int_{0}^{1} x f(x)\,dx + \int_{1}^{\infty} x f(x)\,dx$

$E(x) = 0 + \int_{0}^{1} (x - x^3)\,dx$

$E(x) = K\left[\frac{x^2}{2} - \frac{x^4}{4}\right]_0^1$

$E(x) = K\left[\frac{1}{2} - \frac{1}{4}\right]$

$E(x) = K \times \frac{1}{4}$

$E(x) = \frac{3}{2} \times \frac{1}{4}$

$\boxed{E(x) = \frac{3}{8}}$

(iii) $E(x^2) = \int_{-\infty}^{\infty} x^2 f(x)\,dx$

$E(x^2) = \int_{-\infty}^{0} x^2 f(x)\,dx + \int_{0}^{1} x^2 f(x)\,dx + \int_{1}^{\infty} x^2 f(x)\,dx$

$E(x^2) = 0 + \int_{0}^{1} x^2 K(1 - x^2)\,dx + 0$

$E(x^2) = K \int_{0}^{1} (x^2 - x^4)\,dx$

$E(x^2) = K\left(\frac{x^3}{3} - \frac{x^5}{5}\right)_0^1$

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$$E(x^2) = k\left(\frac{1}{3} - \frac{1}{5}\right)$$

$$E(x^2) = k\left(\frac{5-3}{15}\right)$$

$$E(x^2) = \frac{3}{2} \times \frac{2}{15} = \frac{1}{5}$$

$$\boxed{E(x^2) = \frac{1}{5}}$$

$$\text{Variance} = E(x^2) - [E(x)]^2$$

$$= \frac{1}{5} - \left(\frac{3}{8}\right)^2$$

$$= \frac{1}{5} - \frac{9}{64}$$

$$= \frac{64 - 45}{320}$$

$$\boxed{\text{Var}(x) = \frac{19}{320} = 0.0594}$$

(iv) $P(0.1 < x < 0.2) = \int_{0.1}^{0.2} f(x)\,dx$

$$= \int_{0.1}^{0.2} k(1-x^2)\,dx$$

$$= k\left(x - \frac{x^3}{3}\right)_{0.1}^{0.2}$$

$$= \frac{3}{2}\left[0.2 - \frac{0.008}{3} - \left(0.1 - \frac{0.001}{3}\right)\right]$$

$$= \frac{3}{2}\left[0.1 - \frac{0.008}{3} + \frac{0.001}{3}\right]$$

$$\boxed{P(0.1 < x < 0.2) = 0.1466}$$

(v) $P(0.5 < x < 1) = \int_{0.5}^{1} f(x)\,dx$

$$= k\int_{0.5}^{1} (1-x^2)\,dx$$

$$= \frac{3}{2}\left[x - \frac{x^3}{3}\right]_{0.5}^{1}$$

$$\boxed{= 0.3125}$$

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4 A continous random variable X has the probability density function

$$f(x) = \begin{cases} kx & \text{for } 0 \le x < 2 \\ 2k & \text{for } 2 \le x < 4 \\ -kx+6k & \text{for } 4 \le x \le 6 \\ 0 & \text{el se.where} \end{cases}$$

Find (i) k (ii) Mean

Since $f(x)$ is PDF

$\therefore$ (i) $f(x) \ge 0 \ \forall x$

(ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_{-\infty}^{0} f(x)\,dx + \int_{0}^{2} f(x)\,dx + \int_{2}^{4} f(x)\,dx + \int_{4}^{6} f(x)\,dx + \int_{6}^{\infty} f(x)\,dx = 1$$

$$0 + k\int_0^2 x\,dx + 2k\int_2^4 dx + k\int_4^6 (-x+6)\,dx + 0 = 1$$

$$k\left(\frac{x^2}{2}\right)_0^2 + 2k(x)_2^4 + k\left(\frac{-x^2}{2} + 6x\right)_4^6 = 1$$

$$k(2-0) + 2k(4-2) + k\left[-18+36-(-8+24)\right] = 1$$

$$2k + 4k + k(18+8-24) = 1$$

$$2k + 4k + 2k = 1$$

$$8k = 1$$

$$\boxed{k = \frac{1}{8}}$$

(iii) Mean

$$E(x) = \int_{-\infty}^{\infty} x f(x)\,dx$$

$$E(x) = \int_{-\infty}^{0} x f(x)\,dx + \int_{0}^{2} x f(x)\,dx + \int_{2}^{4} x f(x)\,dx + \int_{4}^{6} x f(x)\,dx + \int_{6}^{\infty} x f(x)\,dx$$

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$$E(x) = k\int_0^2 x^2\,dx + 2k\int_2^4 x\,dx + k\int_4^6 (-x^2+6x)\,dx$$

$$= k\left(\frac{x^3}{3}\right)_0^2 + 2k\left(\frac{x^2}{2}\right)_2^4 + k\left(\frac{-x^3}{3} + \frac{6x^2}{2}\right)_4^6$$

$$= k\left[\frac{8}{3} - 0 + 2(8-2) + \left(-72 + 3\times 36 + \frac{64}{3} - 16\times 3\right)\right]$$

$$= \frac{1}{8}\left[\frac{8}{3} + 12\left(-72 + 108 + \frac{64}{3} - 48\right)\right]$$

$$\boxed{E(x) = 3}$$

Probability Distribution

- Discrete probability Distribution (Based on discrete random variable)
  - (i) Binomial probability distribution
  - (ii) Poisson probability distribution
- Continuous probability Distribution (Based on continuous random variable)
  - (i) Normal probability Distribution

Binomial probability Distribution (Bernoulli's Distribution)

In Binomial distribution:

* Number of trials are finite
* All trials are independent
* Each trial has only two outcomes (Success or failure)

Let a random variable X denote the number of Successes in 'n' independent trials in an experiment.

Let p be the probability of success and q be the probability of failure in a single trial so that

$$p + q = 1$$

r Successes can be obtained in n trials in ${}^nC_r$ ways.

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Probability of r successes in n trials

$$P(x=r) = {}^nC_r\, P(S)\, P(S) \ldots P(S)\; P(F) P(F) \ldots P(F)$$

$$P(x=r) = {}^nC_r\, ppp \ldots p\, qqq \ldots q$$

$$\boxed{P(x=r) = {}^nC_r\, p^r q^{n-r}}$$

where

p = prob. of success
q = prob of failure
n = No. of trials
r = No. of success

where $p+q=1$

And $r = 0, 1, 2, 3 \ldots$

$$P(x=0) = {}^nC_0\, q^n$$
$$P(x=1) = {}^nC_1\, pq^{n-1}$$
$$P(x=2) = {}^nC_2\, p^2 q^{n-2}$$
$$\vdots$$
$$P(x=n) = {}^nC_n\, p^n$$

$\boxed{{}^nC_0\, q^n, {}^nC_1\, pq^{n-1}, {}^nC_2\, p^2 q^{n-2} \ldots {}^nC_n\, p^n \text{ are successive terms of binomial expansion } (q+p)^n}$

If n independent trials constitute one experiment and this experiment is repeated N times

The frequency of sucess is

$$N\, {}^nC_r\, p^r q^{n-r}$$

And Binomial Distribution is $\boxed{N(q+p)^n}$

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Engg. Mathematics – IV **By Gulshan Sir**

Mean, Variance and standard deviation of Binomial distribution

(i) Mean of Binomial Distribution.

$\boxed{\mu = np}$ first moment about origin

Variance of Binomial Distribution

$\boxed{\text{Variance} = npq}$ Second moment about mean.

