EC -III SEM

GATEWAY CLASSES

Digital System Design

Unit-1

Digital Electronics

ONE SHOT



Abhilasha ma'am

BOE310/ BOE410/BOE310H/ BOE410H DIGITAL ELECTRONICS

AKTU : Syllabus

Unit-I

Digital System And Binary Numbers: Number System and its arithmetic Signed binary numbers, Logic simplification and combinational logic design: Binary codes, code conversion, review of Boolean algebra and Demorgans theorem, SOP & POS forms, Canonical forms, Karnaugh maps method up to five variable, Don’t care conditions, POS simplification, NAND and NOR implementation, Quine McClusky method (Tabular method).

DIGITAL SYSTEM DESIGN

UNIT 1 : LOGIC SIMPLIFICATION AND COMBINATIONAL LOGIC DESIGN

AKTU Syllabus

Logic simplification and combinational logic design: Number Systems, Binary arithmetic, signed magnitude representation, Binary codes, code conversion, review of Boolean algebra and Demorgans theorem, SOP & POS forms, Canonical forms, Karnaugh maps up to 5 variables, tabulation method.

Gateway Classes 7455 961294

Number Systems:

- ✓ Decimal (Base: 10)
  - Symbols: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
- ✓ Binary (Base 2)
  - Symbols: 0, 1
- ✓ Octal (Base 8)
  - Symbols: 0, 1, 2, 3, 4, 5, 6, 7
- ✓ Hexadecimal (Base 16)
  - Symbols: 0, 1, 2, ... 9, A, B, C, D, E, F
- ✓ Excess-3 → Binary + 0011
- ✓ Gray ✓
- ✓ BCD ✓

One shot pdf notes
→ App. link given in video description
→ free mat.
→ Result oriented

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Binary Number System:

- Uses two symbols 0, 1
- Base or radix = 2
- 1 nibble = 4 bits
- 1 byte = 8 bits
- 1Kbits (kilo) = 2^{10} = 1024 bits
- 1Mbits (mega) = 2^{20} = 1024 x 1024 bits
- 1Gbits (giga) = 2^{30} = 1024 x 1024 x 1024 bits
- 1 Tbits (tera) = 2^{40}

0/1 = Bit

0110

10100100

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Conversion: Decimal To Binary

- **Successive division by 2:**

$(68)_{10} = (\quad)_2 = (1000100)_2$

2	68	0
2	34	0
2	17	1
2	8	0
2	4	0
2	2	0
2	1	1
	0	

$(68)_{10} \rightarrow (152)_6$

$\rightarrow (\quad)_5$

$\rightarrow (\quad)_7$

6	68	2
6	11	5
6	1	1
	0	

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Example 2 : Decimal Number Conversion

$(0.683)_{10} = (0.1010\ldots)_2$

$0.683 \times 2 = 1.366$ → 1

$0.366 \times 2 = 0.732$ → 0

$0.732 \times 2 = 1.464$ → 1

$0.464 \times 2 = 0.928$ → 0

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$(62.34)_{10} = (111110.0101)_2$

2	62	0
2	31	1
2	15	1
2	7	1
2	3	1
2	1	1
	0	

$(62.34)_{10} \rightarrow (\ )_5$

5	62	2
5	12	2
5	2	2
	0	

$(62.34)_{10} = (222.1322)_5$

$0.34 \times 2 = 0.68$ — 0
$0.68 \times 2 = 1.36$ — 1
$0.36 \times 2 = 0.72$ — 0
$0.72 \times 2 = 1.44$ — 1

$0.34 \times 5 = 1.70$ — 1
$0.70 \times 5 = 3.5$ — 3
$0.5 \times 5 = 2.5$ — 2
$0.5 \times 5 = 2.5$ — 2

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Binary To Decimal Conversion

- The decimal number is equal to the sum of binary digits (d_n) times their power of 2 (2^n):

$$\text{decimal} = d_0 \times 2^0 + d_1 \times 2^1 + d_2 \times 2^2 + \ldots$$

MSB LSB

1 0 1 1

$1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$

$8 + 0 + 2 + 1$

$= (11)_{10}$

$(1011.010)_2$

1 0 1 0 1 1

$1 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$

$32 + 0 + 8 + 0 + 2 + 1$

$= (43)_{10}$

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Example: Find the decimal value of 111001_2

$$(0.111001)_2$$

$$= 0 + 1\times 2^{-1} + 1\times 2^{-2} + 1\times 2^{-3} + 0\times 2^{-4} + 0\times 2^{-5} + 1\times 2^{-6}$$

$$= 0 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + 0 + 0 + \frac{1}{64}$$

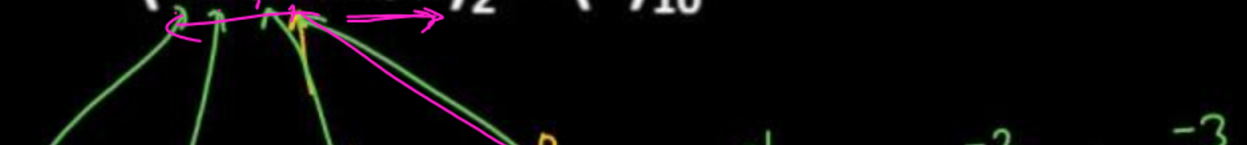
$$= 0.5 + 0.25 + 0.125 + 0 + 0 + 0.$$

$$= (\quad)_{10}$$

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Binary with decimal point

$(1101.101)_2 = (\ )_{10}$



$1\times2^3+1\times2^2+0\times2^1+1\times2^0+1\times2^{-1}+0\times2^{-2}+1\times2^{-3}$

$8+4+0+1+0.5+0+0.125$

$13+0.625$

$=(13.625)_{10}$

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Decimal To Octal

- **Convert decimal number 210 into octal number.**

$(210)_{10} \longrightarrow (1262)_8$

8	210	2
8	26	6
8	10	2
8	1	1
	0	

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Conversion of Decimal Fraction Number in Octal Number

$(0.12)_{10} = (\quad)_8 = (0.07514)_8$

$0.12 \times 8 = 0.96$ → 0

$0.96 \times 8 = 7.68$ → 7

$0.68 \times 8 = 5.24$ → 5

$0.24 \times 8 = 1.92$ → 1

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$(10.56)_{10} = (12.436)_8$

8	10	2
8	1	1
	0	

$0.56 \times 8 = 4.48$ → 4

$0.48 \times 8 = 3.84$ → 3

$0.84 \times 8 = 6.72$ → 6

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Convert binary number 10010110 into octal number.

Method-I

$$10010110$$

$$1\times2^7+0+0+1\times2^4+0\times2^3+1\times2^2+1\times2^1+0\times2^0$$

$$128+16+4+2+0$$

$$(150)_{10}$$

8	150	6
8	18	2
8	2	2
	0	

$$= (226)_8$$

$$(10010110)_2 \rightarrow (226)_8$$

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Convert binary number 10010110 into octal number.

Method-II

$$\underline{010}\,\underline{010}\,\underline{110}$$

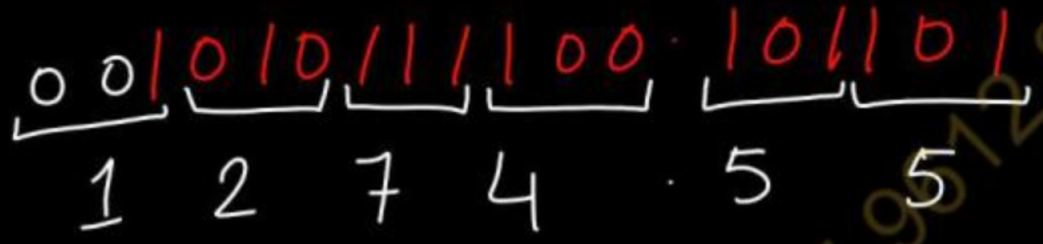
$$(2\ 2\ 6)_8$$

$$(10010110)_2 = (226)_8$$

0 →	0 0 0
1 →	0 0 1
2 →	0 1 0 ✓
3 →	0 1 1
4 →	1 0 0 ✓
5 →	1 0 1 ✓
6 →	1 1 0
7 →	1 1 1

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Convert binary number 1010111100.101101 into octal number.



$$= (1274.55)_8$$

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Decimal to Hexadecimal Conversion:

6 10

1) Divide the decimal by 16 until the result is 15 or less.

2) Take the whole number (before the decimal point) and look up the corresponding hex value in the hex-decimal comparison list above (e.g. 10 = A, 15 = F).

3) Take the remainder (after the decimal point) and multiply by 16.



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Example: Find the Hex equivalent of decimal number 3509

$(3509)_{10} = (\quad)_H = (\quad)_{16}$

16	3509	5
16	219	11 → B
16	13	13 → D
	0	

$(3509)_{10} = (\underline{DB5})_{16}$

A	B	C	D	E	F
10	11	12	13	14	15

0 - - - 9 10 11 12 13

A B C D

14 , 15

E F

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Fractional Decimal to Hex Conversion

$(0.1204)_{10} \rightarrow (\quad)_H$

$0.1204 \times 16 \rightarrow 1.9264$ → 1

$0.9264 \times 16 \rightarrow 14.8224$ → 14 ← E

$0.8224 \times 16 \rightarrow 13.1584$ → 13 ← D

$0.1584 \times 16 \rightarrow 2.5344$ → 2

$(0.1204)_{10} \rightarrow (0.1ED2)_{16}$

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Whole & Fractional Decimal number to Hex Conversion

$(148.62)_{10} \rightarrow (\quad)_H$

16	148	4
16	9	9
	0	

$0.62 \times 16 = 9.92$

$0.92 \times 16 = 14.72$

$0.72 \times 16 = 11.52$

$0.52 \times 16 = 8.32$

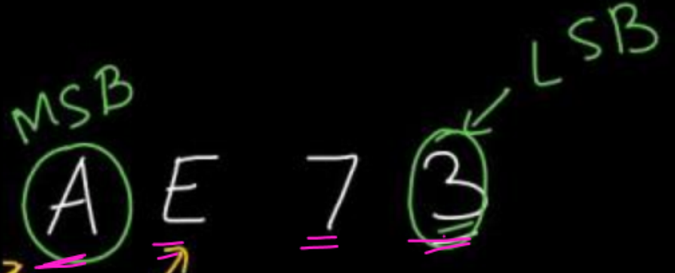
9
14 → E
11 → B
8

$(148.62)_{10} \rightarrow (94.9EB8)_H$

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Hexadecimal To Decimal Conversion:

$(AE73)_{16} = (\ )_{10}$



$10 \times 16^3 + 14 \times 16^2 + 7 \times 16^1 + 3 \times 16^0$

$= 10 \times 4096 + 14 \times 256 + 7 \times 16 + 3 \times 1$

$= 40960 + 3584 + 112 + 3$

$= (44659)_{10}$

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Fractional Hex Number to Decimal Conversion

$(11A \cdot 35B)_{16} \rightarrow (\quad)_{10}$

$1 \times 16^{2} + 1 \times 16^{1} + 10 \times 16^{0} + 3 \times 16^{-1} + 5 \times 16^{-2} + 11 \times 16^{-3}$

$= 256 + 16 + 10 + 0.1875 + 0.01953 + 0.002685$

$= (282.2097)_{10}$

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Binary to Hex Conversion

$(\quad)_2 = (\quad)_{10} = (\quad)_H$

$0101\,0110\,1000\,|\,101\,.\,1100\,1010$

$5\quad 6\quad 8\quad D\quad .\quad C\quad A$

$= (568D \cdot CA)_H$

0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
10	1010
11	1011
12	1100
13	1101
14	1110
15	1111

A B C D E F

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BINARY ARITHMATIC OPERATIONS

- Binary Addition
- Binary Subtraction
- Binary Multiplication
- Binary Division

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Binary Addition

A	B	S	Carry
0	0	0	x
0	1	1	x
1	0	1	x
1	1	0	1
1	1	1	1

$C_i = 0$

$C_i = 1$

```
  1 1 1
  1 0 1 1 0
+ 0 1 1 1 1
-------------
1 0 0 1 0 1
```

Result = 100101

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Binary Subtraction

$0 - 0 = 0$

$1 - 0 = 1$

$1 - 1 = 0$

$0 - 1 = 1$

1 = Borrow

$$\begin{array}{r} {\scriptstyle 1} \\ 217 \\ -\ 9 \\ \hline \end{array}$$

$$\begin{array}{r} 10 \\ -\ 1 \\ \hline \end{array}$$

Ex:

$$\begin{array}{r} {\scriptstyle 0} \quad {\scriptstyle 1} \quad {\scriptstyle 1} \\ 1011 \\ -\ 0100 \\ \hline 0110011 \end{array}$$

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Binary Multiplication

$0 \times 0 = 0$

$0 \times 1 = 0$

$1 \times 0 = 0$

$1 \times 1 = 1$

$$
\begin{array}{r}
123 \\
\times\ 52 \\
\hline
246 \\
+615X \\
\hline
6396
\end{array}
$$

$$
\begin{array}{r}
1011010 \\
\times\ \ 0110 \\
\hline
0000000 \\
1011010X \\
1101010XX \\
+\ 0000000XXX \\
\hline
1000011100
\end{array}
$$

GW GATEWAY CLASSES

GW GATEWAY CLASSES

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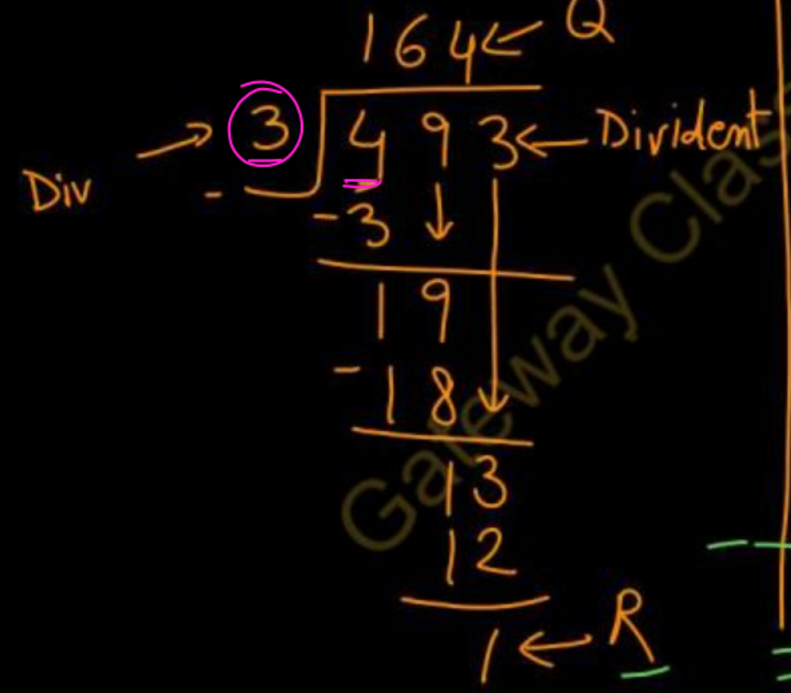
Binary Division

