Engineering Mathematics

UNIT : Partial Differential Equation

Today's Target

- Homogeneous Linear Partial Differential Equation with constant coefficient

  Particular Integral(PI) : Case-I (TYPE-1)

- PYQs
- DPP

Course Details(Paid) : All Subjects

1	Recorded Video Lectures (100 % Syllabus Coverage)
2	Pdf Notes
3	Lecture wise DPP with Answers
4	DPP Solutions
5	Unit wise set of PYQs with Answers
6	Course will be completed by 'Last week of Nov 2023'
	Tentative AKTU Semester Exam- Jan/Feb

Paid Courses are available in Gateway Classes Application
Link in Description

CASE-1: When roots are different

S. No.	Roots of A E	C. F.
1.	m_1, m_2	$f_1(y + m_1x) + f_2(y + m_2x)$
2.	m_1, m_2, m_3	$f_1(y + m_1x) + f_2(y + m_2x) + f_3(y + m_3x)$
3.	$m_1, m_2, m_3 \;\; m_4$	$f_1(y + m_1x) + f_2(y + m_2x) + f_3(y + m_3x) + f_4\,(y + m_4x)$

Note: Where f_1, f_2, f_3 ..... are arbitrary function

CASE-2: When roots are repeated

S. No.	Roots of A E	C. F.
1.	m, m	$f_1(y+mx)+xf_2(y+mx)$
2.	m, m, m	$f_1(y+mx)+xf_2(y+mx)+x^2f_3(y+mx)$

Note: Where f_1, f_2, f_3 ..... are arbitrary function

CASE-3: When D, D', both D and D' are common factor

(i) C F corresponding to $D'\ z = f_1(x)$

(ii) C F corresponding to $D'^{\,2}\ z = f_1(x) + yf_2(x)$

(iii) C F corresponding to $D'^{\,3}\ z = f_1(x) + yf_2(x) + y^2 f_3(x)$

(ii) C F corresponding to $Dz = f_1(y)$

(iii) C F corresponding to $D^2\ z = f_1(y) + xf_2(y)$

(iv) C F corresponding to $D^3\ z = f_1(y) + xf_2(y) + x^2 f_3(y)$

Homogeneous Linear PDE with constant coefficient

(All partial Derivatives are of same order)

Particular Integral

Case – 1

$$F(D, D') = \phi(ax + by)$$

$$\text{PI} = \frac{1}{F(D, D')}\phi(ax + by)$$

Where $F(D, D')$ *is a Homogeneous function of degree n*

Replace

$$D = a\,, D' = b$$

TYPE-I

if $F(a, b) \neq 0$

put $ax + by = u,$

$$\text{PI} = \frac{1}{F(a,b)} \iint \phi(u)du\,du \;\ldots\ldots n \text{ times}$$

TYPE-2

If $F(a,b)=0$ (Next Lecture)

Type - 1	Type - 2
(i) When power of D $\geq$ power of D'	When power of D' $>$ power of D'
(ii) $\text{PI}=x\left[\frac{1\times\phi(ax+by)}{\frac{\partial}{\partial D}[F(D,D')]}\right]$	$\text{PI}=y\left[\frac{1\times\phi(ax+by)}{\frac{\partial}{\partial D'}[F(D,D')]}\right]$
Again Repeat if $\frac{\partial}{\partial D}[F(D,D')]=0$	Again Repeat if $\frac{\partial}{\partial D'}[F(D,D')]=0$

Important Formulae

① $2\sin A\cos B = \sin(A+B) + \sin(A-B)$

② $2\cos A\sin B = \sin(A+B) - \sin(A-B)$

③ $2\cos A\cos B = \cos(A+B) + \cos(A-B)$

④ $2\sin A\sin B = \cos(A-B) - \cos(A+B)$

Q.1. Solve the linear partial differential equation $\frac{\partial^3 u}{\partial x^3} - 3\frac{\partial^3 u}{\partial x^2 \partial y} + 4\frac{\partial^3 u}{\partial y^3} = e^{x+2y}$.

$$\frac{\partial^3 u}{\partial x^3} - 3\frac{\partial^3 u}{\partial x^2 \partial y} + 4\frac{\partial^3 u}{\partial y^3} = e^{x+2y}$$

$$(D^3 - 3D^2D' + 4D'^3)u = e^{x+2y}$$

Put $D = m$, $D' = 1$

$$(m^3 - 3m^2 + 4)u = e^{x+2y}$$

Aux. Eqn

$$m^3 - 3m^2 + 4 = 0$$

$$m = -1, 2, 2$$

$$CF = f_1(y-x) + f_2(y+2x) + x f_3(y+2x)$$

$$PI = \frac{1}{D^3 - 3D^2D' + 4D'^3} e^{x+2y}$$

Put $D = 1$, $D' = 2$, $x + 2y = u$

$$PI = \frac{1}{1 - 3\times1\times2 + 4\times8} \iiint e^u \, du\,du\,du$$

$$PI = \frac{1}{27} e^{u}$$

$$\boxed{PI = \frac{1}{27} e^{x+2y}}$$

complete solution

$$u = CF + PI$$

$$\boxed{u = f_1(y-x) + f_2(y+2x) + x f_3(y+2x) + \frac{1}{27} e^{x+2y}}$$

Q.2 Solve the linear partial differential equation $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = \sin(2x+3y)$.