Standard Deviation,

$\boxed{\sigma = \sqrt{npq}}$

Recurrence or Recursion formula for Binomial Distribution

$$\boxed{p(r+1) = \frac{n-r}{r+1} \frac{p}{q} p(r)}$$

1 Comment on the following statement: For a Binomial distribution mean is 6 and variance is 9

Mean

$$np = 6 \quad (1)$$

Variance

$$npq = 9 \quad (2)$$

Dividing eqn (2) by eqn (1)

$$\frac{npq}{np} = \frac{9}{6}$$

$$q = \frac{3}{2}$$

$\boxed{q = 1.5}$ (impossible)

Since, $0 \leq q \leq 1$

Given statement is false

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2. Ten coins are tossed simultaneously. Find the probability of getting at least seven heads.

$$P(H) = \frac{1}{2}$$

$$p = \frac{1}{2}$$

$$P(T) = \frac{1}{2}$$

$$q = \frac{1}{2}$$

$$n = 10$$

$$P(x = r) = {}^{n}C_r \, p^r q^{n-r}$$

$$P(x \geq 7) = P(x=7) + P(x=8) + P(x=9) + P(x=10)$$

$$= {}^{10}C_7 \left(\frac{1}{2}\right)^7 \times \left(\frac{1}{2}\right)^3 + {}^{10}C_8 \left(\frac{1}{2}\right)^8 \left(\frac{1}{2}\right)^2 + {}^{10}C_9 \left(\frac{1}{2}\right)^9 \left(\frac{1}{2}\right)^1 + {}^{10}C_{10} \left(\frac{1}{2}\right)^{10}$$

$$= \left(\frac{1}{2}\right)^{10} \left[{}^{10}C_7 + {}^{10}C_8 + {}^{10}C_9 + {}^{10}C_{10} \right]$$

$$= \left(\frac{1}{2}\right)^{10} \left[\frac{10 \times 9 \times 8}{3 \times 2} + \frac{10 \times 9}{2} + 10 + 1 \right]$$

$$= \left(\frac{1}{2}\right)^{10} \times \left[120 + 45 + 10 + 1 \right]$$

$$= 176 \times \left(\frac{1}{2}\right)^{10}$$

$$\boxed{P(x \geq 7) = \frac{11}{64}}$$

3. A die is thrown five times. If getting an odd number is a success, find the probability of getting at least four success

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$n = 5$

Getting an odd number is a success.

$S = \{1, 2, 3, 4, 5, 6\}$

$p = \frac{3}{6}$

$\boxed{P = \frac{1}{2}}$ $\boxed{q = \frac{1}{2}}$

$P(x \geq 4) = P(x=4) + P(x=5)$

$= {}^5C_4 \left(\frac{1}{2}\right)^4 \times \frac{1}{2} + {}^5C_5 \left(\frac{1}{2}\right)^5$

$= \left(\frac{1}{2}\right)^5 \left[{}^5C_4 + {}^5C_5\right]$

$= \frac{1}{32} (5+1)$

$= \frac{1}{32} \times 6$

$\boxed{P(x \geq 4) = \frac{3}{16}}$

4. A die is tossed thrice. A success is getting 1 or 6 on a toss. Find the mean and variance of the number of success.

$S = \{1, 2, 3, 4, 5, 6\}$

Prob. of success

$p = \frac{2}{3}$

$\boxed{p = \frac{1}{3}}$

Prob. of failure

$\boxed{q = \frac{2}{3}}$

$n = 3$

Mean

$M = np = 3 \times \frac{1}{3}$

$\boxed{M = 1}$

Variance

$var(x) = npq = 3 \times \frac{1}{3} \times \frac{2}{3}$

$\boxed{Var(x) = \frac{2}{3}}$

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5 A binomial variable X satisfies the relation $9P(X=4)=P(X=2)$ when $n=6$. Find the value of the parameter p and $P(X=1)$

$n=6$

We know that

$P(X=x) = {}^{n}C_x\, p^x\, q^{n-x}$

$P(X=4) = {}^{6}C_4\, p^4 q^2$

$P(X=2) = {}^{6}C_2\, p^2 q^4$

According to question

$9P(X=4) = P(X=$

$9 \times {}^{6}C_4 \times p^4 q^2 = {}^{6}C_2 \times p^2 q^4$

$9 \times \frac{6!}{2!\times 4!} \times p^2 = \frac{6!}{4!\times 2!} \times q^2$

$9p^2 = q^2$

$9p^2 = (1-p)^2$

$9p^2 = 1 + p^2 - 2p$

$8p^2 + 4p - 2p - 1 = 0$

$4p(2p+1) - 1(2p+1) = 0$

$(4p-1)(2p+1) = 0$

$\boxed{p = \frac{1}{4}}$, $p = -\frac{1}{2}$ (Reject)

$q = 1 - p$

$q = 1 - \frac{1}{4} = \frac{3}{4}$

$\boxed{q = \frac{3}{4}}$

$P(X=x) = {}^{n}C_x\, p^x\, q^{n-x}$

$P(X=1) = {}^{6}C_1 \left(\frac{1}{4}\right)^1 \times \left(\frac{3}{4}\right)^5$

$P(X=1) = 6 \times \frac{1}{4} \times \frac{3^5}{4^5}$

$P(X=1) = \frac{729}{2040}$

$\boxed{P(X=1) = 0.3559}$

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6 Fit a binomial distribution to the following frequency data

x:	0	1	2	3	4
f:	24	41	28	5	2

x	f	fx
0	24	0
1	41	41
2	28	56
3	5	15
4	2	8
	$\Sigma f = 100$	$\Sigma fx = 120$

Mean

$$\mu = \frac{\Sigma fx}{\Sigma f}$$

$$= \frac{120}{100}$$

$$\boxed{\mu = 1.2}$$

For Binomial Distribution

$$\mu = np$$

$$1.2 = 4 \times p$$

$$p = 0.3$$

$$q = 0.7$$

Binomial Distribution

$$N(q+p)^n = 100\,(0.7+0.3)^4$$

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Binomial probability Distribution

Let a random variable X denote the number of successes in n independent trials in an experiment.
Let p be the probability of success and q be the probability of failure in a single trial so that

$$p+q=1$$

r successes can be obtained in n trials in ^nC_r ways

Probability of r successes in n trials

$$P(X=r) = {}^nC_r\, P(S)\,P(S)\ldots P(S)\;\; P(F)P(F)\ldots P(F)$$

$$P(X=r) = {}^nC_r\, ppp\ldots p\, qqq\ldots q$$

$$P(X=r) = {}^nC_r\, p^r q^{n-r}$$

If n independent trials constitute one experiment and this experiment is repeated N times.
The frequency of success is

$$\boxed{N\, {}^nC_r\, p^r q^{n-r}}$$

And binomial distribution is $N(q+p)^n$

Mean, variance and Standard deviation of binomial Distribution

(i) Mean of Binomial Distribution

$$\boxed{M=np}$$

iii Standard Deviation

$$\boxed{\sigma=\sqrt{npq}}$$

(ii) Variance of Binomial Distribution

$$\boxed{\text{Variance} = npq}$$

(Third moment about mean = $npq(q-p)$)

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If 10% of bolts produced by a machine are defective. Determine the probability that out of 10 bolts choosen at random (i) 1 (ii) None (iii) at most 2 bolts will be defective

$$P(\text{defective}) = \frac{10}{100}$$

$$\boxed{p = \frac{1}{10}}$$

$$P(\text{non Defective}) = 1 - \frac{1}{10}$$

$$\boxed{q = \frac{9}{10}}$$

$$n = 10$$

(i) $P(X = r) = {}^{n}C_r \, p^r q^{n-r}$

$$P(X = 1) = {}^{10}C_1 \left(\frac{1}{10}\right)^1 \times \left(\frac{9}{10}\right)^9$$

$$P(X = 1) = 10 \times \frac{1}{10} \times (0.9)^9$$

$$\boxed{P(X = 1) = 0.3874}$$

(ii) $P(X = r) = {}^{n}C_r \, p^r q^{n-r}$

$$P(X = 0) = {}^{10}C_0 \left(\frac{1}{10}\right)^0 \times \left(\frac{9}{10}\right)^{10}$$

$$P(X = 0) = 1 \times 1 \times (0.9)^{10}$$

$$\boxed{P(X = 0) = 0.3487}$$

(iii) $P(r \leq 2) = P(X = 0) + P(X = 1) + P(X = 2)$ (1)

$$P(X = 2) = {}^{10}C_2 \, p^2 q^8$$

$$P(X = 2) = \frac{10!}{8! \times 2!} \times \left(\frac{1}{10}\right)^2 \times \left(\frac{9}{10}\right)^8$$

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$$= \frac{10 \times 9 \times 8!}{8! \times 2} \times \frac{1}{100} \times (0.9)^8$$

$$\boxed{P(x=2) = 0.1937}$$

Required probability

$$P(x \leq 2) = 0.3487 + 0.3874 + 0.1937$$

$$\boxed{P(x \leq 2) = 0.9298}$$

2. If the probability of hitting a target is 10% and 10 shots are fired independently. What is the probability that the target will be hit at least once?

$$P(\text{Hitting a target}) = \frac{10}{100}$$

$$\boxed{p = \frac{1}{10}}$$

$$P(\text{Not Hitting target}) = 1 - \frac{1}{10}$$

$$\boxed{q = \frac{9}{10}}$$

$$n = 10$$

$$P(x \geq 1) = 1 - P(X=0)$$

$$P(x \geq 1) = 1 - {}^{n}C_x \, p^x q^{n-x}$$

$$P(x \geq 1) = 1 - {}^{10}C_0 \left(\frac{1}{10}\right)^0 \times \left(\frac{9}{10}\right)^{10}$$

$$P(x \geq 1) = 1 - 1 \times 1 \times (0.9)^{10}$$

$$\boxed{P(x \geq 1) = 0.6513}$$

3. In 800 families with 5 children each. how many families would be expected to have (i) 3 boys and 2 girls (ii) 2 boys and 3 girls (iii) no girl (iv) at the most two girls. Assume probabilities for boys and girls to be equal.