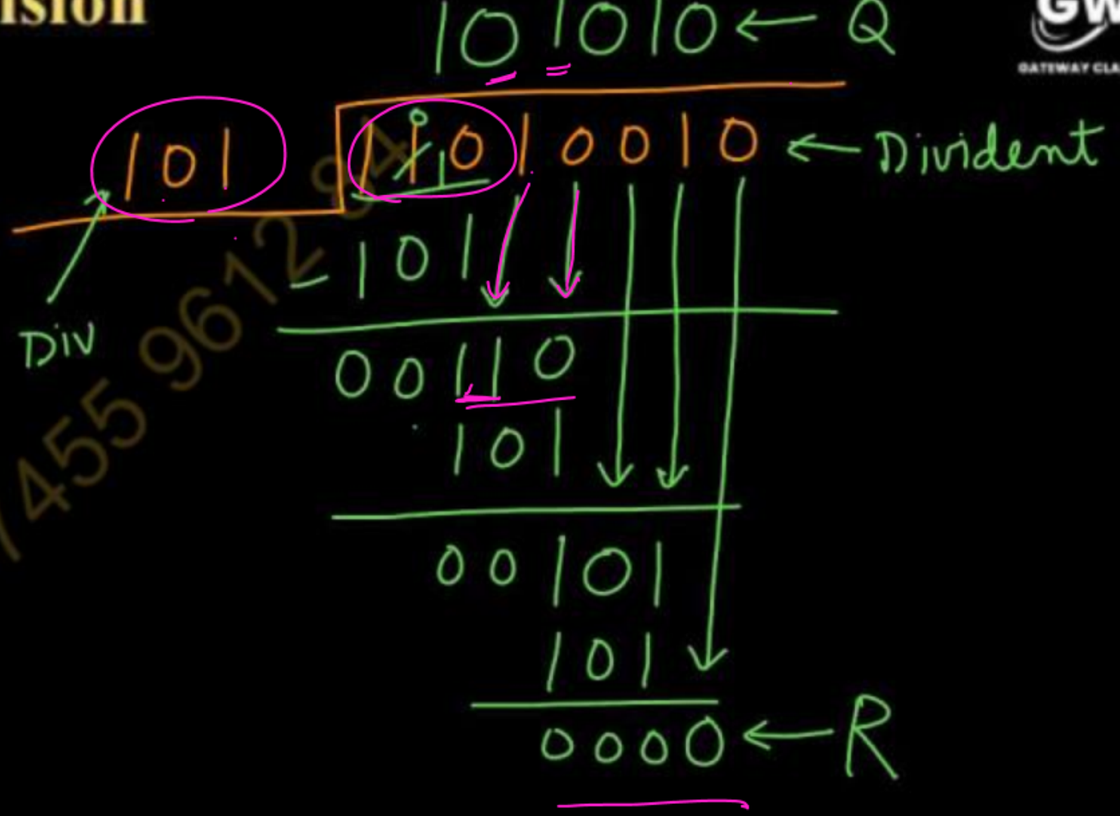
$0 \div 0 = 0$

$0 \div 1 = 0$

$1 \div 0 = ND$

$1 \div 1 = 1$





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$$(33)_6 \rightarrow (\quad)_{10}$$
$$+(45)_6 \rightarrow (\quad)_{10}$$
$$((1)22)_6$$

$$(\quad)_6$$

8
6 | 8
2 | 6
2

$$(42)_5 + (13)_5 = (110)_5$$

6 —
7 —
8 —

0 1 2 3 4 5 10 11 12 13 14 15 20 21 25

0 1 2 3 4 9 10 11 12 13 19 20

99
100

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UNIVERSITY QUESTIONS

YEAR	QUESTIONS	MARKS
22-23	Convert $(454.523)_{10}$ to an Hexadecimal number	2 marks
20-21	Convert the following: (i) Hexadecimal equivalent of decimal number 256 (ii) Decimal equivalent of $(123)_9$ (iii) $(378.93)_{10}$ to octal (iv) Convert A3BH and 2F3H into binary	10 marks
19-20	The solution to the quadratic equation $k^2 - 11k + 22 = 0$ are k=3 and k=6. What is the base of the number system?	2 marks
18-19	Determine the value of base x, if $(193)_x = (623)_8$	2 marks
17-18	What is (33)6 + (45)6	2 marks
17-18	Convert the following (i) $(562.13)_7 = (?)_{10}$ (ii) $(467.342)_8 = (?)_{10}$	2 marks

$(256)_{10} = (\quad)_H$

$(123)_9$

$1\times9^2 + 2\times9^1 + 3\times9^0 = 81 + 18 + 3 = (102)_{10}$

1010 0011 1011

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$$(193)_x = (623)_8$$

$$1 \times x^2 + 9 \times x^1 + 3 \times x^0 = 6 \times 8^2 + 2 \times 8^1 + 3 \times 8^0$$

$$x^2 + 9x + 3 =$$

$K^2 - 11K + 22 = 0 \Rightarrow (K = 3, K = 6)$

Suppose base = B

For $K = 3$

$3^2 - (1 \times B^1 + 1 \times B^0)3 + (2 \times B^1 + 2 \times B^0) = 0$

$9 - (B + 1)3 + (2B + 2) = 0$

$9 - 3B - 3 + 2B + 2 = 0$

$-B + 8 = 0$

$B = 8$

For $K = 6$

$6^2 - (1 \times B^1 + 1 \times B^0)6 + (2B^1 + 2B^0) = 0$

$36 - (B + 1)6 + (2B + 2) = 0$

$36 - 6B - 6 + 2B + 2 = 0$

$(15)_6$

$1 \times 6^1 + 5 \times 6^0$

$32 - 4B = 0$

$4B = 32$

$B = 8$

Gateway Classes : 7455 961284

$(454.523)_{10} = (\quad)_4$

16	454	6
16	28	12=C
16	1	1
	0	

$0.523 \times 16 = 8.368$ → 8

$0.368 \times 16 = 5.888$ → 5

$0.888 \times 16 = 14.208$ → 14(E)

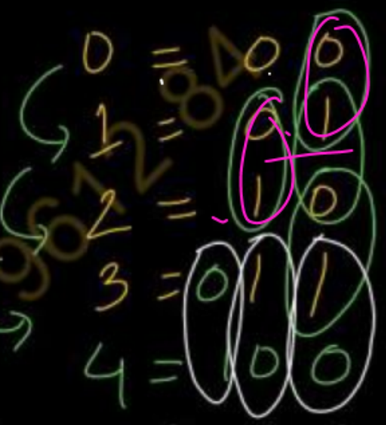
$(1C6.85E)_{16}$

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Gray code

- The reflected binary code (RBC)
- reflected binary (RB) or Gray code
- after Frank Gray,
- two successive values differ in only one bit





- widely used to prevent spurious output from electromechanical switches
- to facilitate error correction in digital communications such as digital terrestrial television and some cable TV systems.
- Many devices indicate position by closing and opening switches. If that device uses natural binary codes, positions 3 and 4 are next to each other but all three bits of the binary representation differ:

• 3=011 & 4=100

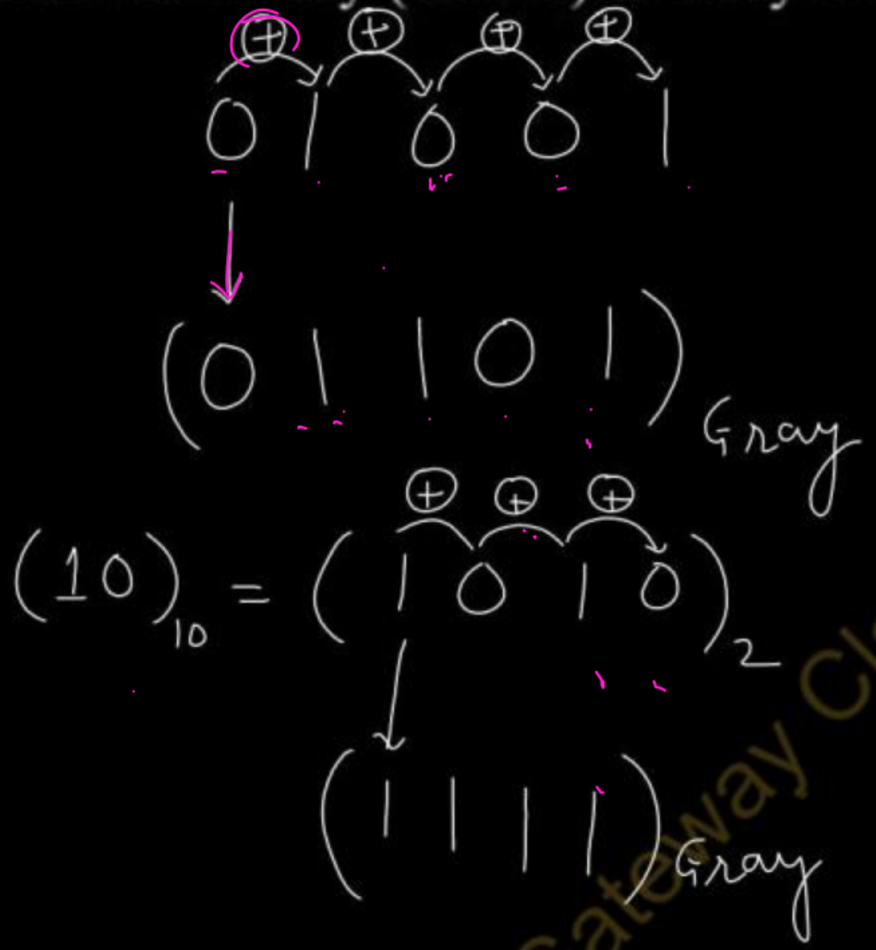
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DECIMAL	BINARY	GRAY
0	0000	0000
1	0001	0001
2	0010	0011
3	0011	0010
4	0100	0110
5	0101	0111
6	0110	0101
7	0111	0100
8	1000	1100
9	1001	1101
10	1010	1111
11	1011	1110
12	1100	1010
13	1101	1011
14	1110	1001
15	1111	1000

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How to Convert Binary to Gray Code

- Convert binary (01001) to Gray Code



Ex- OR

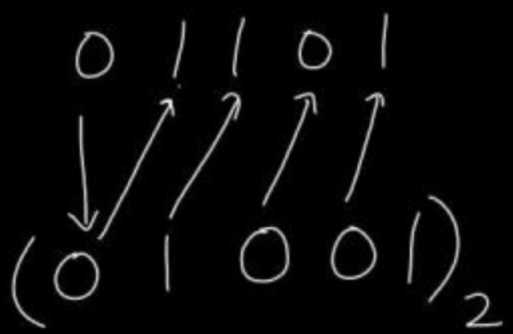
$Y = \bar{A}B + A\bar{B}$

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

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Gray to Binary Code Converter

- Convert Gray number 01101 to binary





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$(43)_{10} \Rightarrow (\quad)_{Gray}$

$(43)_{10} = (101011)_2 = (111110)_{Gray}$

2	43	1
2	21	1
2	10	0
2	5	1
2	2	0
2	1	1
	0	



Gateway Classes : 7455 96 12 84

1's Complement & 2's Complement

- 1's complement of (-19) is:

no = 10011

1's = 01100

2's = 1's + 1

1's = 01100

\+ 1

2's ⇒ 01101

- 2's complement of (-19) is:

no = 10011

2's = 01101

10100

2's 01100

2	19	1
2	9	1
2	4	0
2	2	0
2	1	1
	0	

$(19)_{10} = (10011)_2$

16	8	4	2	1
1	0	0	1	1

Gateway Classes : 7455961284

no ⇒ 1 0 0 1 0 0

2's ⇒ 0 1 1 0 0

no ⇒ 1 0 0 1 0 0

1's ⇒ 0 1 1 0 1 1

\+ 1

2's ⇒ 0 1 1 0 0

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7's

no. 65

7's 77

− 65

12

7's of → 12

9's

99

− 65

34

9's → 34

8's

65

7's = 12

+ 1

8's 13

10's

9's of 65 = 34

+ 1

10's → 35

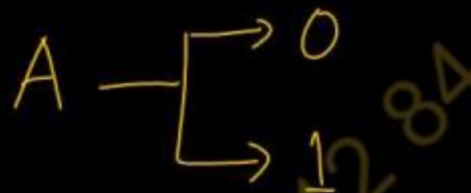
UNIVERSITY QUESTIONS

YEAR	QUESTION	MARKS	ANSWER
20-21	Convert binary code (00011011) to gray code.	2 marks	(00010110)
18-19	Define cyclic codes	2 marks	First slide

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Rule in Boolean Algebra

- Variable used can have only two values.



- Binary 1 = HIGH and Binary 0 = LOW.
- Complement of a variable is represented by an overbar (-)/(‘). $\bar{A} = A'$
- Thus if B = 0 then B’= 1 and if B = 1 then B’= 0.

$A + B = Y = 1$

- ORing of the variables is represented by a plus (+) sign between them. Ex- A + B
- Logical ANDing of the two or more variable is represented by a dot between them. Ex- A.B.C. or ABC.

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Boolean Laws

- There are six types of Boolean Laws.