$$\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = \sin(2x+3y)$$

$$(D^2 - 2DD' + D'^2)z = \sin(2x+3y)$$

Put $D = m$, $D' = 1$

$$(m^2 - 2m + 1)z = \sin(2x+3y)$$

Aux. Eqn

$$m^2 - 2m + 1 = 0$$

$$(m-1)^2 = 0$$

$$m = 1, 1$$

$$CF = f_1(y+x) + x f_2(y+x)$$

$$PI = \frac{1}{(D^2 - 2DD' + D'^2)} \sin(2x+3y)$$

Put $D = 2$, $D' = 3$, $2x+3y = u$

$$PI = \frac{1}{4 - 12 + 9} \iint \sin u \, du \, du$$

$$PI = \int\int \sin u \, du \, du$$

$$PI = -\int \cos u \, du$$

$$PI = -\sin u$$

$$PI = -\sin(2x+3y)$$

complete solution

$$z = CF + PI$$

$$\boxed{z = f_1(y+x) + x f_2(y+x) - \sin(2x+3y)}$$

Q.3 Solve: $r + s - 2t = \sqrt{2x + y}$. **(UPTU-2015)**

$$r + s - 2t = \sqrt{2x+y}$$

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 2\frac{\partial^2 z}{\partial y^2} = (2x+y)^{1/2}$$

$$(D^2 + DD' - 2D'^2)z = (2x+y)^{1/2}$$

Put $D = m$, $D' = 1$

$$(m^2 + m - 2)z = (2x+y)^{1/2}$$

Aux Eqn

$$m^2 + m - 2 = 0$$

$$m = 1, -2$$

$$CF = f_1(y+x) + f_2(y-2x)$$

$$PI = \frac{1}{(D^2 + DD' - 2D'^2)}(2x+y)^{1/2}$$

Put $D = 2$, $D' = 1$, $2x+y = u$

$$PI = \frac{1}{(4+2-2)} \int\int u^{1/2}\, du\, du$$

$$PI = \frac{1}{4}\int\int u^{1/2}\,du\,du$$

$$PI = \frac{1}{4}\int \frac{u^{3/2}}{\frac{3}{2}}\,du$$

$$PI = \frac{1}{4} \times \frac{2}{3} \times \frac{u^{5/2}}{5}$$

$$PI = \frac{1}{4} \times \frac{2}{3} \times \frac{2}{5} \times u^{5/2}$$

$$\boxed{PI = \frac{1}{15}u^{5/2}}$$

complete solution

$$z = CF + PI$$

$$\boxed{z = f_1(y+x) + f_2(y-2x) + \frac{1}{15}(2x+y)^{5/2}}$$

Q.4 Solve the linear partial differential equation $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x \partial y} = \sin x \cos 2y$

(AKTU-2018)

$$\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x \partial y} = \sin x \cos 2y$$

$$(D^2 - 2DD')z = \sin x \cos 2y$$

$$D(D - 2D')z = \sin x \cos 2y$$

$Dz = 0$

$CF = f_1(y)$

$(D - 2D')z = 0$

Put $D = m$, $D' = 1$

$(m - 2)z = 0$

$m - 2 = 0$

$m = 2$

$CF = f_2(y + 2x)$

$$\boxed{CF = f_1(y) + f_2(y + 2x)}$$

$$PI = \frac{1}{(D^2 - 2DD')} \sin x \cos 2y$$

$$PI = \frac{1}{2(D^2 - 2DD')} 2 \sin x \cos 2y$$

$$PI = \frac{1}{2(D^2-2DD')}\left[\sin(x+2y)+\sin(x-2y)\right]$$

$$PI = \frac{1}{2(D^2-2DD')}\sin(x+2y)+\frac{1}{2(D^2-2DD')}\sin(x-2y)$$

$$PI = P_1 + P_2$$

$$P_1 = \frac{1}{2(D^2-2DD')}\sin(x+2y)$$

Put $D=1$, $D'=2$, $x+2y=u$

$$P_1 = \frac{1}{2(1-4)}\int\int \sin u \, du\, dy$$

$$P_1 = -\frac{1}{6}(-\sin u)$$

$$P_1 = \frac{1}{6}\sin(x+2y)$$

$$P_2 = \frac{1}{2(D^2-2DD')}\sin(x-2y)$$

Put $D=1$, $D'=-2$

$x-2y=u$

$$P_2 = \frac{1}{2(1+4)}\int\int \sin u \, du\, dy$$

$$P_2 = \frac{1}{10}(-\sin u)$$

$$P_2 = -\frac{1}{10}\sin(x-2y)$$

$$PI = \frac{1}{6}\sin(x+2y) - \frac{1}{10}\sin(x-2y)$$

Complete solution

$$z = CF + PI$$

$$z = f_1(y) + f_2(y+2x) + \frac{1}{6}\sin(x+2y) - \frac{1}{10}\sin(x-2y)$$

Q.5 Solve the linear partial differential equation $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \cos mx \cos ny + 30(2x + y)$.