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$N = 800$

$n = 5$

$P(\text{Having a girl}) = \frac{1}{2}$

$\boxed{p = \frac{1}{2}}$

$P(\text{Having a boy}) = \frac{1}{2}$

$\boxed{q = \frac{1}{2}}$

(i) Expected number of families having 3 boys and 2 Girls

$$NP(X=2) = N^nC_x p^x q^{n-x}$$

$$= 800 \times {}^5C_2 \left(\frac{1}{2}\right)^2 \times \left(\frac{1}{2}\right)^3$$

$$= 800 \times \frac{5!}{3!2!} \times \left(\frac{1}{2}\right)^5$$

$$\boxed{NP(X=2) = 250}$$

(ii) No. of families having 2 boys and 3 girls

$$NP(X=3) = N^nC_x p^x q^{n-x}$$

$$= 800 \times {}^5C_3 \left(\frac{1}{2}\right)^3 \times \left(\frac{1}{2}\right)^2$$

$$\boxed{NP(X=3) = 250}$$

(iii) No. of families having no girl

$$NP(X=0) = N^nC_x p^x q^{n-x}$$

$$= 800 \times {}^5C_0 \left(\frac{1}{2}\right)^0 \times \left(\frac{1}{2}\right)^5$$

$$\boxed{NP(X=0) = 25}$$

(iv) No. of families having atmost two girls

$$NP(x \leq 2) = N[P(X=0) + P(X=1) + P(X=2)]$$

$$P(X=1) = {}^5C_1 p^x q^{n-x}$$

$$= 5 \times \left(\frac{1}{2}\right)^1 \times \left(\frac{1}{2}\right)^4$$

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$= 5 \times \frac{1}{2^5}$

$= \frac{5}{32}$

$NP(x \leq 2) = NP(X=0) + NP(X=1) + NP(X=2)$

$= 25 + 800 \times \frac{5}{32} + 250$

$= 25 + 125 + 250$

$\boxed{NP(x \leq 2) = 400}$

4. A student is given a true false examination with 8 questions. If he correct at least 7 questions, he passes the examination. Find the probability that he will pass given that he guesses all questions

$n = 8$

$p = \frac{1}{2}$

$q = \frac{1}{2}$

Prob. that he will pass

$P(x \geq 7) = P(X=7) + P(X=8)$

$= {}^8C_7 \left(\frac{1}{2}\right)^7 \times \left(\frac{1}{2}\right)^1 + {}^8C_8 \left(\frac{1}{2}\right)^8 \left(\frac{1}{2}\right)^0$

$= 8 \times \left(\frac{1}{2}\right)^8 + 1 \times \left(\frac{1}{2}\right)^8$

$= \left(\frac{1}{2}\right)^8 (8+1)$

$= \frac{9}{2^8}$

$\boxed{P(x \geq 7) = 0.035}$

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Six dies are thrown 729 times. How many times do you expect at least three dice to show a five or six?

$N = 729$

$n = 6$

$S = \{1, 2, 3, 4, 5, 6\}$

$P(\text{having } 5 \text{ or } 6) = \frac{2}{6}$

$$\boxed{p = \frac{1}{3}}$$

$P(\text{having not } 5 \text{ or } 6) = \frac{4}{6}$

$$\boxed{q = \frac{2}{3}}$$

$$NP(x \geq 3) = N[P(x=3) + P(x=4) + P(x=5) + P(x=6)]$$

$$= 729\left[{}^6C_3\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^3 + {}^6C_4\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^2 + {}^6C_5\left(\frac{1}{3}\right)^5\left(\frac{2}{3}\right)^1 + {}^6C_6\left(\frac{1}{3}\right)^6\left(\frac{2}{3}\right)^0\right]$$

$$NP(x \geq 3) = 729\left[{}^6C_3 \times \frac{8}{3^6} + {}^6C_4 \times \frac{4}{3^6} + {}^6C_5 \times \frac{2}{3^6} + {}^6C_5 \times \frac{2}{3^6} + {}^6C_6 \times \frac{1}{3^6}\right]$$

$$= \frac{729}{36}\left[{}^8C_3 + {}^4C_4 + 2\,{}^6C_5 + {}^6C_6\right]$$

$$= \frac{729}{729}\left[\frac{8 \times 6!}{3! \times 3!} + 4 \times \frac{6!}{4! \times 2!} + 2 \times 6 + 1\right]$$

$$= 160 + 60 + 12 + 1$$

$$\boxed{NP(x \geq 3) = 233}$$

Poisson Probability Distribution

* Poisson Distribution is a limiting case of binomial distribution

$$P(X=r) = \frac{\lambda^r e^{-\lambda}}{r!} \quad (r=0,1,2,3,\cdots)$$

* Recurrence formula for the Poisson Distribution

$$P(r+1) = \frac{\lambda}{(r+1)} P(r) \quad (r=0,1,2,3,\cdots)$$

* Mean and variance of the poisson Distribution

$$\text{Mean} = \lambda$$
$$\text{variance} = \lambda \quad \{\lambda = np\}$$

Important Derivations:

1. Show that Poisson Distribution is a limiting case of binomial distribution. (AKTU 2021)

In Binomial Distribution

n = no. of trials

p = Prob. of sucess

If n is very large and p is very small then Binomial distribution become complicated

If we assume that $n \to \infty$ and $p \to 0$ such that $np = \lambda$ is finite

Then we get poisson approximation to binomial distribution.

$$P(X=r) = \frac{e^{-\lambda} \lambda^r}{r!}$$

Proof:

$$P(X=r) = {}^nC_r p^r q^{n-r}$$

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$$P(X=r) = {}^nC_r\, p^r (1-p)^{n-r} \quad \{p+q=1\}$$

$$P(X=r) = \frac{n!}{(n-1)!\,r!}\; p^r \times (1-p)^n (1-p)^{-r}$$

$$P(X=r) = \frac{n!}{(n-r)!} \times \left(\frac{\lambda}{n}\right)^r \times \frac{\left(1-\frac{\lambda}{n}\right)^n}{\left(1-\frac{\lambda}{n}\right)^r}$$

$$P(X=r) = \frac{\lambda^r \times n(n-1)(n-2)\ldots(n-r+1)(n-r)!\left(1-\frac{\lambda}{n}\right)^n}{r!\times(n-r)!\times n^r \times\left(1-\frac{\lambda}{n}\right)}$$

$$P(X=r) = \frac{\lambda^r}{r!} \times \frac{n}{n} \times \left(\frac{n-1}{n}\right)\left(\frac{n-2}{n}\right)\ldots\left(\frac{n-r+1}{n}\right) \frac{\left(1-\frac{\lambda}{n}\right)^n}{\left(1-\frac{\lambda}{n}\right)^r}$$

$$P(X=r) = \frac{\lambda^r}{r!} \times 1 \times \left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\ldots\left(1-\frac{r}{n}+\frac{1}{n}\right) \frac{\left(1-\frac{\lambda}{n}\right)^n}{\left(1-\frac{\lambda}{n}\right)^r}$$

$$P(X=r) = \lim_{n\to\infty} \frac{\lambda^r}{r!}\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\ldots\left(1-\frac{r}{n}+\frac{1}{n}\right) \frac{\left(1-\frac{\lambda}{n}\right)^n}{\left(1-\frac{\lambda}{n}\right)^r}$$

$$= \frac{\lambda^r}{r!}(1-0)(1-0)\ldots(1-0+0) \frac{\lim_{n\to\infty}\left(1-\frac{\lambda}{n}\right)^n}{\lim_{n\to\infty}\left(1-\frac{\lambda}{n}\right)^r}$$

$$= \frac{\lambda^r}{r!} \times 1 \times 1 \ldots \times 1 \times \frac{e^{-\lambda}}{1}$$

$$= \frac{\lambda^r}{r!} \times e^{-\lambda}$$

$$\boxed{P(X=r) = \frac{\lambda^r e^{-\lambda}}{r!}}$$ Poisson Distribution

$$\lim_{n\to\infty}\left(1-\frac{x}{n}\right)^n = e^{-x}$$

$$\lim_{n\to\infty}\left(1-\frac{x}{n}\right)^r = 1$$

Poisson Distribution

$$\boxed{P(X=r) = \frac{\lambda^r e^{-\lambda}}{r!}} \quad (r=0,1,2,3,\ldots)$$

λ is called the parameter of the distribution.