Addition | Multiplication

1) **Commutative law:** $A+B = B+A$ | $A \cdot B = B \cdot A$

2) **Associative law:** $A(B+C) = AB+AC$

3) **Distributive law:** $A+(B+C) = (A+B)+C$ | $A \cdot (B \cdot C) = (A \cdot B) \cdot C$

4) **AND law:** $A+0 = A$, $A+1 = 1$ | $A+A = A$, $A+\bar{A} = 1$

5) **OR law:** $A \cdot 0 = 0$, $A \cdot 1 = A$ | $A \cdot A = A$, $A \cdot \bar{A} = 0$

6) **INVERSION law:** $\bar{\bar{A}} = A$

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De- Morgan's Theorem

1) $(A+B)' = A'.B'$ ✓ $\quad \overline{A+B} = \bar{A}\cdot\bar{B}$

invert and replace OR with AND

2) $(A.B)' = A' + B'$ $\quad \overline{A\cdot B} = \bar{A}+\bar{B}$

invert and replace AND with OR

DMT1 DMT2

A	B	$\overline{A+B}$	$\bar{A}$	$\bar{B}$	$\bar{A}\cdot\bar{B}$	$\overline{A}\cdot B$	$\bar{A}+\bar{B}$
0	0	1	1	1	1	1	1
0	1	0	1	0	0	1	1
1	0	0	0	1	0	1	1
1	1	0	0	0	0	1	0

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Simplify the following expression: $(AB)'(A' + B)(B' + B)$

$\overline{AB}\,(\bar{A}+B)(B+\bar{B})$

De morgan's th^{m} $(\bar{A}+\bar{B})(\bar{A}+B)\,1$

$(\bar{A}+\bar{B})(\bar{A}+B)$

$\bar{A}(\bar{A}+B)+\bar{B}(\bar{A}+B)$

$\bar{A}\bar{A}+\bar{A}B+\bar{A}\bar{B}+B\bar{B}$

$\bar{A}+\bar{A}B+\bar{A}\bar{B}+0$

$\bar{A}(1+B)+\bar{A}B$

$\bar{A}+\bar{A}B$

$\bar{A}(1+B)$

$\bar{A}$

AND law

$A\cdot\bar{A}=0$

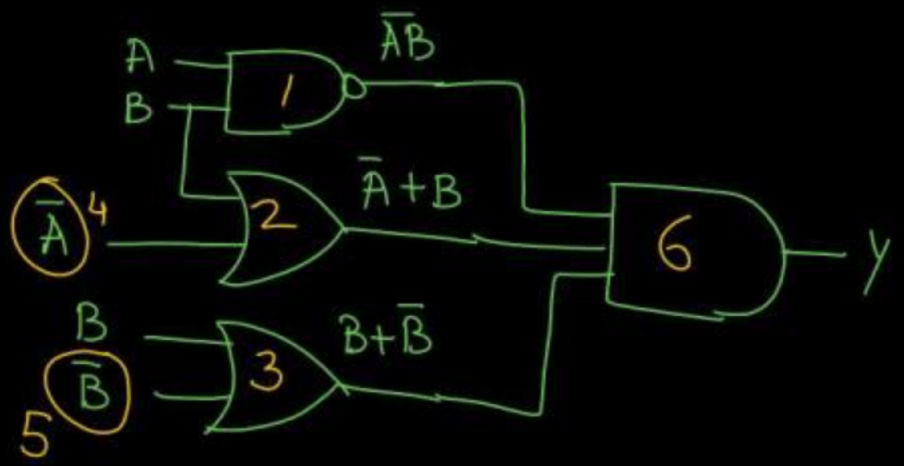
$A+A=A$

OR law

$A+\bar{A}=1$

AND law

$1\cdot A=A$





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Simplify the following expression: A + B(A + C) + AC

$$A + B(A + C) + AC$$

$$A + AB + BC + AC$$

$$A(1 + B) + BC + AC$$

$$A + BC + AC$$

$$A(1 + C) + BC$$

$$A + BC$$

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UNIVERSITY QUESTIONS

YEAR	QUESTION	MARKS	ANSWER
20-21	State and Prove Demorgan's theorem	10 marks	

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Binary-coded decimal

- **Binary-coded decimal** (**BCD**) is: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
- a class of binary encodings of decimal numbers
- each digit is represented by a fixed number of bits, usually four or eight.
- Sometimes, special bit patterns are used for a sign or other indications (e.g. error or overflow).

+9 = 0 1001 ⊖9 = 1 1001

-9 =

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BCD Representation for Decimal Digits

dec.	BCD
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001

dec. BCD

10 → 0001 0000

1 0

11 = 0001 0001

435 → 0100 0011 0101

78 → 0111 1000

(1001 0110)$_{BCD}$ = (96)$_{10}$

9 6

Gateway Classes : 7455 9612 84

BCD ARITHMATIC

- BCD Addition
- BCD Subtraction

Gateway Classes : 7455 9612 84

BCD Addition

1) At first the given number are to be added using the rule of binary.

2) Result of addition consider two cases:

$0 + 0 = 0$
$0 + 1 = 1$
$1 + 0 = 1$
$1 + 1 = 0$ | 1 = carry
$1 + 1 + 1 = 1$ | 1 = carry

- **Case 1:** the result of addition is greater than 9 or if a carry bit is present in the result which is not valid for BCD number => Add 6 whose binary equivalent is $(0110)_2$ to the result of addition.

- **Case 2:** The result of addition in is less than 9, which is valid for BCD numbers.

Gateway Classes 7455 961234

Examples: (i) 1010 + 0101
(ii) 0001 + 0101
(iii) 0001 0001 + 0010 0110

```
  1 0 1 0   = (10)
+ 0 1 0 1   = (5)
-----------
  1 1 1 1  ⇐ Invalid BCD
+ 0 1 1 0
-----------
0001 | 0101
-----------
  1      5
```

```
    1
  0 0 0 1
+ 0 1 0 1
---------
  0 1 1 0  ⇐ Result
```

```
  0001  0001  = 11
+ 0010  0110  +26
---------------
  0011  0111    37
---------------
  3     7
```

Gateway Classes: 7455961284

BCD Subtraction

- Method of BCD Subtraction : using 1's complement method

- Method of BCD Subtraction: using 9's complement method

1's complement

$1 \rightarrow 0$

$0 \rightarrow 1$

1's ⇒ 0 1 0 1 → 0 0 1 0

9's Complement

47

99

− 47

52 ← 9's of no.

Gateway Classes : 7455961264

Example : (i) 0101 0100 0001 - 0010 0001 0110 **(ii)** $(49)_{10} - (51)_{10}$

(iii) $(57)_{10} - (78)_{10}$

0101 0100 0001
-0010 0001 0110

541
-216
325

I : 0101 0100 0001
1's +1101 1110 1001

! 0011 0010 1010 ← Result +ve

⊗ 1010

0011 0010 0100

→ 1

0011 0010 0101
3 2 5

0110 ← 6
1010 ← 2's of 6

49 0100 1001
-51 0101 0001

I: 0100 1001
1's +1010 1110

X 1111 0111 ← Result - ve

0000 1000 ← 1's of Result

⊗ 1010 ← 2's of 6

0000 0010

-(0 2)

Gateway Classes : 7455 9612 84

$(57)_{10} - (78)_{10}$

0101 0111

$-$ 0111 1000

I: 0101 0111

1's of II: $+$ 1000 0111

1101 1110 ⇐ No carry, Result -ve

0010 0001 ⇐ 1's of Result

$-$ (2 1)

1) Take I no.
2) Add 1's of II no. (-ve no.)
3) If carry is generated, Result is +ve
   ⇒ If all digits are valid BCD ≥ Result OK
   ⇒ If digit is invalid BCD add 2's of 6 (i.e. 1010) to invalid digit
4) If no carry → Result is -ve
   ⇒ take 1's of Result
   ⇒ Check if carry was generated from I nibble to 2 nibble
      → If carry generated → add 0110 to lower nibble
      → otherwise take result as it is.

Gateway Classes : 7455 961 784

BCD Subtraction: using 9's complement method

- (0101 0001) – (0010 0001)

51
−21

99
−21
78 ⇐ 9's of 21

I: 51
9's of II: 78
11 29 ⇐ Result +ve
↳ 1
30

21 − 51

99
−51
48 ⇐ 9's of 51

I: 21
9's of II: 48
69 ⇐ Result -ve

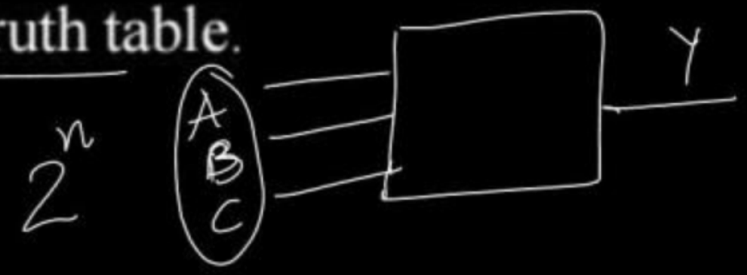
−(30) ⇐ 9's of Result

99
−69
30

Gateway Classes : 7498612 84

Truth Table Formation

- A truth table represents a table having all combinations of inputs and their corresponding result.
- It is possible to convert the switching equation into a truth table.
- For example, F(A,B,C) = A + BC
- The output will be high (1) if A = 1 or BC = 1 or both are 1
- The number of rows in the truth table is 2^n where n is the number of input variables
- Here there are $2^3 = 8$ possible input combination of inputs.



Gateway Classes : 7455961284

(A+BC) INPUTS			OUTPUT
A	B	C	Y
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

Maxterms

M0, M1, M2

Minterms

m3, m4, m5, m6, m7

Gateway Classes : 7455 9612 84

Generate Truth Table for : $Y = (A' + B) + C$

A	B	C	Y
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

Gateway Classes : 7455 9612 84

CANONICAL AND STANDARD FORMS

- **Canonical Form –**
- In Boolean algebra,Boolean function can be expressed as Canonical Disjunctive Normal Form known as **minterm**
- Some are expressed as Canonical Conjunctive Normal Form known as **maxterm** .
- In Minterm, we look for the functions where the output results in "1" & perform **Sum of minterm = SOP**

$Y = \bar{A}B + C$ $\quad Y = (A+B)(\bar{A}+\bar{B})$

- In Maxterm we look for function where the output results in "0" & perform **Product of Maxterm = POS**

$Y = AB\bar{C} + \bar{A}BC$

- Boolean functions expressed as a sum of minterms or product of maxterms are said to be in canonical form.

Gateway Classes : 7455 961 284

Standard Form

- In standard form Boolean function will contain all the variables in either true form or complemented form while in canonical number of variables depends on the output of SOP or POS.

- AB +BC = Canonical SOP
- ABC' + ABC + A'BC = Standard SOP
- (A+B) (B+C) = Canonical POS
- (A+B+C')(A+B+C)(A'+B+C) = Standard POS

Gateway Classes : 7455 9612 8

Truth table representing minterm and maxterm

X	Y	Z	Minterms	Maxterms
0	0	0	$m_0 = \bar{x}\bar{y}\bar{z}$	$M_0 = (x+y+z)$
0	0	1	$m_1 = \bar{x}\bar{y}z$	$M_1 = (x+y+\bar{z})$
0	1	0	$m_2 = \bar{x}y\bar{z}$	$M_2 = (x+\bar{y}+z)$
0	1	1	$m_3 = \bar{x}yz$	$M_3 = (x+\bar{y}+\bar{z})$
1	0	0	$m_4 = x\bar{y}\bar{z}$	$M_4 = (\bar{x}+y+z)$
1	0	1	$m_5 = x\bar{y}z$	$M_5 = (\bar{x}+y+\bar{z})$
1	1	0	$m_6 = xy\bar{z}$	$M_6 = (\bar{x}+\bar{y}+z)$
1	1	1	$m_7 = xyz$	$M_7 = (\bar{x}+\bar{y}+\bar{z})$

Gateway Classes : 7455 96 1284

Example – Express the Boolean function $F = A + B'C$ as standard sum of minterms.

$B + \bar{B}$

$A + A = A$

$$F = A + \bar{B}C$$

$$= A \cdot 1 \cdot 1 + 1 \cdot \bar{B}C$$

$$= A(B + \bar{B})(C + \bar{C}) + (A + \bar{A})\bar{B}C$$

$$= (AB + A\bar{B})(C + \bar{C}) + A\bar{B}C + \bar{A}\bar{B}C$$

$$= ABC + AB\bar{C} + A\bar{B}C + A\bar{B}\bar{C} + A\bar{B}C + \bar{A}\bar{B}C$$

$$= ABC + AB\bar{C} + A\bar{B}C + A\bar{B}\bar{C} + \bar{A}\bar{B}C$$

$$= \Sigma m(7, 6, 5, 4, 1)$$

Gateway Classes: 7455 9612 84

Express the Boolean function F = xy + x'z as a product of maxterms

$F = xy + \bar{x}z$

$= (xy + \bar{x})(xy + z)$

$= (x + \bar{x})(y + \bar{x})(x + z)(y + z)$

$= (\bar{x} + y)(x + z)(y + z)$

$= (\bar{x} + y + 0)(x + 0 + z)(0 + y + z)$

$= (\bar{x} + y + z\bar{z})(x + y\bar{y} + z)(x\bar{x} + y + z)$

$= (\bar{x} + y + z)(\bar{x} + y + \bar{z})(x + y + z)(x + \bar{y} + z)(x + y + z)(\bar{x} + y + z)$

$= (\bar{x} + y + z)(\bar{x} + y + \bar{z})(x + y + z)(x + \bar{y} + z)$

$= \Pi M(4, 5, 0, 2)$

$A + BC = (A + B)(A + C)$

$A \cdot A = A$

Gateway Classes : 7455961284

Example –Convert F(A, B, C) = ∑m(1,4,5,6,7) to POS FORM

$$F(A,B,C) = \Sigma m(1,4,5,6,7)$$

$$= \pi M(0,2,3)$$

$$= (A+B+C)(\bar{A}+\bar{B}+C)(A+\bar{B}+\bar{C})$$

001
100
101
110

$$= (\bar{A}\bar{B}C + A\bar{B}\bar{C} + A\bar{B}C + AB\bar{C} + ABC)$$

Gateway Classes : 7455961284

Methods to simplify the boolean function

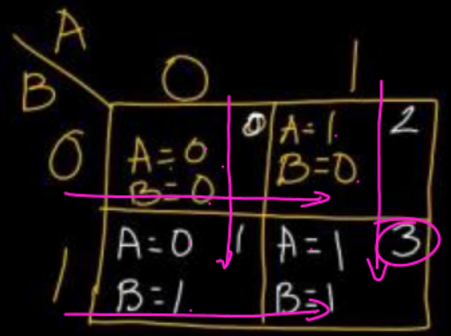
The methods used for simplifying the Boolean function are as follows –

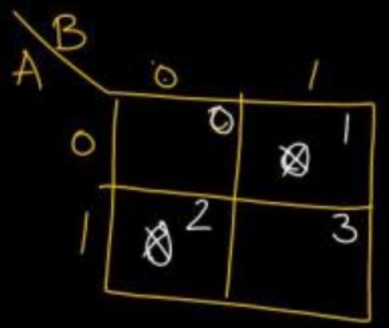
- Karnaugh-map or K-map
- NAND gate method
- Tabular method

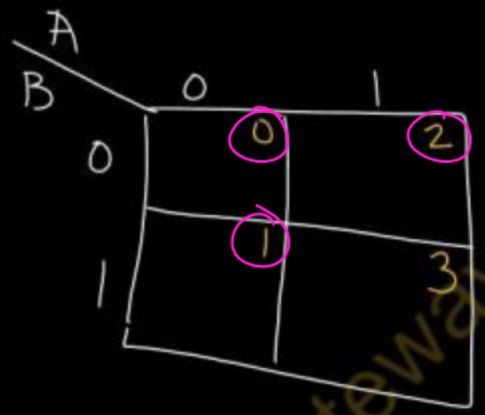
Gateway Classes : 7455 9612 84

2 Variable K-Map

- The number of cells is four, since the number of variables is two.
- **2 variable K-Map.**