(GBTU-2011, 2012)

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \cos mx \cos ny + 30(2x+y)$$

$$(D^2 + D'^2)z = \cos mx \cos ny + 30(2x+y)$$

Put $D = m$, $D' = 1$

$$(m^2+1)z = \cos mx \cos ny + 30(2x+y)$$

Aux. Eq^n

$$m^2 + 1 = 0$$

$$m = \pm i$$

$$CF = f_1(y+ix) + f_2(y-ix)$$

$$PI = \frac{1}{D^2+D'^2}\left[\cos mx \cos ny + 30(2x+y)\right]$$

$$PI = \frac{1}{(D^2+D'^2)}\left[\frac{2}{2}\cos mx \cos ny + 30(2x+y)\right]$$

$$PI = \frac{1}{(D^2+D'^2)}\left[\frac{1}{2}\{\cos(mx+ny) + \cos(mx-ny)\} + 30(2x+y)\right]$$

$$PI = \frac{1}{2(D^2+D'^2)}\cos(mx+ny) + \frac{1}{2(D^2+D'^2)}\cos(mx-ny) + \frac{1}{(D^2+D'^2)} \times 30(2x+y)$$

$$PI = P_1 + P_2 + P_3$$

$$P_1 = \frac{1}{2(D^2+D'^2)}\cos(mx+ny)$$

Put $D = m$, $D' = n$, $mx+ny = u$

$$P_1 = \frac{1}{2(m^2+n^2)}\int\int \cos u \, du \, du$$

$$P_1 = \frac{-1}{2(m^2+n^2)}\cos u$$

$$\boxed{P_1 = \frac{-1}{2(m^2+n^2)}\cos(mx+ny)}$$

$$P_2 = \frac{1}{2(D^2+D'^2)} \cos(mx-ny)$$

Put $D = m$, $D' = n$, $mx-ny = u$

$$P_2 = \frac{1}{2(m^2+n^2)} \iint \cos u \, du \, du$$

$$P_2 = \frac{-1}{2(m^2+n^2)} \cos u$$

$$\boxed{P_2 = \frac{-1}{2(m^2+n^2)} \cos(mx-ny)}$$

$$P_3 = \frac{1}{D^2+D'^2} 30(2x+y)$$

Put $D = 2$, $D' = 1$, $2x+y = u$

$$P_3 = \frac{1}{u+1} \times 30 \iint u \, du \, du$$

$$P_3 = \frac{1}{5} \times \overset{6}{\cancel{30}} \times \int \frac{u^2}{2} \, du$$

$$P_3 = \cancel{6} \times \frac{u^3}{\cancel{6}}$$

$$\boxed{P_3 = (2x+y)^3}$$

$$PI = P_1 + P_2 + P_3$$

$$PI = \frac{-1}{2(m^2+n^2)}\cos(mx+ny) - \frac{1}{2(m^2+n^2)}\cos(mx-ny) + (2x+y)^3$$

$$PI = \frac{-1}{2(m^2+n^2)}\left[\cos(mx+ny) + \cos(mx-ny)\right] + (2x+y)^3$$

$$PI = -\frac{1}{2(m^2+n^2)}\left[2\cos mx \cos ny\right] + (2x+y)^3$$

$$PI = \frac{-1}{m^2+n^2}(\cos mx \cos ny) + (2x+y)^3$$

complete solution

$$Z = CF + PI$$

Q.6 Solve the linear partial differential equation $(D^2 + 7DD' + 12D'^2)z = \sinh x$. (AKTU-2017)

$$(D^2+7DD'+12D'^2)z = \sinh x$$

Put $D = m, D' = 1$

$$(m^2+7m+12)z = \sinh(x+0y)$$

Aux. Eqn

$$m^2+7m+12 = 0$$

$$m = -3, -4$$

$$CF = f_1(y-3x) + f_2(y-4x)$$

$$PI = \frac{1}{(D^2+7DD'+12D'^2)} \sinh(x+0y)$$

Put $D = 1, D' = 0, x+0y = u$

$$PI = \frac{1}{(1+0+0)} \iint \sinh u \, du \, du$$

$$PI = \int \cosh u \, du$$

$$PI = \sinh u$$

$$PI = \sinh x$$

Complete solution

$$z = CF + PI$$

$$\boxed{z = f_1(y-3x) + f_2(y-4x) + \sinh x}$$

Particular Integral(PI) : Case-I

Q .1 *Solve the linear partial differential equation* $\frac{\partial^2 z}{\partial x^2} + 3\frac{\partial^2 z}{\partial x \partial y} + 2\frac{\partial^2 z}{\partial y^2} = x + y.$

$$z = f_1(y-x) + f_2(y-2x) + \frac{(x+y)^3}{36}$$

Q .2 *Solve :* $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = \sin x.$ *(MTU-2013)*

$$z = f_1(y+x) + x f_2(y+x) - \sin x$$

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