$\lambda = np$ (Finite)

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The sum of the probability $P(x)$ for $x = 0, 1, 2, 3, \ldots$ is 1

$$\Sigma P(x) = 1$$

$$P(X=0) + P(X=1) + P(X=2) + P(X=3) \ldots = 1$$

2. Recurrence formula for the Poisson Distribution

By poisson Distribution

$$P(x) = \frac{\lambda^x e^{-\lambda}}{x!} \quad -(1)$$

Replace 'x' by '$x+1$'

$$P(x+1) = \frac{\lambda^{x+1} e^{-\lambda}}{(x+1)!}$$

$$P(x+1) = \frac{\lambda^x \times \lambda^1 \times e^{-\lambda}}{(x+1) \times x!} \quad -(2)$$

Divide (2) by (1)

$$\frac{P(x+1)}{P(x)} = \frac{\lambda^x \times \lambda \times e^{-\lambda}}{(x+1) \times \lambda^x}$$

$$\frac{P(x+1)}{P(x)} = \frac{\lambda}{x+1}$$

$$\boxed{P(x+1) = \frac{\lambda}{x+1} P(x)}$$

3. Mean and Variance of the Poisson Distribution

By Poisson Distribution

$$P(x) = \frac{e^{-\lambda} \lambda^x}{x!}$$

Mean

$$\mu = E(x) = \sum_{x=0}^{\infty} x p(x)$$

$$\mu = \sum_{x=0}^{\infty} \frac{x e^{-\lambda} \lambda^x}{x!}$$

$$\mu = e^{-\lambda} \sum_{x=1}^{\infty} \frac{\lambda^x}{x(x-1)!}$$

$$\mu = e^{-\lambda} \sum_{x=1}^{\infty} \frac{\lambda^x}{(x-1)!}$$

$$\mu = e^{-\lambda} \left(\frac{\lambda}{1} + \frac{\lambda^2}{1!} + \frac{\lambda^3}{2!} + \frac{\lambda^4}{3!} + \ldots \right)$$

$$\mu = e^{-\lambda} \lambda \left(1 + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} + \frac{\lambda^3}{3!} + \ldots \right)$$

$$\text{Mean} = e^{-\lambda} \times \lambda \times e^{\lambda} = \lambda \times e^{0}$$

$$\boxed{\text{Mean} = \lambda}$$

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Variance

$\sigma^2 = E(x^2) - [E(x)]^2$

$\sigma^2 = \sum_{x=0}^{\infty} x^2 P(x) - (\lambda)^2$

$\sigma^2 = \sum_{x=0}^{\infty} \frac{x^2 e^{-\lambda} \lambda^x}{x!} - \lambda^2$

$\sigma^2 = e^{-\lambda} \sum_{x=1}^{\infty} \frac{x^2 \lambda^x}{x!} - \lambda^2$

$\sigma^2 = e^{-\lambda}\left(\frac{1^2 \times \lambda}{1!} + \frac{2^2 \times \lambda^2}{2!} + \frac{3^2 \times \lambda^3}{3!} + \cdots\right) - \lambda^2$

$\sigma^2 = e^{-\lambda} \times \lambda\left(1 + \frac{2\lambda}{1!} + \frac{3\lambda^2}{2!} + \frac{4\lambda^3}{3!} + \cdots\right) - \lambda^2$

$\sigma^2 = \lambda e^{-\lambda}\left[1 + \frac{\lambda + \lambda}{1!} + \frac{\lambda^2 + 2\lambda^2}{2!} + \frac{\lambda^3 + 3\lambda^3}{3!} + \cdots\right] - \lambda^2$

$\sigma^2 = \lambda e^{-\lambda}\left[1 + \frac{\lambda}{1!} + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} + \frac{2\lambda^2}{2!} + \frac{\lambda^3}{3!} + \frac{3\lambda^3}{3!} + \cdots\right] - \lambda^2$

$\sigma^2 = \lambda e^{-\lambda}\left[\left(1 + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} + \cdots\right) + \left(\frac{\lambda}{1!} + \frac{2\lambda^2}{2!} + \frac{3\lambda^3}{3!} + \cdots\right)\right] - \lambda^2$

$\sigma^2 = \lambda e^{-\lambda}\left[\left(1 + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} + \frac{\lambda^3}{3!} + \cdots\right) + \lambda\left(1 + \frac{2\lambda}{2!} + \frac{3\lambda^2}{3!} + \frac{4\lambda^3}{4!} + \cdots\right)\right] - \lambda^2$

$\sigma^2 = \lambda e^{-\lambda}\left[e^{\lambda} + \lambda\left(1 + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} + \frac{\lambda^3}{3!} + \cdots\right)\right] - \lambda^2$

$\sigma^2 = \lambda e^{-\lambda}\left[e^{\lambda} + \lambda e^{\lambda}\right] - \lambda^2$

$\sigma^2 = \lambda e^{-\lambda} e^{\lambda} + \lambda^2 e^{-\lambda} e^{\lambda} - \lambda^2$

$\sigma^2 = \lambda + \lambda^2 - \lambda^2$

$\sigma^2 = \lambda$

Variance = λ

1. If a random variable X follow a poisson distribution such that $P(X=2) = 9P(X=4) + 90P(X=6)$. Find mean and variance of X (AKtu 2023)

For poisson Distribution

$$P(X=x) = \frac{e^{-\lambda}\lambda^x}{x!}$$

$P(X=2) = 9P(X=4) + 90P(X=6)$

$\frac{e^{-\lambda}\lambda^2}{2!} = \frac{9 \times e^{-\lambda}\lambda^4}{4!} + \frac{90 \times e^{-\lambda}\lambda^6}{6!}$

$\frac{e^{-\lambda}\lambda^2}{2!} = e^{-\lambda}\left(\frac{9\lambda^4}{4!} + \frac{90\lambda^6}{6!}\right)$

$\frac{\lambda^2}{2} = \lambda^2\left(\frac{9\lambda^2}{4\times3\times2\times1} + \frac{90\lambda^4}{6\times5\times4\times3\times2\times1}\right)$

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$$\frac{1}{2} = \frac{3\lambda^2}{8} + \frac{\lambda^4}{8}$$

$$\frac{1}{2} = \frac{3\lambda^2 + \lambda^4}{8}$$

$$4 = 3\lambda^2 + \lambda^4$$

$$\lambda^4 + 3\lambda^2 - 4 = 0$$

Put $\lambda^2 = x$

$$x^2 + 3x - 4 = 0$$

$$x^2 + 4x - x - 4 = 0$$

$$x(x+4) - 1(x+4) = 0$$

$$(x-1)(x+4) = 0$$

$x = 1$ { $x = -4$ } → Reject

$\lambda^2 = 1$ { $\lambda^2 = -4$ }

$$\lambda = \pm 1$$

$$\lambda = 1$$

Hence $\lambda = 1$

Mean = 1
Variance = 1

2. If the variance of the poisson Distribution is 2 find the probabilities for $x = 1, 2, 3, 4$ from the recurrence relation of the poisson