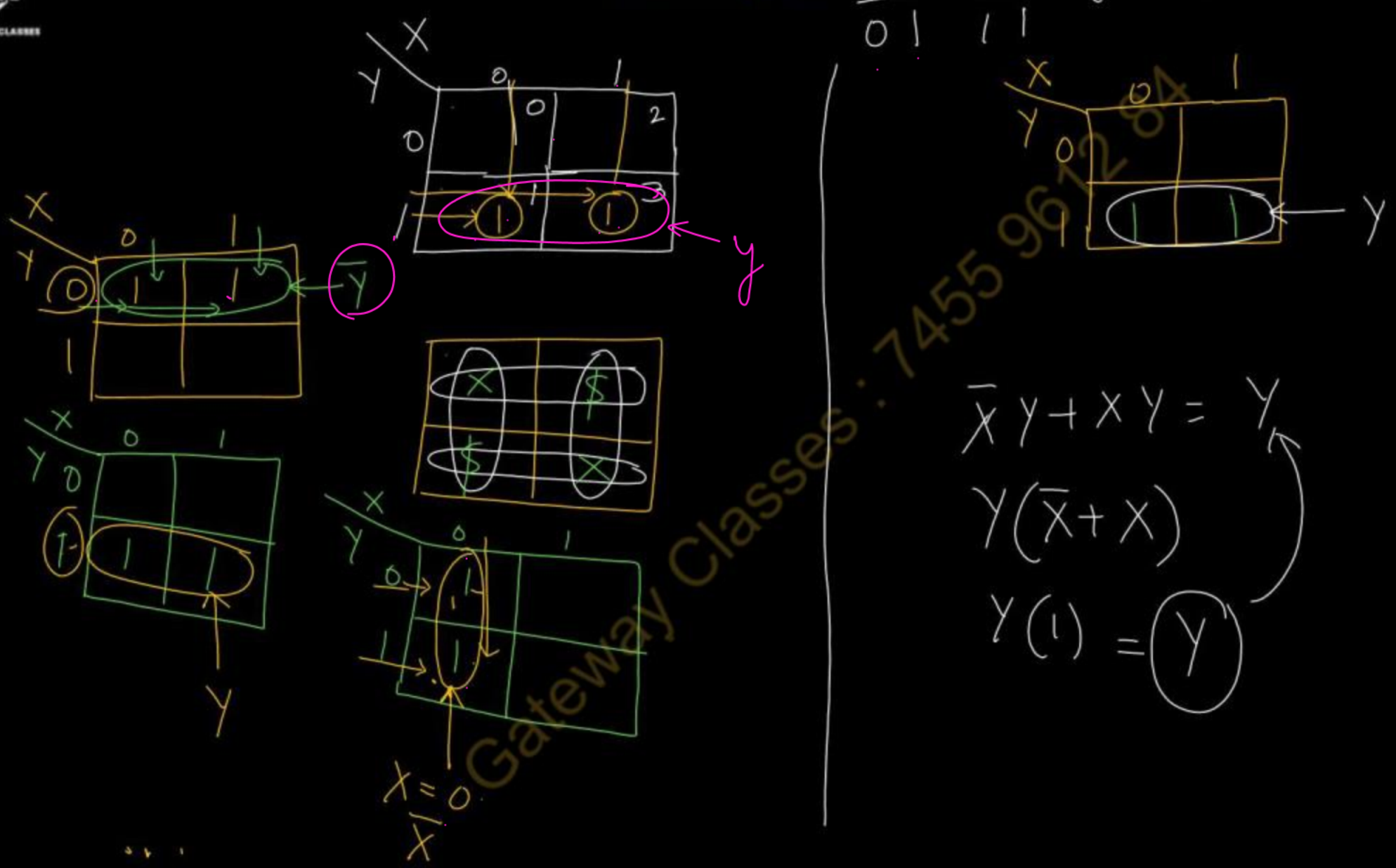


variable = 2

Combination = $2^n = 2^2 = 4$

Gateway Classes : 7455961284

Example - X'Y+XY (SOP)



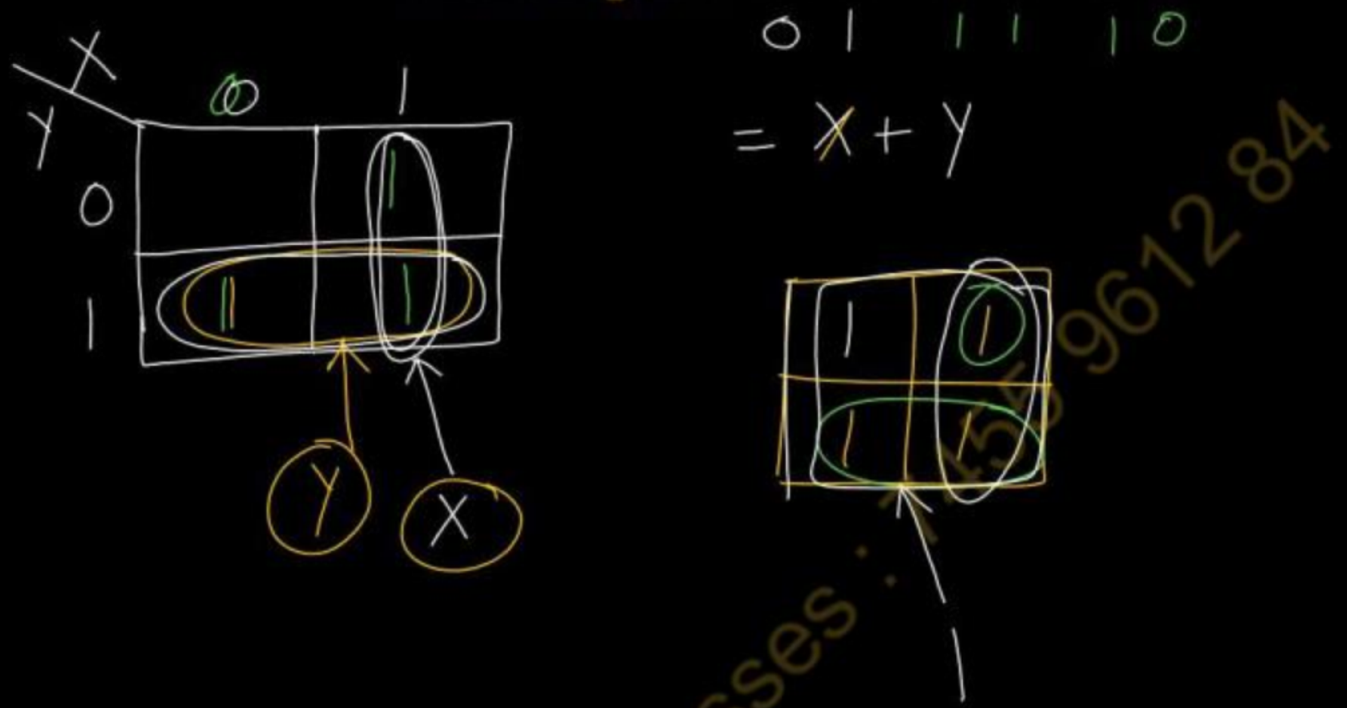
$$\bar{X}Y + XY = Y$$

$$Y(\bar{X} + X)$$

$$Y(1) = Y$$

Gateway Classes : 7455 9612 84

Example - X'Y+XY+XY'



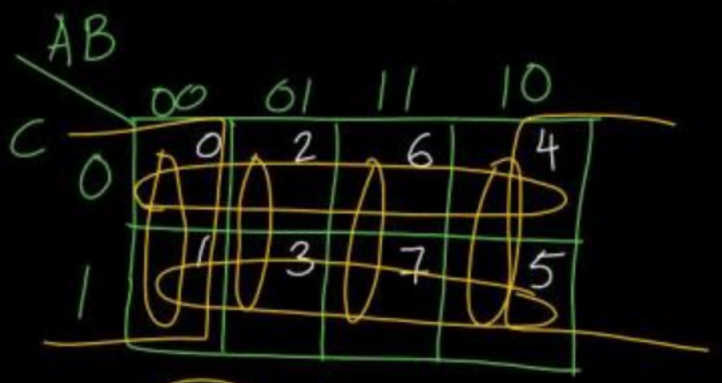
0 1 1 1 1 0

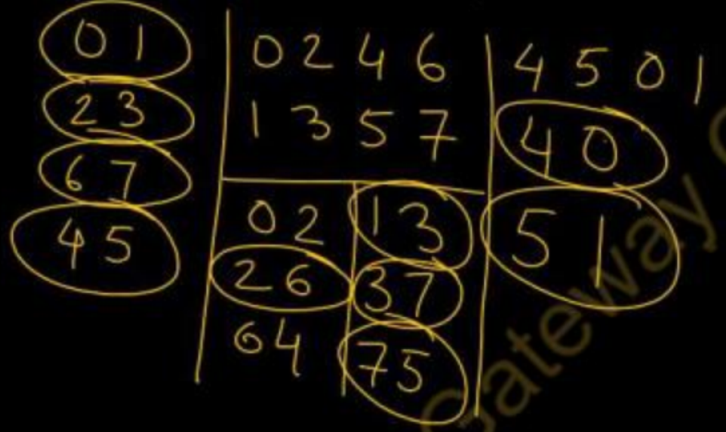
= X + Y

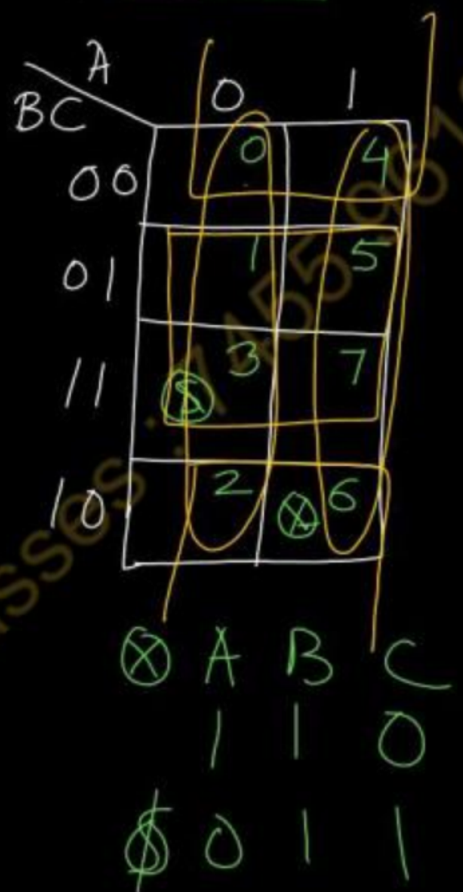
Gateway Classes : 7439612 84

3 Variable K-Map

- The number of cells in 3 variable K-map is eight, since the number of variables is three. 2^3
- **3 variable K-Map**





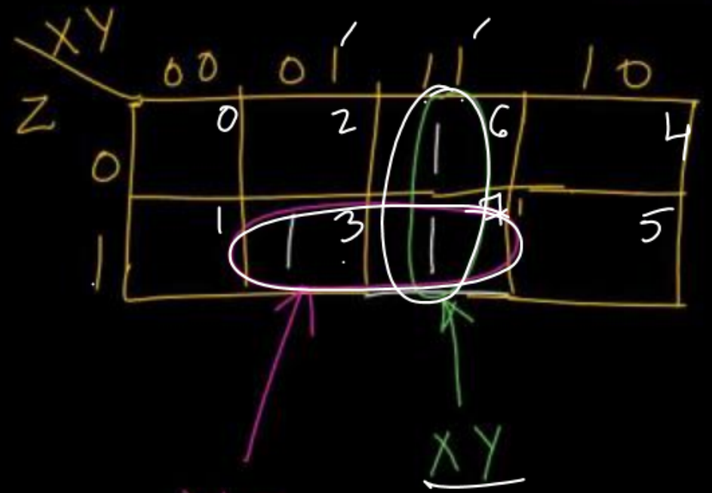


⊗ A B C
1 1 0
$ 0 1 1

Gateway Classes

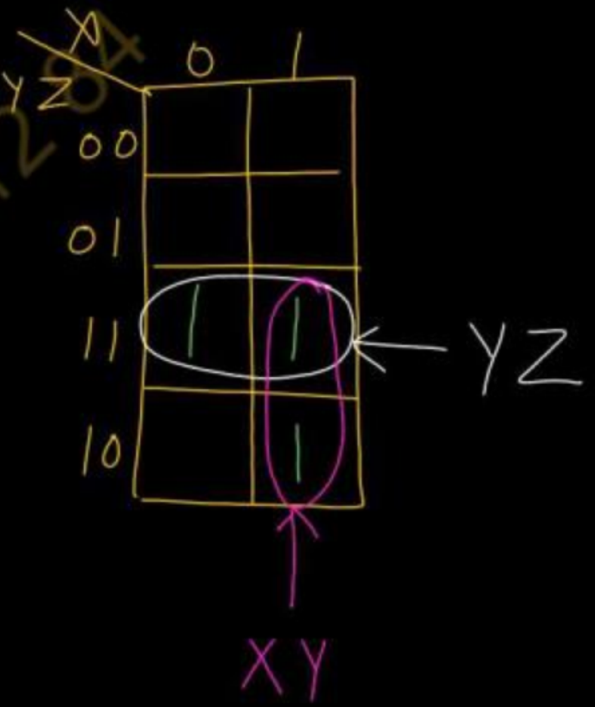
Example- F = XYZ'+XYZ+X'YZ (SOP)

110 | 111 | 011



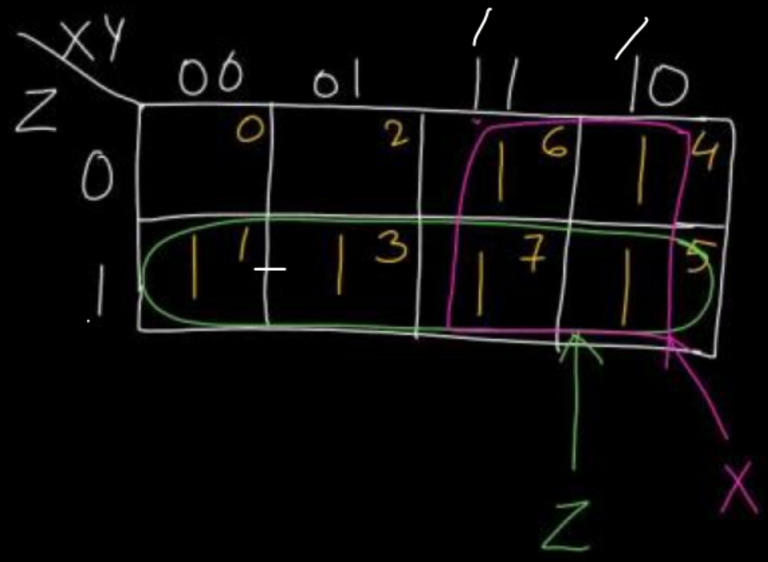
$F = XY + YZ$





Gateway Classes : 7455 961254

Example- F(X,Y,Z) = (1,3,4,5,6,7)



$$F = X + Z$$

Gateway Classes : 7455 9612 84

4 Variable K-Map

- number of cells in 4 variable K-map is sixteen $2^4 = 16$
- **4 variable K-Map.**