By Poisson Distribution

$$P(x) = \frac{e^{-\lambda} \lambda^x}{x!}$$

$$P(0) = \frac{e^{-2} \times \lambda^0}{0!}$$

$$P(0) = e^{-2}$$

$P(0) = 0.1353$

By Recurrence Relation

$$P(x+1) = \frac{\lambda}{x+1} P(x)$$

$$P(0+1) = \frac{2}{0+1} P(0)$$

$$P(1) = 2\,P(0)$$

$P(1) = 0.2706$

$$P(1+1) = \frac{2}{1+1} P(1)$$

$$P(2) = P(1)$$

$P(2) = 0.2706$

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$$P(2+1) = \frac{2}{2+1} P(2)$$

$$P(3) = \frac{2}{3} \times 0.2706$$

$$\boxed{P(3) = 0.1804}$$

$$P(3+1) = \frac{2}{3+1} P(3)$$

$$P(4) = \frac{1}{2} \times 0.1804$$

$$\boxed{P(4) = 0.0902}$$

$$P(x \geq 4) = 1 - P(x < 4)$$

$$P(x \geq 4) = 1 - [P(0) + P(1) + P(2) + P(3)]$$

$$P(x \geq 4) = 1 - 0.8569$$

$$\boxed{P(x \geq 4) = 0.1431}$$

3. Using Poisson distribution. Find the probability that the ace of spades will be drawn from a pack of well-shuffled cards at least once in 104 consecutive trials (UPTU 2015)

$$P(\text{ace of spade}) = \frac{1}{52}$$

$$p = \frac{1}{52}, \quad n = 104$$

$$\lambda = np$$

$$\lambda = 104 \times \frac{1}{52}$$

$$\lambda = 2$$

By poisson Distribution

$$P(x) = \frac{e^{-\lambda} \lambda^x}{x!}$$

$$P(x \geq 1) = 1 - P(X = 0)$$

$$= 1 - \frac{e^{-2} \times \lambda^0}{0!}$$

$$= 1 - e^{-2}$$

$$\boxed{P(x \geq 1) = 0.8647}$$

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Poisson Probability Distribution:

1. It is given that 2% of the electric bulbs manufactured by a company are defective. Using Poisson distribution. Find the probability that a sample of 200 bulbs will contain (i) no defective bulb (ii) two defective bulbs (iii) at the most three defective bulbs (GBTU 2011)

Prob. of Defective bulbs

$$p = \frac{2}{100}$$

$$p = 0.02$$

$$n = 200$$

$$\lambda = np$$

$$\lambda = 0.02 \times 200$$

$$\boxed{\lambda = 4}$$

By poisson distribution

$$P(X = x) = \frac{e^{-\lambda} \lambda^x}{x!}$$

(i) Prob. of no defective bulb $(x = 0)$

$$P(X = 0) = \frac{e^{-4} \times (4)^0}{0!} = e^{-4}$$

$$\boxed{P(X = 0) = 0.018315}$$

(ii) Prob of two defective $(x = 2)$

$$P(X = 2) = \frac{e^{-4} (4)^2}{2!} = \frac{16}{2} e^{-4}$$

$$P(X = 2) = 8e^{-4}$$

$$\boxed{P(X = 2) = 0.146525}$$

(iii) Prob. of at most 3 defective bulbs $(x \leq 3)$

$$P(x \leq 3) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3)$$

$$= e^{-4} + \frac{e^{-4} \times 4}{1!} + 8e^{-4} + \frac{e^{-4} (4)^3}{3!}$$

$$= e^{-4} + 4e^{-4} + 8e^{-4} + \frac{32}{3} e^{-4}$$

$$= e^{-4} \left(13 + \frac{32}{3} \right)$$

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$$= e^{-4}\left(\frac{39+32}{3}\right)$$

$$= \frac{71}{3}e^{-4}$$

$$\boxed{P(x \leq 3) = 0.43347}$$

2. A manufacturer knows that the condensors he makes contain on an average 1% of defectives. He packes them in boxes of 100. What is the probability that a box picked at random will contain 4 or more faulty condensors?

Probability of defective condensors

$$P = \frac{1}{100}$$

$$P = 0.01$$

$$n = 100$$

$$\lambda = np = 100 \times 0.01$$

$$\boxed{\lambda = 1}$$

By poisson distribution

$$P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}$$

Probability of 4 or more faulty condensor $(x \geq 4)$

$$P(x \geq 4) = 1 - P(x < 4)$$

$$P(x \geq 4) = 1 - [P(X=0) + P(X=1) + P(X=2) + P(X=3)]$$

$$= 1 - \left[\frac{e^{-1}(1)^0}{0!} + \frac{e^{-1} \times (1)^1}{1!} + \frac{e^{-1} \times 1^2}{2!} + \frac{e^{-1} \times 1^3}{3!}\right]$$

$$= 1 - \left[e^{-1} + e^{-1} + \frac{1}{2}e^{-1} + \frac{1}{6}e^{-1}\right]$$

$$= 1 - e^{-1}\left(2e^{-1} + \frac{e^{-1}}{2} + \frac{e^{-1}}{6}\right)$$

$$= 1 - e^{-1}\left(\frac{12e^{-1} + 3e^{-1} + e^{-1}}{6}\right)$$

$$= \left(1 - \frac{8}{3}e^{-1}\right)$$

$$\boxed{P(x \geq 4) = 0.018988}$$

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3. In a certain factory turning out razor blades, there is a small chance of 0.002 for any blade to be defective. The blades are supplied in packets of 10. Calculate the approximate number of packets containing no defective, one defective and two defective blades in a consignment of 10,000 packets. (Given: $e^{-0.02} = 0.9802$) (AKTU 2018)

Prob. of defective blade

$p = 0.002$

$n = 10$

$N = 10,00$

$\lambda = np = 0.002 \times 10$

$\boxed{\lambda = 0.02}$

By poisson Distribution

$$P(X = x) = \frac{e^{-\lambda} \lambda^x}{x!}$$

(i) No. of packets containing no defective blades

$$P(X=0) = \frac{e^{-0.02} \times (0.02)^0}{0!}$$

$P(X=0) = e^{-0.02}$

$P(X=0) = 0.9802$

Number of packets $= 10,000 \times 0.9802$

$= 9802$

(ii) $P(X=1) = \frac{e^{-0.02} \times (0.02)^1}{1!}$

$P(X=1) = 0.0196$

No. of packets having 1 defective blades $= 0.0196 \times 10.000$

$= 196$

(iii) $P(X=2) = \frac{e^{-0.02} \times (0.02)^2}{2!} = 0.000196$

No. of packets having 2 defective blades $= 0.000196 \times 10,000$

$= 1.96$

$= 2$

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4 The number of accidents in a year involving taxi drivers in a city follows a Poisson distribution with mean equal to 3 Out of 1000 taxi drivers, find approximately the number of drivers with (i) no accidents (ii) more than 3 accidents in a year (AKTU 2021)

$\lambda = 3$, $N = 1000$

By poisson Distribution

$$P(X=x) = \frac{e^{-\lambda}\lambda^x}{x!}$$

(i) $P(X=0) = \frac{e^{-3}(3)^0}{0!} = e^{-3}$

$P(X=0) = 0.04978$

Number of drivers having no accidents $= 0.04978 \times 1000$

$= 49.7$

$= 50$

(ii) $P(x>3) = 1-[P(X=0)+P(X=1)+P(X=2)+P(X=3)]$

$$= 1-\left[e^{-3} + \frac{e^{-3}\times(3)^1}{1!} + \frac{e^{-3}\times(3)^2}{2!} + \frac{e^{-3}\times(3)^3}{3!}\right]$$

$$= 1-\left[e^{-3} + 3e^{-3} + \frac{9}{2}e^{-3} + \frac{9}{2}e^{-3}\right]$$

$= 1 - e^{-3}(4+4.5+4.5)$

$= 1-13e^{-3}$

$= 0.3527$

Number of drivers having more than 3 accidents

$= 0.3527 \times 1000$

$= 352.7$

$= 353$

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5. The following table gives the no. of days in a 50 day period during which automobile accidents occured in a city

No. of accidents : 0 1 2 3 4

No. of days : 21 18 7 3 1

Fit a Poisson distribution to the data (AKTU 2022)

x	f	fx	Theoritical frequencies
0	21	0	20
1	18	18	18
2	7	14	8
3	3	9	3
4	1	4	1
	$\Sigma f = 50$	$\Sigma fx = 45$	

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\lambda = \frac{45}{50}$$

$$\boxed{\lambda = 0.9}$$

$N = 50$

$$\text{Poisson Distribution} = \frac{N e^{-\lambda} \lambda^x}{x!}$$

$$\boxed{= \frac{50 \times e^{-0.9} (0.9)^x}{x!}}$$

when $x = 0$

$= 20.32 = 20$

when $x = 0$

$= 18.29 = 18$

when $x = 2$

$= 8.2 = 8$

when $x = 3$

$= 2.46 = 3$

when $x = 4$

$= 0.555 = 1$

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Engg. Mathematics – IV **By Gulshan Sir**

Normal Distribution

* The normal Distribution is a continous distribution based on continuous random variable
* A continous random variable X is said to have a normal distribution with parameters μ (mean) and σ^2 (variance) if the probability density function is given by

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

(i) $f(x) \geq 0$

(ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

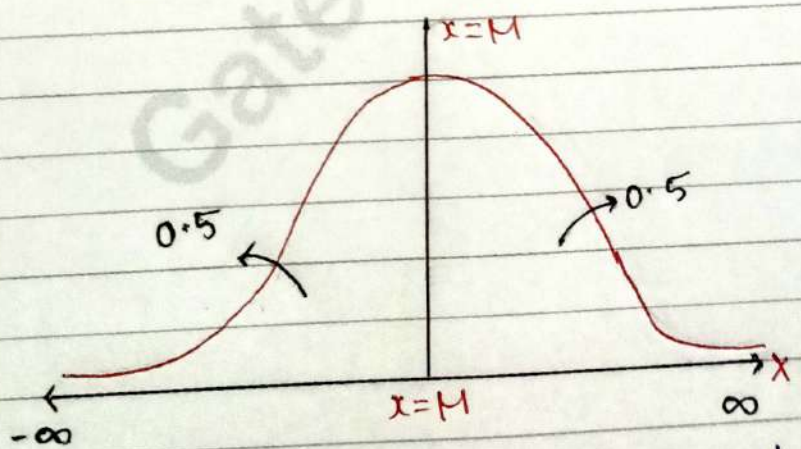
where

μ = Mean

σ = Standard deviation

Normal Curve

* The graph of normal Distribution is called normal curve
* It is well shaped and symmetrical about Mean μ.



* In normal distribution mean, median and mode coincide
* Total area under the curve above x-axis is 1.

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Engg. Mathematics – IV **By Gulshan Sir**

Standard Normal Distribution

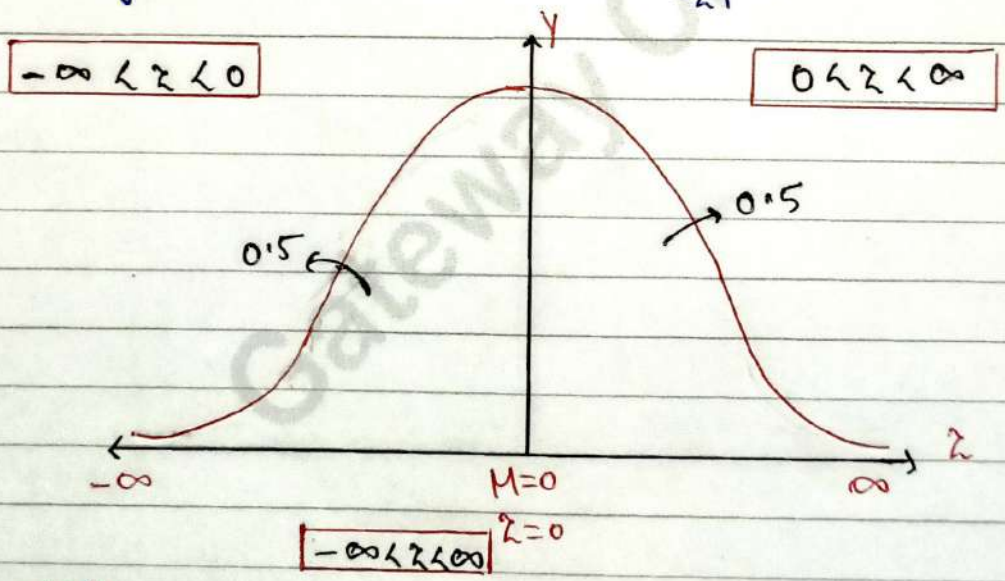
* If X is a Normal random variable with mean μ and standard deviation σ then a random variable z with mean = 0 and standard deviation = 1 is called Standard normal random variable

$$z = \frac{x-\mu}{\sigma}$$

The probability density function for the normal distribution in standard form is given by

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(z)^2}$$

Probability $P(z_1 < z < z_2) = \int_{z_1}^{z_2} f(z)\,dz$



Probability

$P(z_1 < z < z_2)$ = Area under Standard normal curve between $z = z_1$ and $z = z_2$

Total area under curve = 1

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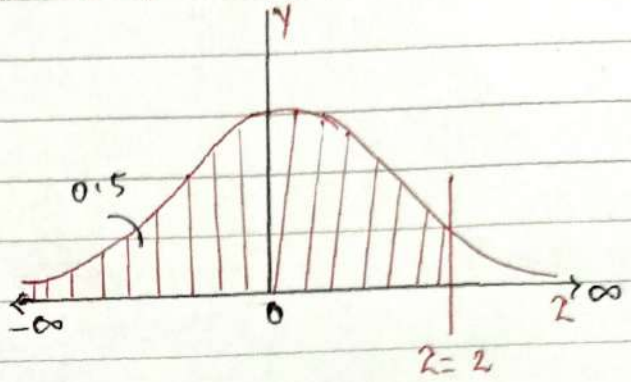
1. If X is a normal variate with mean 80 and standard deviation 10, find probability $P(X \leq 100)$

$M = 80, \sigma = 10, x = 100$

$$z = \frac{x - M}{\sigma}$$

$$z = \frac{100 - 80}{10}$$

$$\boxed{z = 2}$$



$$P(x \leq 100) = P(z \leq 2)$$
$$= P(-\infty < z < 0) + P(0 < z \leq 2)$$
$$= 0.5 + P(0 < z \leq 2)$$
$$= 0.5 + 0.4772$$
$$\boxed{P(X \leq 100) = 0.9772}$$

2. The life of army shoes is normally distributed with mean 8 months & standard deviation 2 months. If 5000 pairs are insured how many pairs would be expected to need replacement after 12 months? [Given that $P(z \geq 2) = 0.0228$ and $z = \frac{x-M}{\sigma}$] [AKTU 2018]

$M = 8, \sigma = 2, N = 5000$

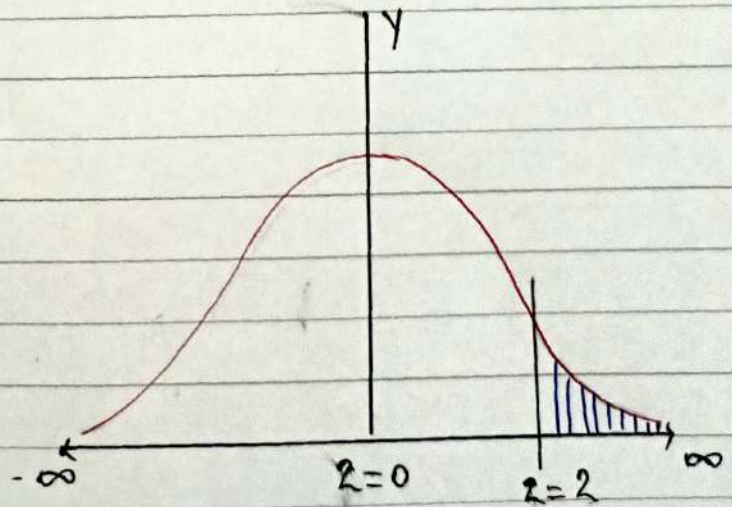
Let $x = 12$

$P(x \geq 12) = ?$

$$z = \frac{x - M}{\sigma}$$

$$z = \frac{12 - 8}{2}$$

$$\boxed{z = 2}$$



$$P(x \geq 12) = P(0 < z < \infty) - P(0 < z < 2)$$

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$P(x \geq 12) = 0.5 - 0.4772$

$= 0.0228$

Number of pairs having life more than 12 months

$= 5000 \times 0.0228$

$= 114$

Number of pairs replaced after 12 months $= 5000 - 114$

$= 4886$

3. Assume mean height of Soldiers to be 68.22 inches with a variance of 10.8 inches square. How many soldiers in a regiment of 1,000 would you expect to be over 6 feet tall given that the area under the standard normal curve between $z=0$ and $z=0.35$ is 0.1368 and between $z=0$ and $z=1.15$ is 0.3746 [AKTU(C.O) 2021]