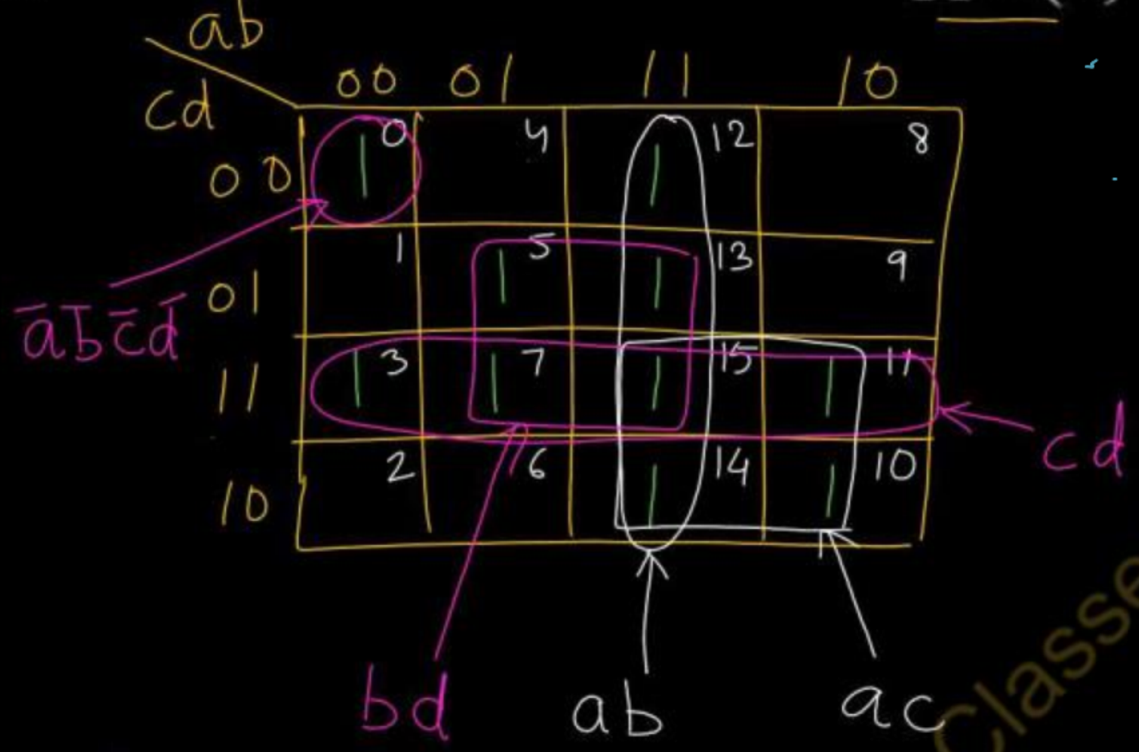
CD \ AB	00	01	11	10
00	0	4	12	8
01	1	5	13	9
11	3	7	15	11
10	2	6	14	10

ABCD

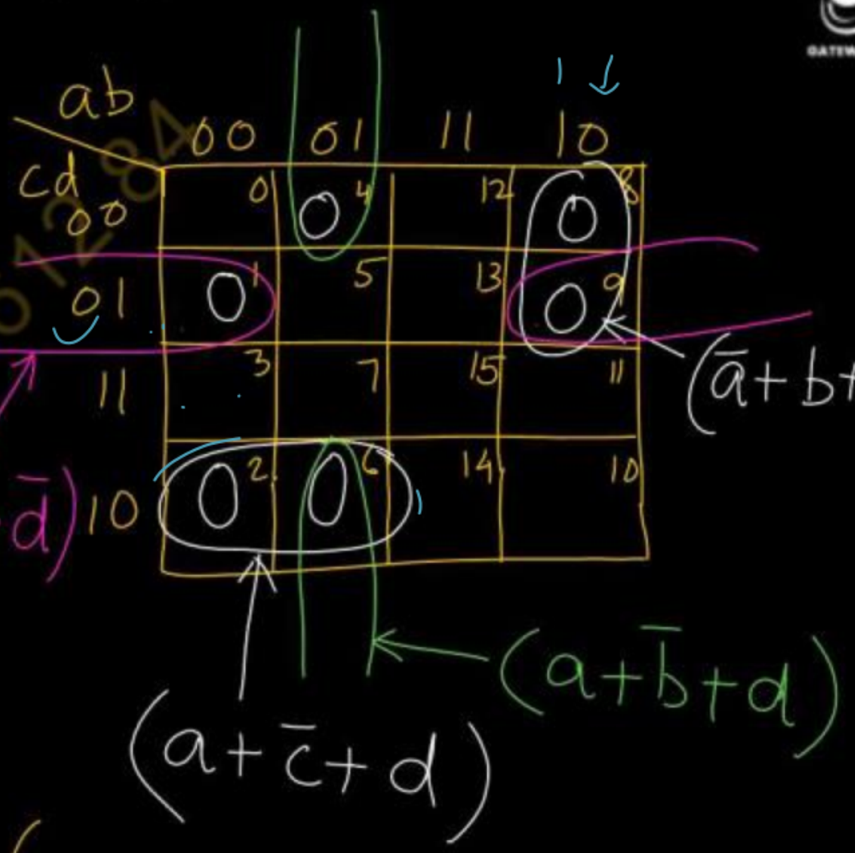
AB \ CD	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

Gateway Classes : 7455 9612 84

Example-f(a,b,c,d) = $\sum m(0,3,5,7,10,11,12,13,14,15)$ = $\prod M(1,2,4,6,8,9)$



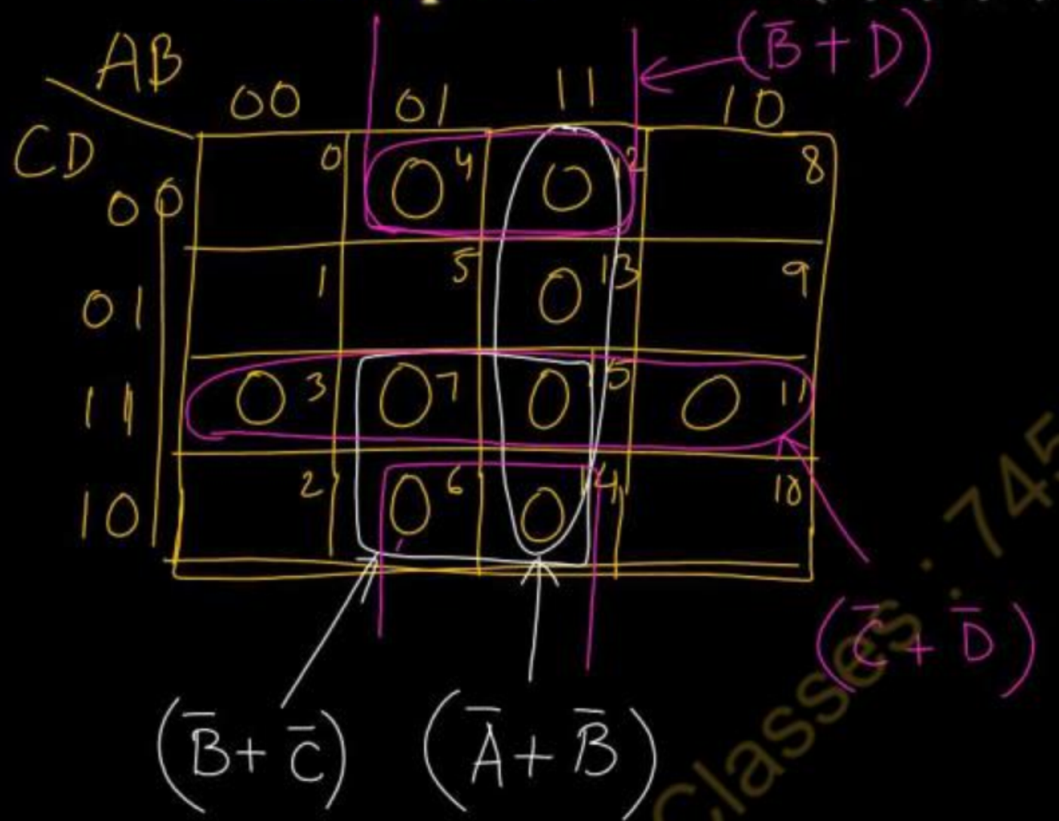
$$f = bd + ab + ac + cd + \bar{a}\bar{b}\bar{c}\bar{d}$$



$$f = (b+c+\bar{d})(a+\bar{c}+d)(a+\bar{b}+d)(\bar{a}+b+c)$$

Gateway Classes : 7455 961 284

Example: $F = \Pi M(3,4,6,7,11,12,13,14,15) \Rightarrow$ POS



$$F = (\bar{B}+\bar{C})(\bar{A}+\bar{B})(\bar{C}+\bar{D})(\bar{B}+D)$$

Gateway Classes : 7455 9612 84

UNIVERSITY QUESTIONS

YEAR	QUESTION	MARKS	ANSWER
21-22	Simplify the expression F(A,B,C) =AB+BC+A' by K map	2 marks	A'+B
21-22	Minimize the following Boolean function F(A,B,C,D)= Σm(0,3,4,5,7,9,13,14,15)	10 marks	BD+A'C'D'+ A'CD+AC'D+ ABC
20-21	Simplify using K map to obtain a minimum POS expression: (A'+B'+C+D)(A+B'+C+D)(A+B+C'+D') (A'+B+C'+D') (A+B+C'+D)	10 marks	(B'+C+D) (A+B+C') (B+C'+D)

Gateway Classes . 7455 9612 84

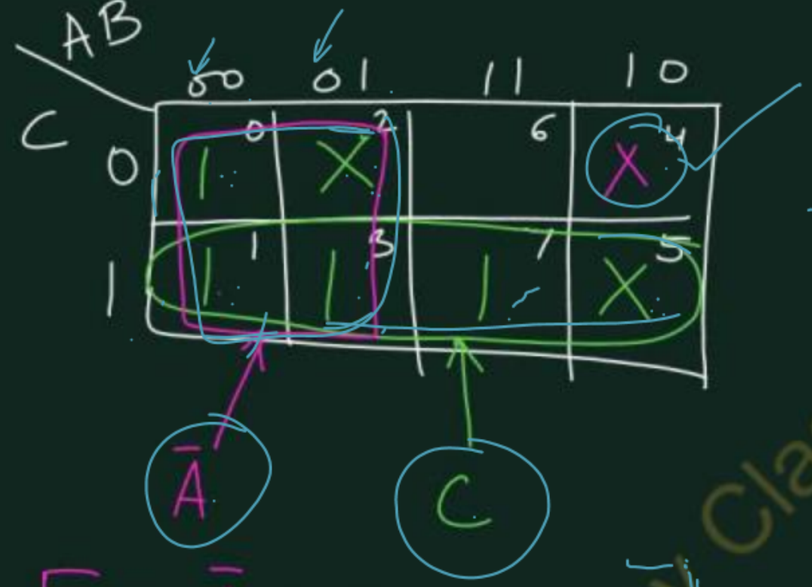
Don't Care (Incompletely Specified) Conditions

- Some times, not all values of a function are defined
- Some input conditions will never occur
- We don't care what the output is for that input condition
- In these cases, we can choose the output to be either 0 or 1, whichever simplifies the circuit.

Gateway Classes: 7455 9612 84

$$F = \Sigma m(0,1,3,7) + d(2,5)$$

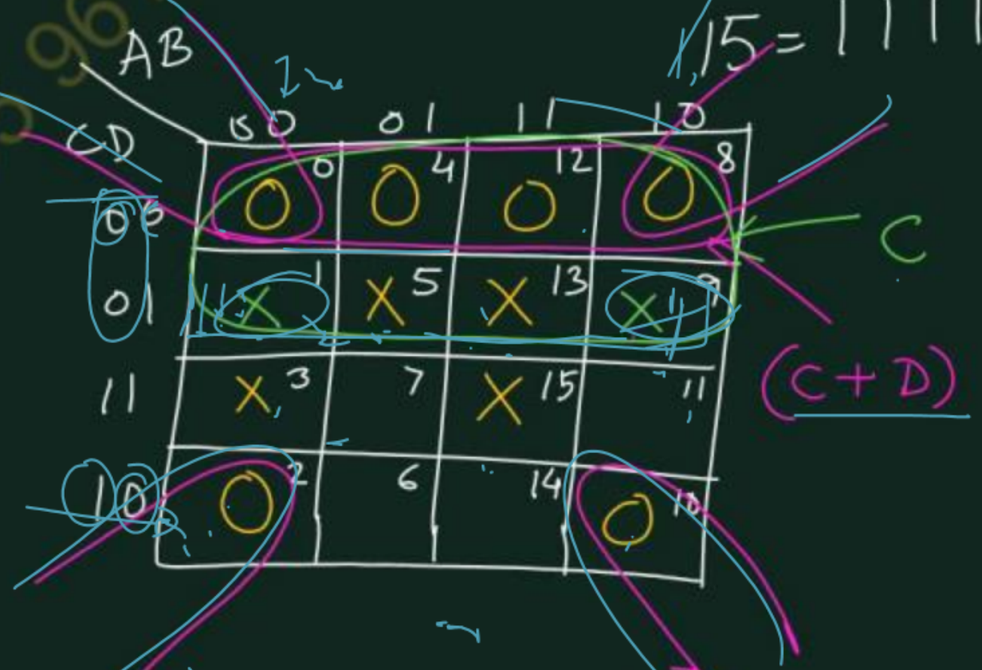
$7 = 1\,1\,1$



$$F = \bar{A} + C$$

$$F = \pi M(0,2,4,8,10,12) + d(3,5,13,15)$$

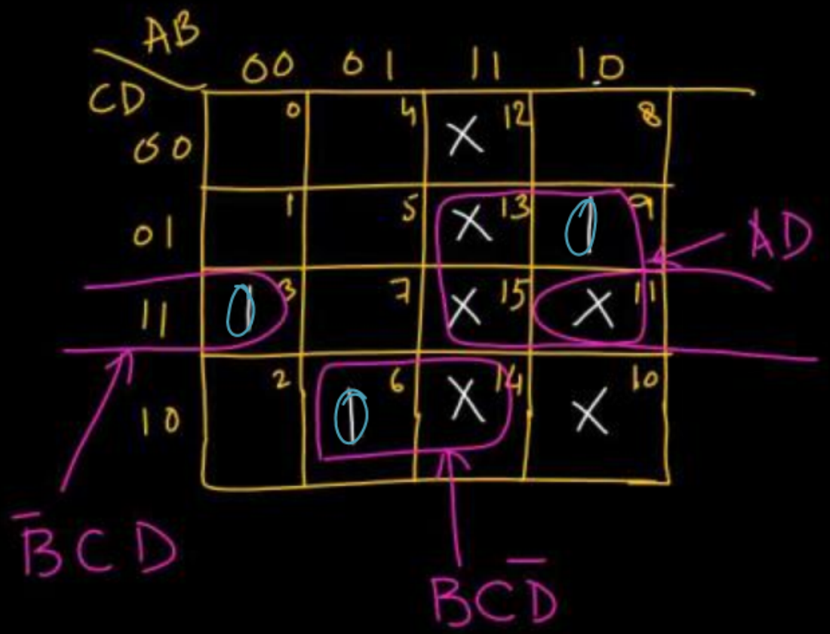
$15 = 1\,1\,1\,1$



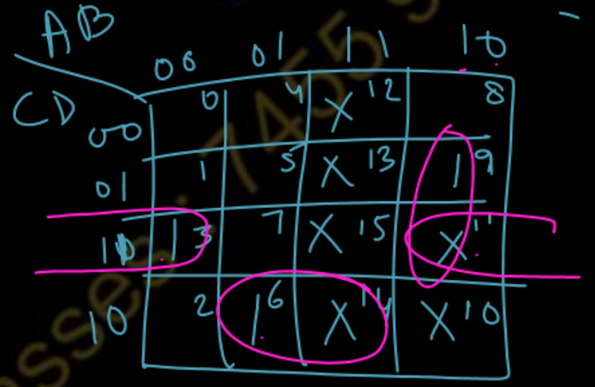
$$F = (B+D)(C+D)$$

Gateway Classes : 7455 9612 84

Example: A circuit is to have an output of 1 if a binary coded decimal (BCD) digit is a multiple of 3



$$F = \pi M(0, 1, 2, 4, 5, 7, 8) + d(10, 11, 12, 13, 14, 15)$$



A	B	C	D	Y
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	1
0	1	0	0	1
0	1	0	1	0
0	1	1	0	1
0	1	1	1	0
1	0	0	0	0
1	0	0	1	1
1	0	1	0	X
1	0	1	1	X
1	1	0	0	X
1	1	0	1	X
1	1	1	0	X
1	1	1	1	X