$M = 68.22$, $\sigma^2 = 10.8$, $\sigma = \sqrt{108}$

$N = 1000$, $x = 6$ feet

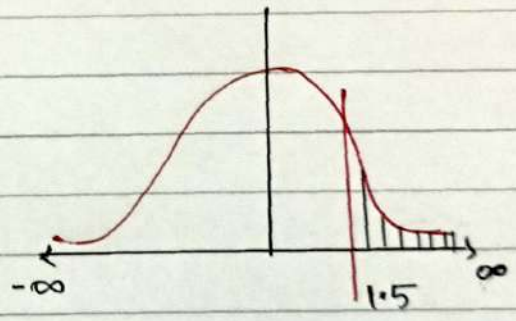
$x = 6 \times 12$

$x = 72$ inches

$z = \frac{x - M}{\sigma}$

$z = \frac{72 - 68.22}{\sqrt{10.8}}$

$z = 1.15$



$P(x > 72) = P(z > 1.15)$

$= P(0 < z < \infty) - P(0 < z < 1.15)$

$= 0.5 - 0.3746$

$P(x > 72) = 0.1254$

No. of Soldiers $= 1000 \times 0.1254$

$= 125$ (approx)

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4 The daily wages of 1000 workers are distributed around a mean of Rs. 140 and with a standard deviation of Rs. 10. Estimate the number of workers whose daily wages

(i) between 140 and 144

(ii) Less than 126

(iii) More than 160

[AKTU 2021]

$N = 1000,\ M = 140,\ \sigma = 10,$

$$z = \frac{x - M}{\sigma}$$

$$z = \frac{x - 140}{10} \quad (1)$$

(i) $x_1 = 140,\ x_2 = 144$

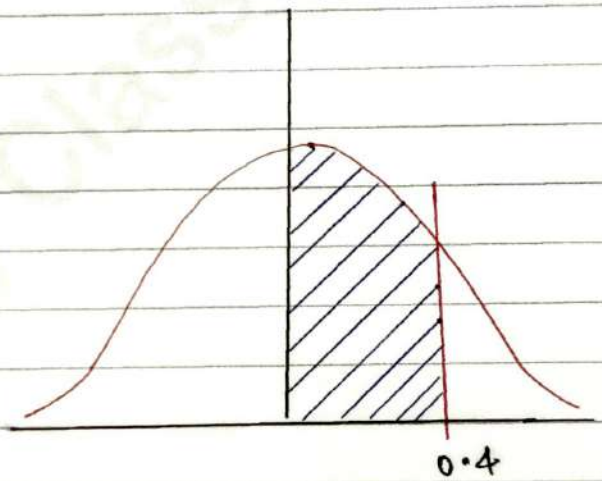
$$z_1 = \frac{140 - 140}{10}$$

$$\boxed{z_1 = 0}$$

$$z_2 = \frac{144 - 140}{10}$$

$$z_2 = \frac{4}{10}$$

$$\boxed{z_2 = 0.4}$$



$P(140 < x < 144) = P(0 < z < 0.4)$

$= 0.1554$

Expected number of workers $= 0.1554 \times 1000$

$= 155.4$

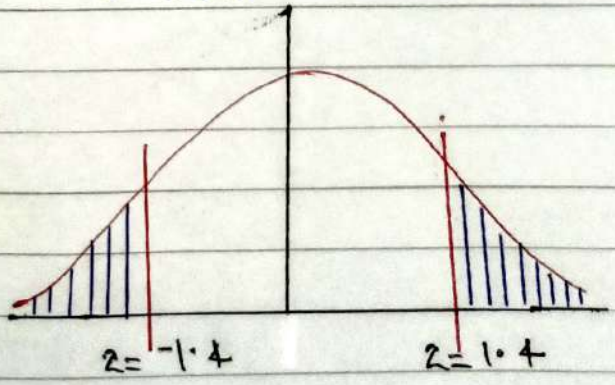
$= 155$

(ii) $x = 126$

$$z = \frac{x - 140}{10}$$

$$z = \frac{126 - 140}{10}$$

$$\boxed{z = -1.4}$$



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$P(X<126) = P(Z<-1.4)$

$P(x<126) = P(Z>1.4)$

$= P(0<Z<\infty) - P(0<Z<1.4)$

$P(X<126) = 0.5 - 0.4192$

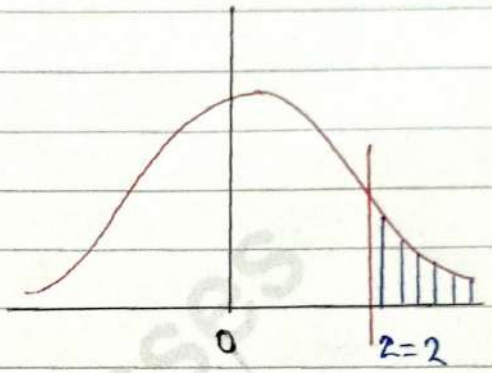
$= 0.0808$

Expected no. of workers

$= 1000 \times 0.0808$

$= 80.8$

$= 81$ (approx)



(iii) $x = 160$

$Z = \frac{160-140}{10} = \frac{20}{10}$

$\boxed{Z = 2}$

$P(x>160) = P(Z>2)$

$= P(0<Z<\infty) - P(0<Z<2)$

$= 0.5 - 0.4772$

$= 0.0228$

Expected no of workers $= 0.0228 \times 1000$

$= 22.8$

$= 23$ (approx)

Standard Normal Curve :-

1. A sample of 100 dry battery cells tested to find the length of life produced the following results: $\bar{x} = 12$ hours, $\sigma = 3$ hours. Assuming the data to be normally distributed what % of battery cells are expected to have life (i) more than 15 hours (ii) less than 6 hours.
(iii) between 10 and 14 hours? [AKTU 2018, 2021]

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$N = 100$, $M = 12$, $\sigma = 3$

$$z = \frac{x - M}{\sigma}$$

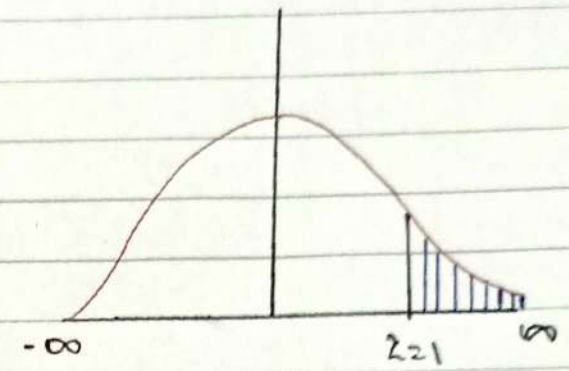
$$z = \frac{x - 12}{3} \quad \ldots\ldots (1)$$

(i) $x = 15$

$$z = \frac{15 - 12}{3}$$

$$z = 1$$

$P(x > 15) = P(z > 1)$



$P(x > 15) = P(0 < z < \infty) - P(0 < z < 1)$

$= 0.5 - 0.3413$

$= 0.1587$

$P(x > 15) = 0.1587 \times 100$

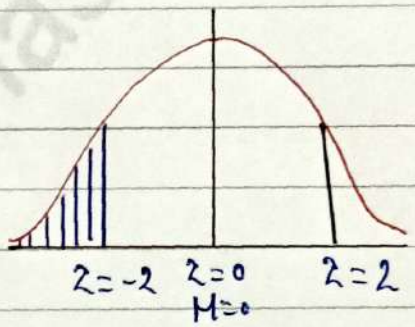
$= 15.87\%$

(ii) $x = 6$

$$z = \frac{x - 12}{3}$$

$$z = \frac{6 - 12}{3}$$

$$z = -2$$



$P(x < 6) = P(z < -2)$

$= P(z > 2)$

$P(x < 6) = P(0 < z < \infty) - P(0 < z < 2)$

$= 0.5 - 0.4772$

$= 0.0228$

$P(x < 6) = 0.0228 \times 100$

$= 2.28\%$

$P(x < 6) = 2.28\%$

(iii) $x_1 = 10$

$x_2 = 14$

$$z_1 = \frac{x_1 - 12}{3}$$

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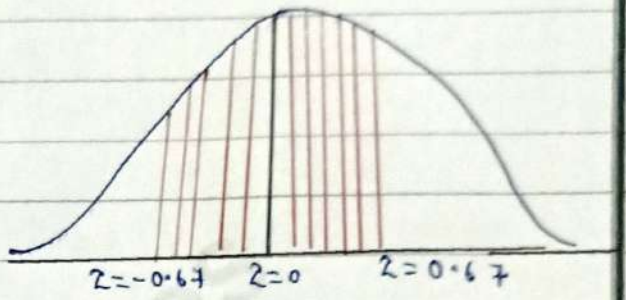
$$z_1 = \frac{10-12}{3}$$

$$z_1 = -0.67$$

$$z_2 = \frac{x_2-12}{3}$$

$$z_2 = \frac{14-12}{3}$$

$$z_2 = 0.67$$



$$P(10<x<14) = P(0.67<z<0.67)$$
$$= 2P(0<z<0.67)$$
$$= 2 \times 0.2485$$
$$= 0.497$$

$$\boxed{P(10<x<14) = 49.7\%}$$

2. In a sample of 1000 cases, the mean of a certain test is 14 and standard deviation is 2.5. Assuming the distribution to be normal, find

(i) How many students score between 12 and 15?