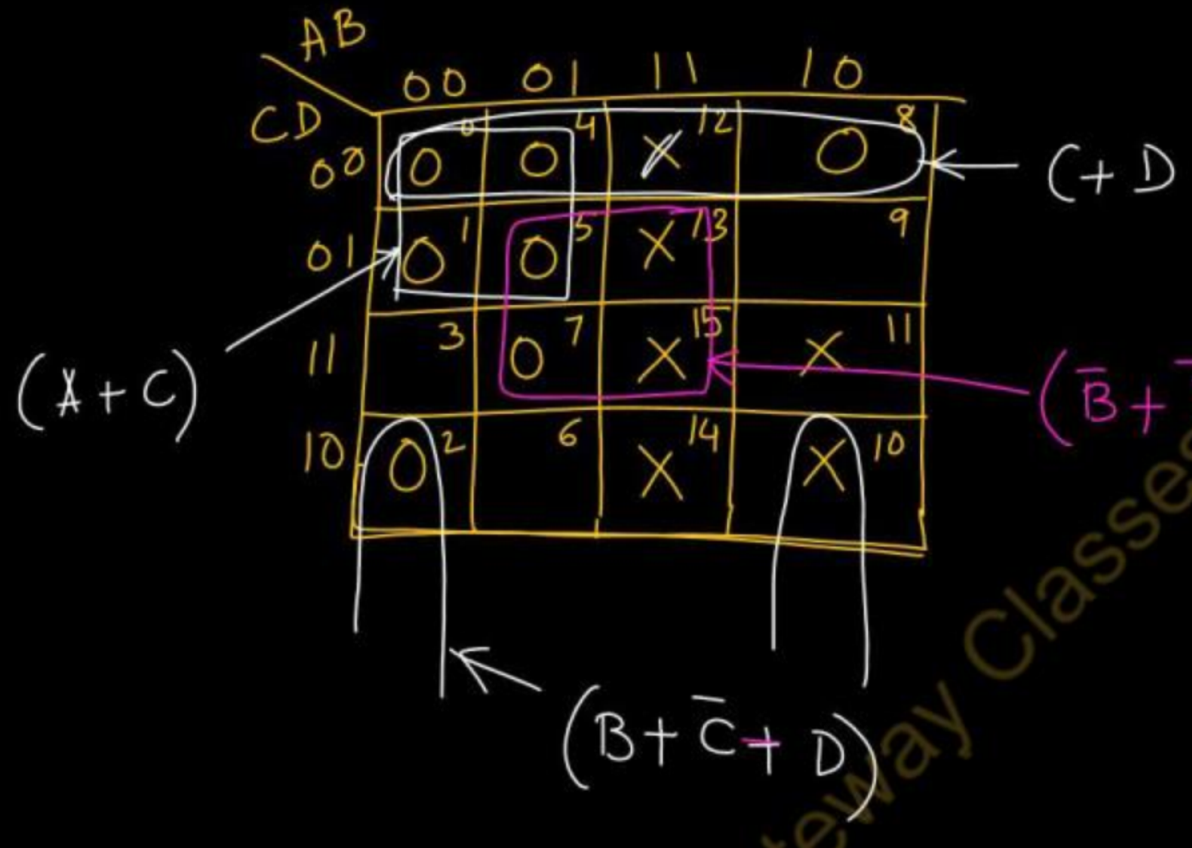
$$F = AD + B\bar{C}\bar{D} + \bar{B}CD$$
$$= AD + C(B\bar{D} + \bar{B}D)$$
$$= AD + C(B \oplus D)$$

$A\bar{B}\bar{D} + B\bar{C}\bar{D} + \bar{B}CD$

GatewayClasses 7455961264

Product of sums:

$F = \Pi M(0,1,2,4,5,7,8) + d(10,11,12,13,14,15)$



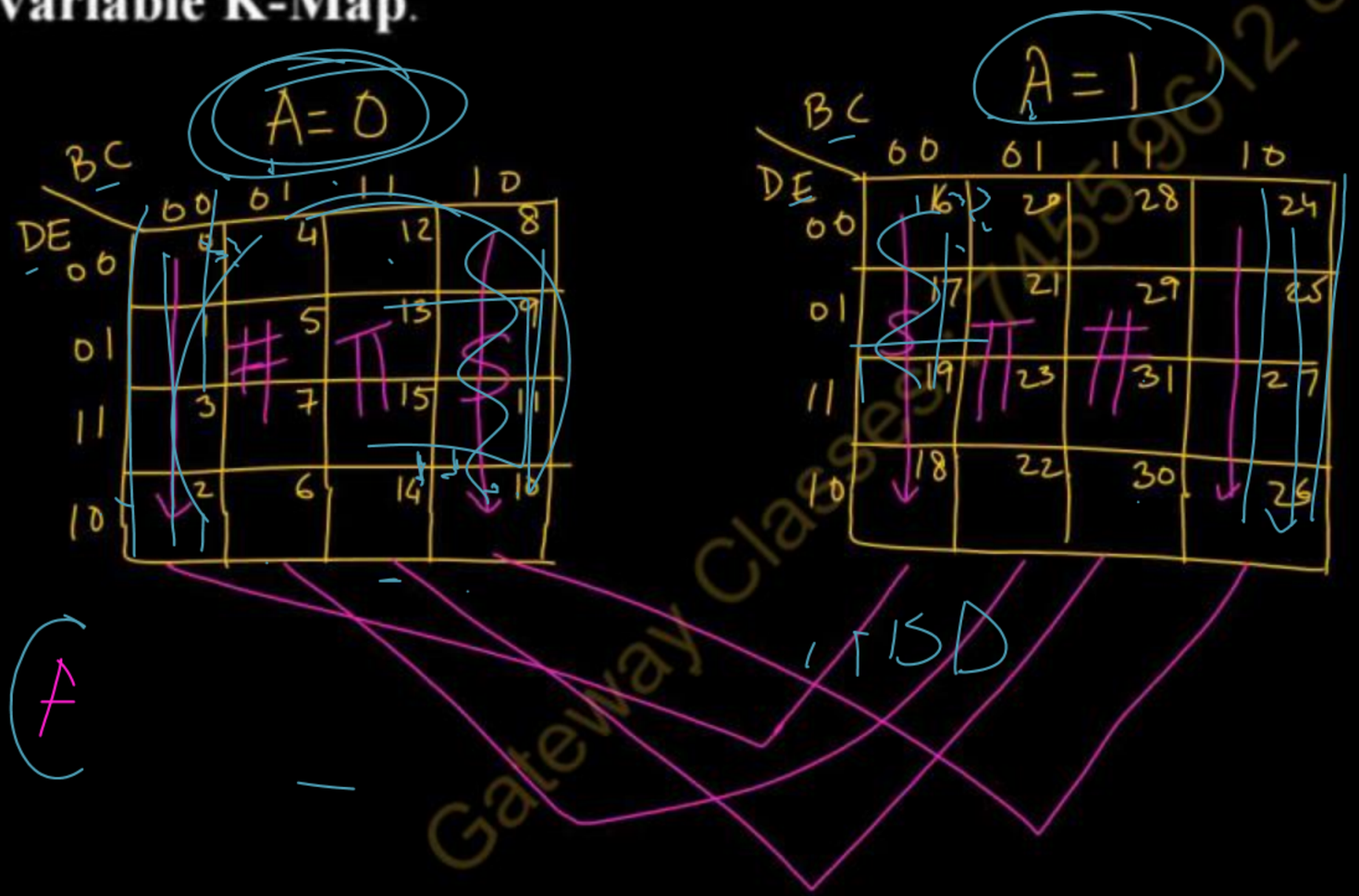
$F = (C+D)(\bar{B}+\bar{D})(B+\bar{C}+D)$

$(A+C)$

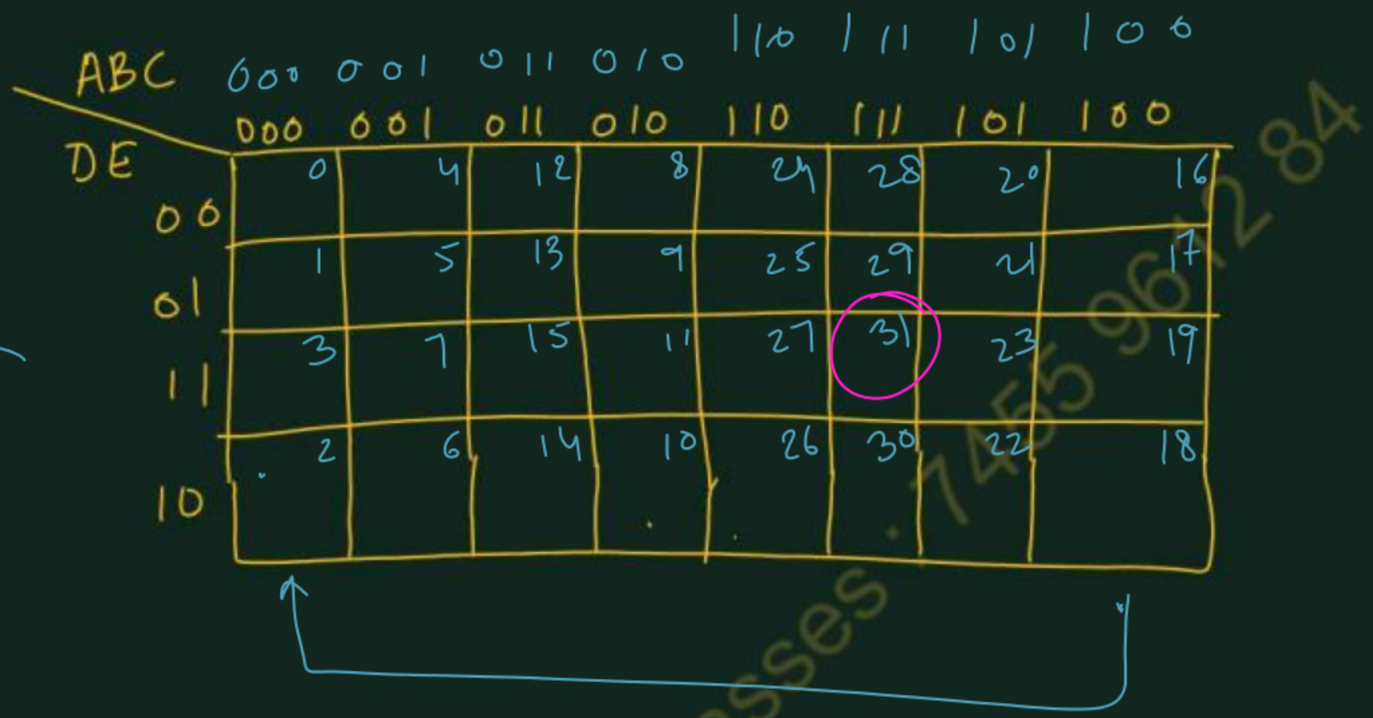
Gateway Classes: 7455 96 12 84

5 Variable K-Map

- The number of cells in 5 variable K-map is thirty-two $2^5 = 32$
- **5 variable K-Map.**

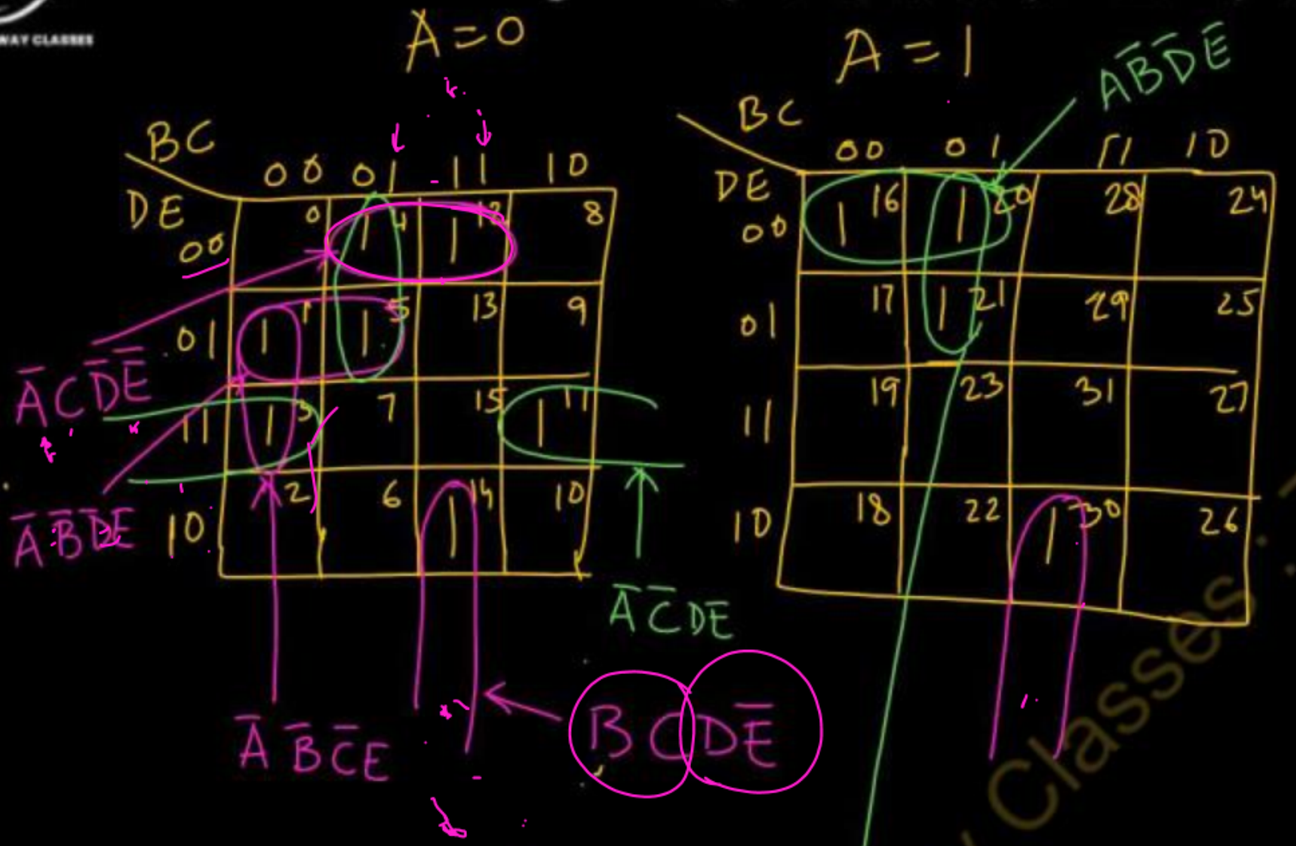


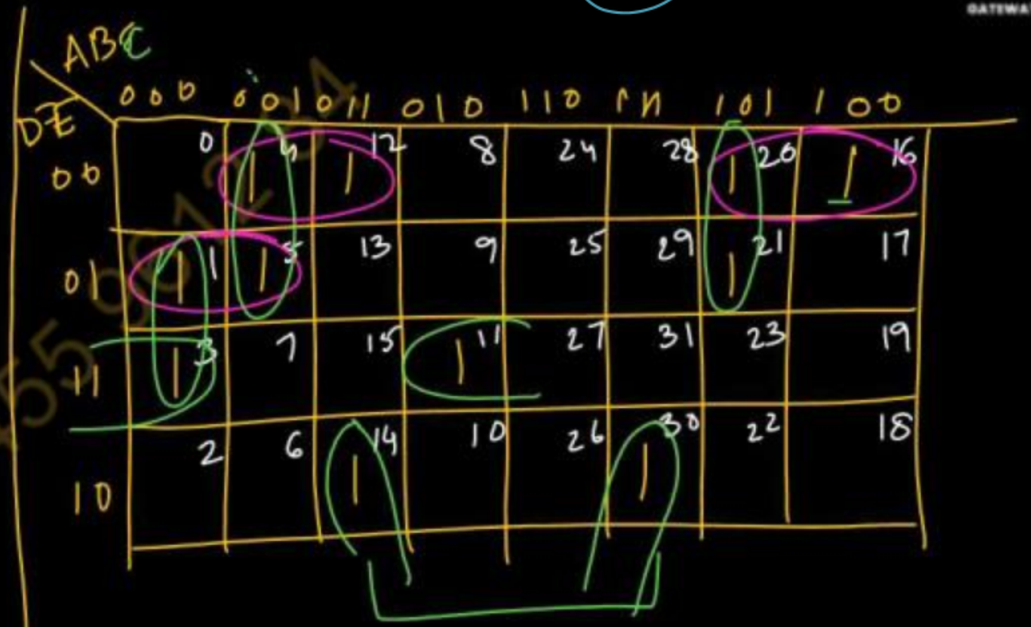
Gateway Classes 7455 9612 84



Gateway Classes . 7455 96 2 84

Example: $Y(A,B,C,D,E) = \sum m(1,3,4,5,11,12,14,16,20,21,30)$





$$F = \underset{①}{\bar{A}C\bar{D}\bar{E}} + \underset{②}{\bar{A}\bar{B}\bar{D}\bar{E}} + \underset{③}{\bar{A}\bar{B}\bar{C}E} + \underset{④}{BC\bar{D}\bar{E}} + \underset{⑤}{\bar{B}C\bar{D}} + \underset{⑥}{\bar{A}\bar{C}DE} + A\bar{B}\bar{D}\bar{E}$$

$$F = \underset{⑤}{\bar{B}C\bar{D}} + \underset{②}{A\bar{B}\bar{D}\bar{E}} + \underset{①}{\bar{A}C\bar{D}\bar{E}} + \underset{③}{\bar{A}\bar{B}\bar{C}E} + \underset{⑥}{\bar{A}\bar{C}DE} + \underset{④}{BC\bar{D}\bar{E}}$$

Gateway Classes : 7455 6612 14

6 Variable K-Map

$2^6 = 64$

AB = 00

CD \ EF	00	01	11	10
00	0	4	12	8
01	1	5	13	9
11	3	7	15	11
10	2	6	14	10

AB = 01

16	20	28	24
17	21	29	25
19	23	31	27
18	22	30	26

AB = 11

48	52	60	56
49	53	61	57
51	54	63	59
50	55	62	58

AB = 10

32	36	44	40
33	37	45	41
35	39	47	43
34	38	46	42

DEF \ ABC	000	001	011	010	110	111	101	100
000	0	8	24	16	48	56	40	32
001	1	9	25	17	49	57	41	33
011	3	11	27	19	51	59	43	35
010	2	10	26	18	50	58	42	34
110	6	14	30	22	54	62	46	38
111	7	15	31	23	55	63	47	39
101	5	13	29	21	53	61	45	37
100	4	12	28	20	52	60	44	36

A

F

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UNIVERSITY QUESTIONS

YEAR	QUESTION	MARKS	ANSWER
22-23	Simplify the logic function using K-map Y = Σm (0, 2, 3, 4, 6, 7, 9, 11,16,18,19,20,22,23,25,27)	10 marks	
19-20	Simplify the logic expression using K map F(A,B,C,D,E,F) = Σm(0,5,7,8,9,12,13,23,24,25, 28,29, 37, 40,42,44,46,55,56,57,60,61)	10 marks	

Gateway Classes 7265 9612 84

Tabulation Method

STEP 1:

- Convert Minterm List to Prime Implicant List

STEP 2:

- Choose All Essential Prime Implicants
- If all minterms are covered, stop

STEP 3:

- Formulate the Reduced Cover Table

Gateway Classes : 7455 962 284

EXAMPLE: $f^{on} = \{m_0, m_1, m_2, m_3, m_5, m_8, m_{10}, m_{11}, m_{13}, m_{15}\}$
$= \sum(0, 1, 2, 3, 5, 8, 10, 11, 13, 15)$

m_0	0 0 0 0
m_1	0 0 0 1
m_2	0 0 1 0
m_3	0 0 1 1
m_5	0 1 0 1
m_8	1 0 0 0
m_{10}	1 0 1 0
m_{11}	1 0 1 1
m_{13}	1 1 0 1
m_{15}	1 1 1 1

G_1	m_0	0 0 0 0	✓
G_2	m_1	0 0 0 1	✓
	m_2	0 0 1 0	✓
	m_8	1 0 0 0	✓
G_3	m_3	0 0 1 1	✓
	m_5	0 1 0 1	✓
	m_{10}	1 0 1 0	✓
G_4	m_{11}	1 0 1 1	✓
	m_{13}	1 1 0 1	✓
G_5	m_{15}	1 1 1 1	✓

0,1	0 0 0 –	✓
0,2	0 0 – 0	✓
0,8	– 0 0 0	✓
1,3	0 0 – 1	✓
1,5	0 – 0 1	(PI)
2,3	0 0 1 –	✓
2,10	– 0 1 0	✓
8,10	1 0 – 0	✓
3,11	– 0 1 1	✓
5,13	– 1 0 1	(PI)
10,11	1 0 1 –	✓
11,15	1 – 1 1	(PI)
13,15	1 1 – 1	(PI)

A B C D	
0,1,2,3	0 0 – – (EPI)
0,2,8,10	– 0 – 0 (EPI)
2,3,10,11	– 0 1 – (EPI)

	0	1	2	3	5	8	10	11	13	15
$\bar{A}\bar{B}$	X	X	X	X						
$\bar{B}\bar{D}$	X		X			X	X			
$\bar{B}C$			X	X			X	X		
$\bar{A}\bar{C}D$		X			X					
$B\bar{C}D$					X				X	
ACD								X		X
$A\bar{B}D$									X	X
	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓

$$f = \bar{B}\bar{D} + \bar{A}\bar{B} + B\bar{C}D + ACD$$

Gateway Classes: 7455 gc12@gmail

Simplify the following expression using Tabulation Method

$$F(A,B,C,D) = \sum m(0,4,8,10,12,13,15) + d(1,2)$$

m_0	0 0 0 0
m_4	0 1 0 0
m_8	1 0 0 0
m_{10}	1 0 1 0
m_{12}	1 1 0 0
m_{13}	1 1 0 1
m_{15}	1 1 1 1
d_1	0 0 0 1
d_2	0 0 1 0

G_1	m_0	0000 ✓	0,4	0 _ 0 0 ✓
			0,8	_ 0 0 0 ✓
G_2	m_4	0 1 0 0 ✓	0,1	0 0 0 _ (PI)
	m_8	1 0 0 0 ✓	0,2	0 0 _ 0 ✓
	d_1	0 0 0 1 ✓	4,12	_ 1 0 0 ✓
	d_2	0 0 1 0 ✓	8,10	1 0 _ 0 ✓
G_3	m_{10}	1 0 1 0 ✓	8,12	1 _ 0 0 ✓
	m_{12}	1 1 0 0 ✓	2,10	_ 0 1 0 ✓
G_4	m_{13}	1 1 0 1 ✓	12,13	1 1 0 _ (PI)
G_5	m_{15}	1 1 1 1 ✓	13,15	1 1 _ 1 (PI)

A B C D	
0,4,8,12	_ _ 0 0 (EPI)
0,8,2,10	_ 0 _ 0 (EPI)

	m: 0	4	8	10	12	13	15	d: 1	2
$\bar{C}\bar{D}$	X	(X)	X		X				
$\bar{B}\bar{D}$	X		X	(X)					X
$\bar{A}\bar{B}\bar{C}$	X							X	
$AB\bar{C}$					X	X			
ABD						X	(X)		
	✓	✓	✓	✓	✓	✓	✓		✓

$$F = \bar{C}\bar{D} + \bar{B}\bar{D} + ABD$$

Gateway Classes 7455 9612 84

$F = \Sigma m(0, 8, 12, 15, 21, 26, 30, 31)$

	16	8	4	2	1
	A	B	C	D	E
m_0	0	0	0	0	0
m_8	0	1	0	0	0
m_{12}	0	1	1	0	0
m_{15}	0	1	1	1	1
m_{21}	1	0	1	0	1
m_{26}	1	1	0	1	0
m_{30}	1	1	1	1	0
m_{31}	1	1	1	1	1

Gateway Classes : 7455 9612 84

UNIVERSITY QUESTIONS

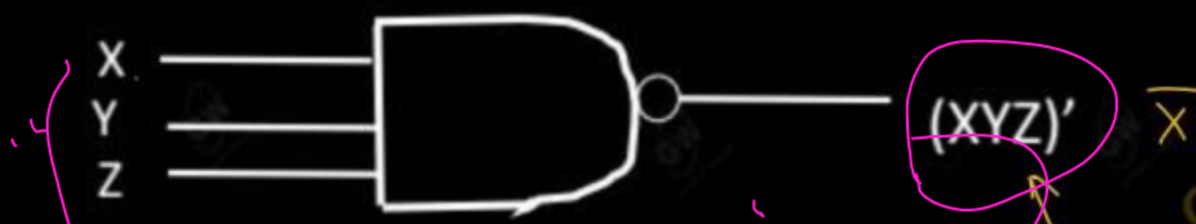
YEAR	QUESTION	MARKS	ANSWER
22-23	Minimize y = Σm(0,1,2,3,5,6,7,8,14,15) + d(4,11,13) using tabular method.	10 marks	Y= A' +BC + B'C'D'
19-20	Minimize the logic function using Quine-McCluskey Method $F(A, B,C,D,E)$ = Σm(8,9,10,11,13,15, 16, 18,21,24,25,26,27,30,31)	10 marks	F=BC'+ABD+ AC'E' + A'BE + AB'CD'E
18-19	Minimize the following using Quine-McCluskey meth0d: F(A,B, C, D)= Σm(0,1, 9,15, 24, 29, 30) + d(8,11, 31)	10 marks	F= A'B'C'D' + A'BDE + A'BC'E + BC'D'E' + ABCE + ABCD

Gateway Classes 7455 961 284

NAND GATE IMPLEMENTATION OF CIRCUIT

- NAND gate is a universal gate
- Function should be in SOP form
- Obtain simplified Boolean function in terms of Boolean operators and then convert the function to NAND logic.
- Implement the function with AND & OR gates.
- Replace AND with NAND gate.
- Replace OR gate by invert OR (NAND) gate.
- Two bubbles on same line denote double complement & both can be removed.

Gateway Classes : 7455 9612 84



$(\bar{x}+\bar{y}+\bar{z})$



$\overline{A+B} = \bar{A} \cdot \bar{B}$

$\overline{A \cdot B} = \bar{A} + \bar{B}$



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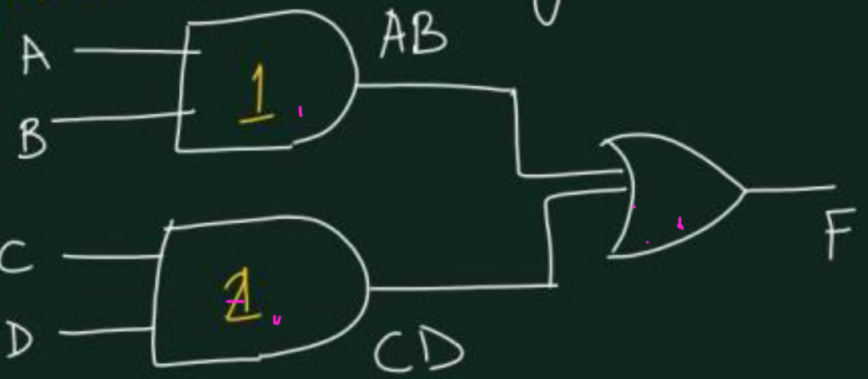
Example: F = AB + CD Implement using NAND gates only.

- Step 1: Implement using Basic gates.
- Step 2: Convert AND to NAND and OR to invert OR.
- Step 3: Replace invert OR by NAND gate.

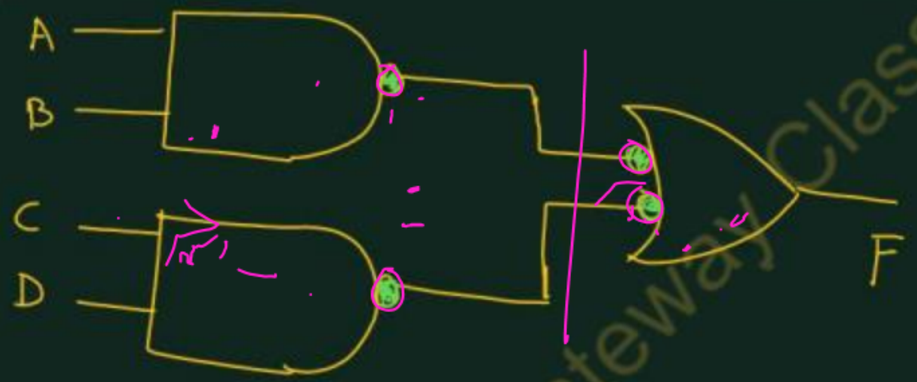
Gateway Classes: 7455 9612 84

$F = AB + CD$

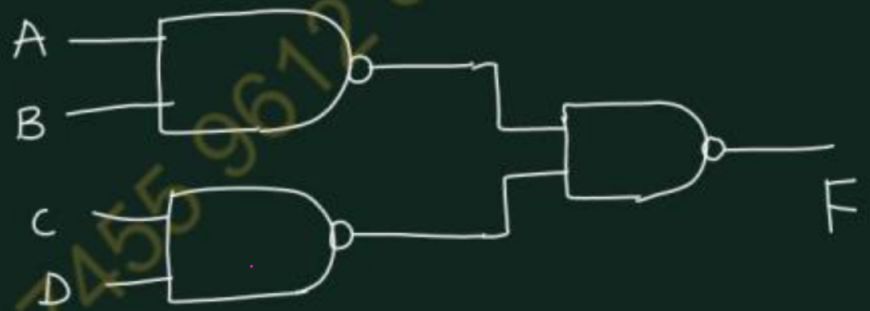
Step 1: Implement using basic Gates



Step 2: AND → NAND & OR → Invert OR



Step 3: Invert OR → NAND



Gateway Classes : 7455 9612 84

Example: F = (A+B) (A+C) Implement using NAND gates only.