(ii) how many score above 18?

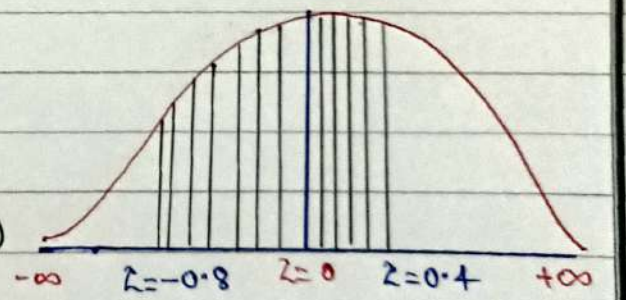
(iii) how many score below 8?

(iv) how many score 16?

$\mu = 14$, $\sigma = 2.5$, $N = 1000$

$$z = \frac{x-\mu}{\sigma}$$

$$z = \frac{x-14}{2.5} \quad \ldots\ldots (1)$$



(i) $x_1 = 12$, $x_2 = 15$

$$z_1 = \frac{x_1-14}{2.5} = \frac{12-14}{2.5}$$

$$z_1 = -0.8$$

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$z_2 = \frac{x_2 - 14}{2.5} = \frac{15-14}{2.5}$

$z_2 = 0.4$

$P(12 < x < 15) = P(-0.8 < z < 0.4)$

$P(12 < x < 15) = P(0 < z < 0.4) + P(0 < z < 0.8)$

$= 0.1554 + 0.2881$

$= 0.4435$

Number of students $= 1000 \times 0.4435$

$= 443.5$

$= \boxed{444 \text{ (approx)}}$

(iii) $x = 18$

$z = \frac{x-14}{2.5} = \frac{18-14}{2.5}$

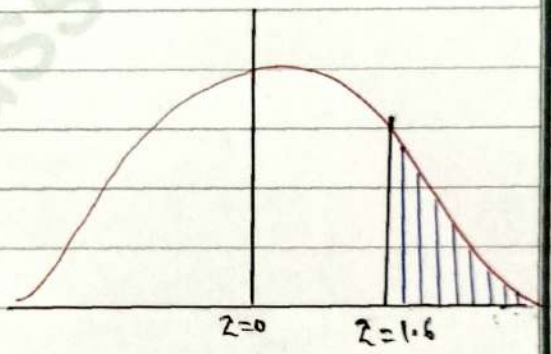
$z = 1.6$

$P(x < 18) = P(z > 1.6)$

$= P(0 < z < \infty) - P(0 < z < 1.6)$

$= 0.5 - 0.4452$

$P(x > 18) = 0.0548$



Number of students $= 1000 \times 0.0548$

$= 54.8$

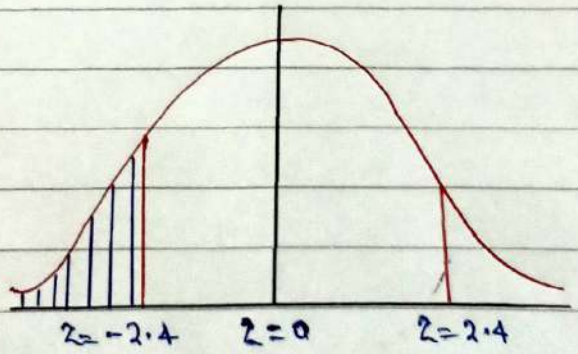
$= \boxed{55 \text{ (approx)}}$

(iii) $x = 8$

$z = \frac{x-14}{2.5}$

$z = \frac{8-14}{2.5}$

$z = -2.4$



$P(x < \infty) = P(z < -2.4)$

$= P(z > 2.4)$

$P(x < 8) = P(0 < z < \infty) - P(0 < z < 2.4)$

$= 0.5 - 0.4918 = 0.0082$

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Number of Students $= 0.0082 \times 1000$

$= 8.2$

$\boxed{= 8 \text{ (approx)}}$

(iv) $x = 16$

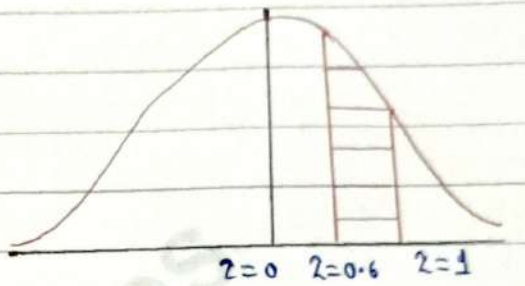
$x_1 = 15.5$, $x_2 = 16.5$

$z_1 = \frac{x_1 - 14}{2.5} = \frac{15.5 - 14}{2.5}$

$\boxed{z_1 = 0.6}$

$z_2 = \frac{x_2 - 14}{2.5} = \frac{16.5 - 14}{2.5}$

$\boxed{z_2 = 1}$



$P(15.5 < x < 16.5) = P(0.6 < z < 1)$

$P(15.5 < x < 16.5) = P(0 < z < 1) - P(0 < z < 0.6)$

$= 0.3413 - 0.2257$

$= 0.1156$

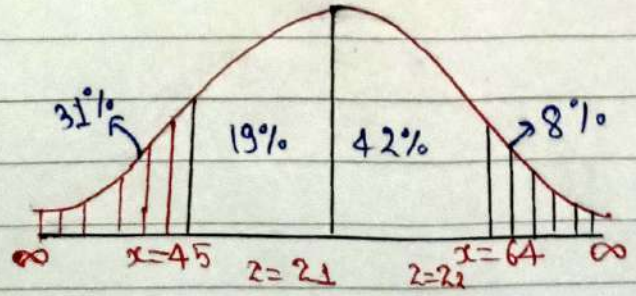
$\boxed{\text{Number of students} = 0.1156 \times 1000 = 116}$

3. In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and standard deviation of the distribution. It is given that if

$f(t) = \frac{1}{\sqrt{2\pi}} \int_0^t e^{-\frac{1}{2}x^2} dx$ then $f(0.5) = 0.19$ and $f(1.4) = 0.42$

(AKTU 2020)

Let mean $= \mu$, Standard deviation $= \sigma$



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when $x_1 = -45$ $P(-45 < x < 0) = \frac{19}{100} = 0.19$

$z_1 = -0.5$

when $x_2 = 64$, $P(0 < x < 64) = \frac{42}{100} = 0.42$

$z_2 = 1.4$

$$z = \frac{x - M}{\sigma}$$

$$z_1 = \frac{x_1 - M}{\sigma}$$

$$-0.5 = \frac{45 - M}{\sigma}$$

$$-0.5\sigma = 45 - M$$

$$-0.5\sigma + M = 45 \quad ..(1)$$

$$z_2 = \frac{x_2 - M}{\sigma}$$

$$1.4 = \frac{64 - M}{\sigma}$$

$$1.4\sigma = 64 - M$$

$$1.4\sigma + M = 64 \quad ..(2)$$

Subtract (1) and (2)

$$-0.5\sigma + M = 45$$
$$1.4\sigma + M = 64$$
$$- \quad - \quad -$$

$$-1.9\sigma = -19$$

$$\boxed{\sigma = 10}$$

$$\boxed{M = 50}$$

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