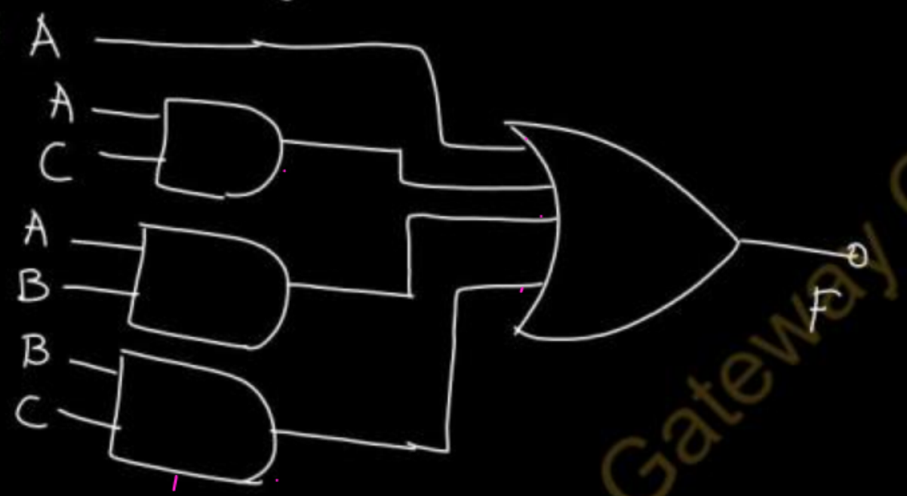
$F = (A+B)(A+C)$

$= A(A+C) + B(A+C)$

$= A \cdot A + A \cdot C + B \cdot A + B \cdot C$

$= A + AC + AB + BC$

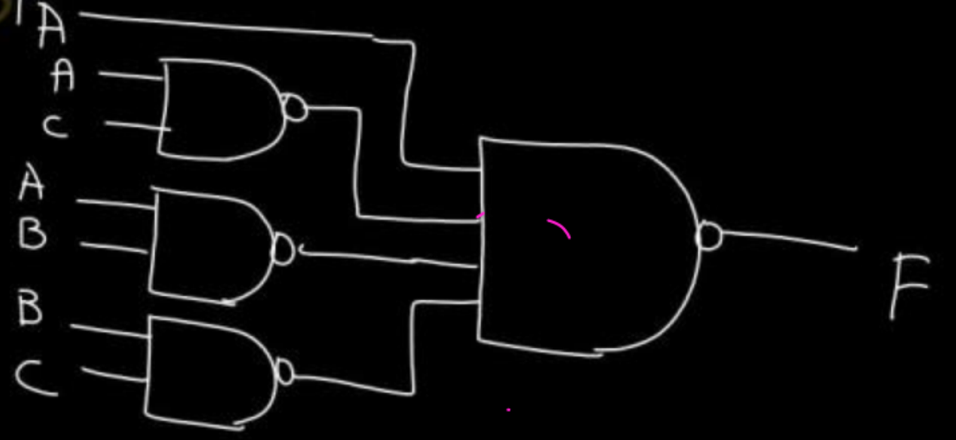
step 1: Using Basic Gates



step 2:



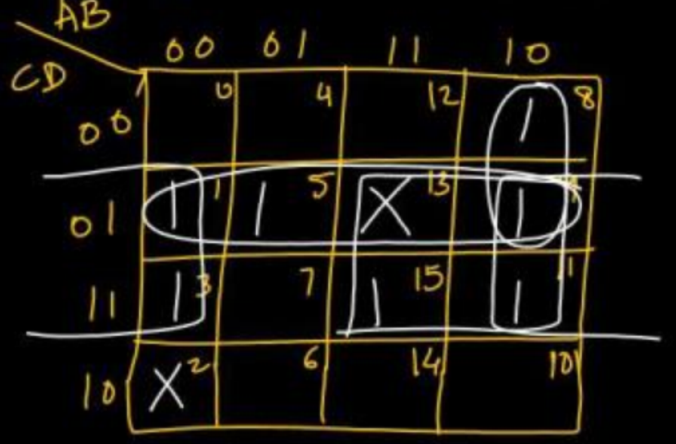
step 3:



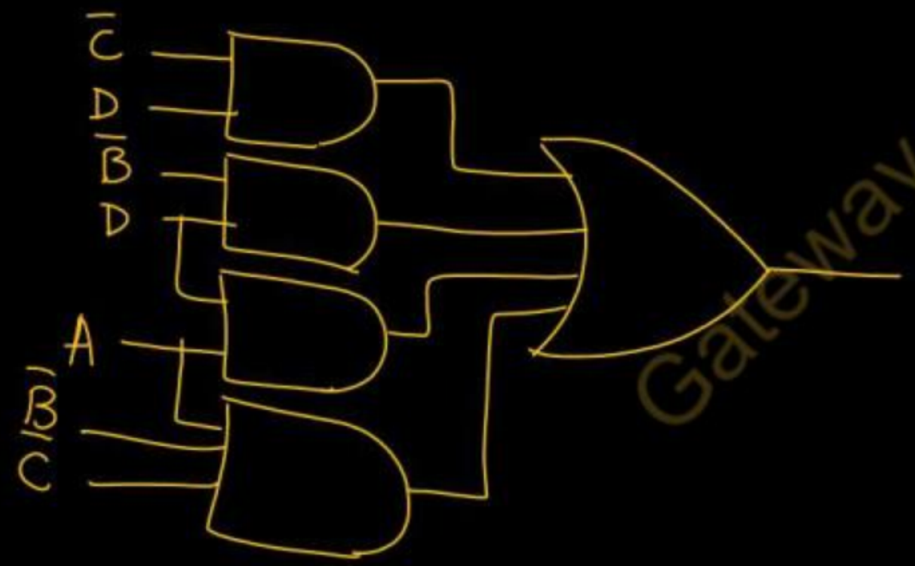
Gateway Classes: 7458961284

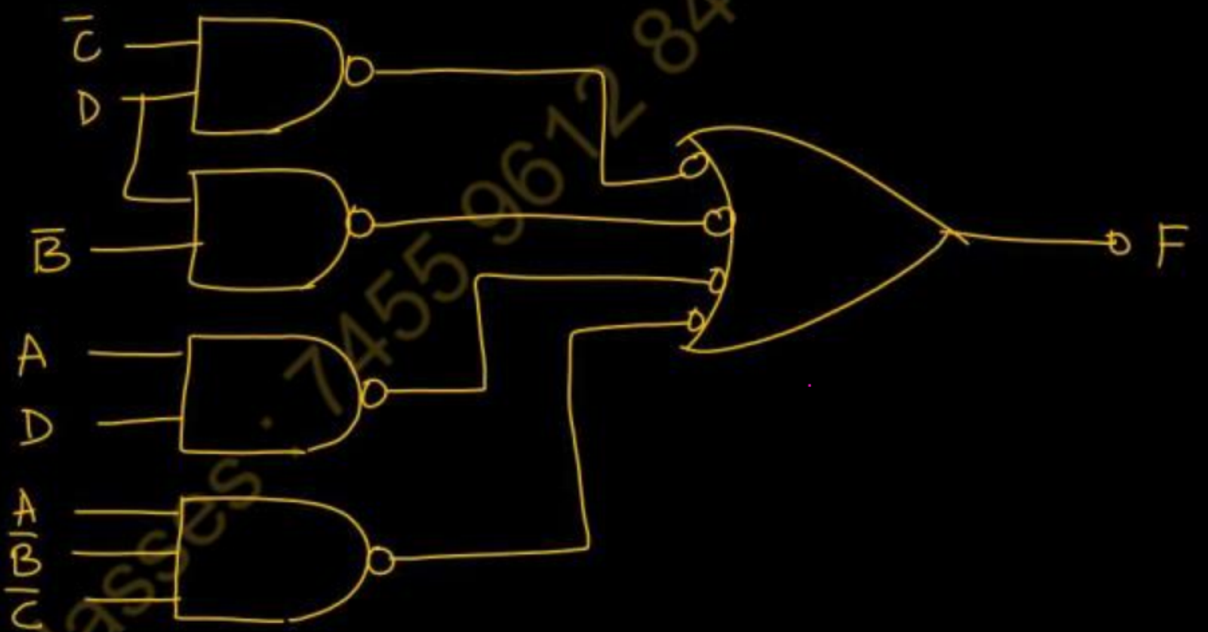
Example: $F = \Sigma m(1,3,5,8,9,11,15) + d(2,13)$

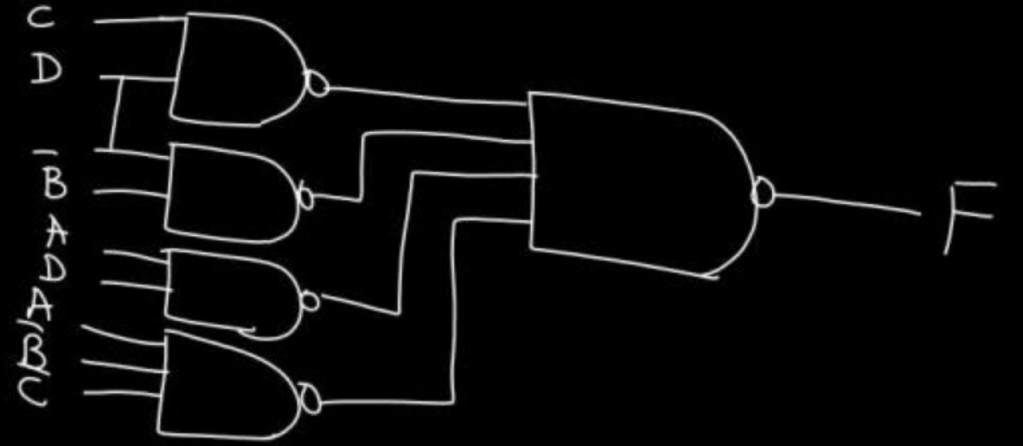
Implement using NAND gates only.



$$F = \bar{C}D + \bar{B}D + A\bar{D} + A\bar{B}\bar{C}$$







Gateway Classes · 7455 9612 84

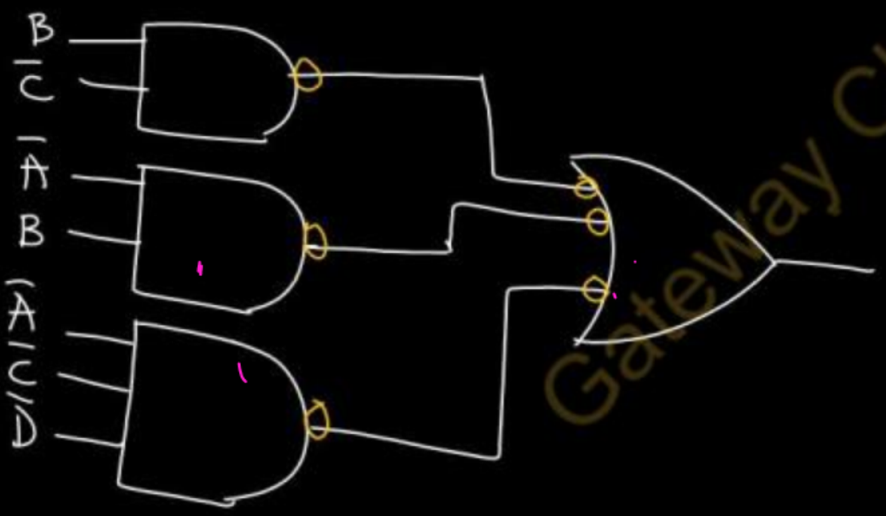
Example: f(A,B,C,D) = πM(1,2,3,8,9,10,11,14) . d(7,15)

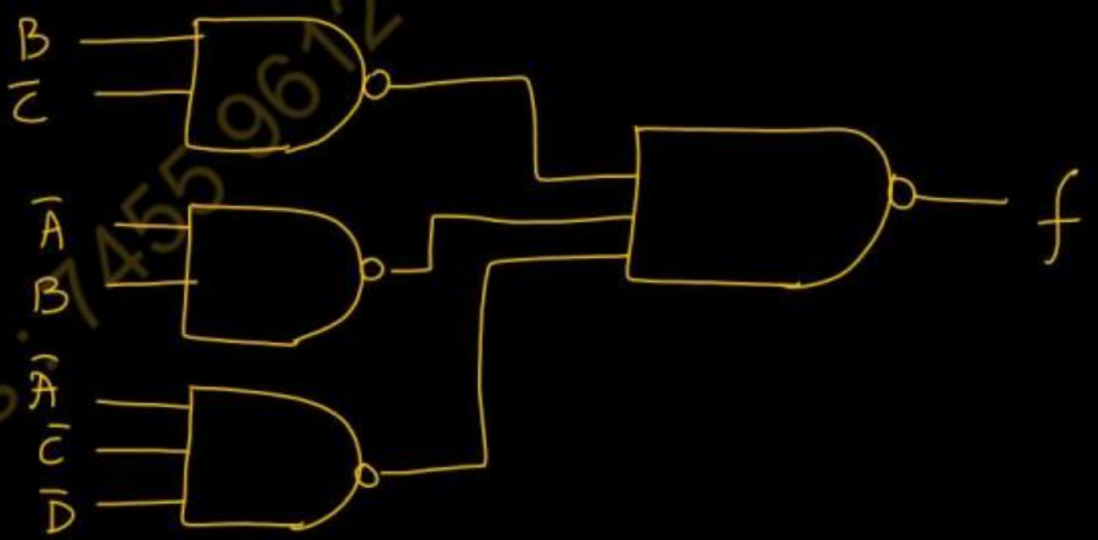
Implement using NAND gates only.

$f(A,B,C,D) = \Sigma m(0,4,5,6,12,13) + d(7,15)$



$f = B\bar{C} + \bar{A}B + \bar{A}\bar{C}\bar{D}$





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UNIVERSITY QUESTIONS

YEAR	QUESTION	MARKS	ANSWER
22-23	Give the general procedure for converting a multilevel AND-OR diagram into an all NAND diagram. Implement the following Boolean function with NAND gates only. $F(x, y, z) = \Sigma m(1,2,3,4,5,7)$	10 marks	F= z+x'y + xy'
17-18	Implement the Expression Y = ABC' + BD + E using Nand gate only.	2 marks	

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Thank You