DPP - 7 Topic - Charpit Method

Q-1: Solve: $2zx - px^2 - 2qxy + pq = 0$

$2zx - px^2 - 2qxy + pq = 0$

Let $f(x, y, z, p, q) = 2zx - px^2 - 2qxy + pq = 0$ - (1)

$\frac{\delta f}{\delta x} = 2z - 2px - 2qy$, $\frac{\delta f}{\delta y} = -2qx$

$\frac{\delta f}{\delta z} = 2x$, $\frac{\delta f}{\delta p} = -x^2 + q$, $\frac{\delta f}{\delta q} = -2xy + q$

Charpit's Auxiliary Equation:-

$$\frac{dx}{-\frac{\delta f}{\delta p}} = \frac{dy}{-\frac{\delta f}{\delta q}} = \frac{dz}{-p\frac{\delta f}{\delta p} - q\frac{\delta f}{\delta q}} = \frac{dp}{\frac{\delta f}{\delta x} + p\frac{\delta f}{\delta z}} = \frac{dq}{\frac{\delta f}{\delta y} + q\frac{\delta f}{\delta z}}$$

$$\frac{dx}{x^2 - q} = \frac{dy}{2xy - q} = \frac{dz}{px^2 - 2pq + 2qxy} = \frac{dp}{2z - 2qy} = \frac{dq}{0}$$

$dq = 0$

$\boxed{q = a}$

Put $q = a$ in eq (1)

$2zx - px^2 - 2axy + pa = 0$

$p(a - x^2) = 2axy - 2zx$

$p = \frac{2x(ay - z)}{a - x^2}$

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$$\boxed{p = \frac{2x(z-ay)}{x^2-a}}$$

We know that, $dz = p\,dx + q\,dy$

$$dz = \frac{2x(z-ay)}{x^2-a}dx + a\,dy$$

$$dz - a\,dy = \frac{2x(z-ay)}{x^2-a}dx$$

$$\int \frac{dz - a\,dy}{z-ay} = \int \frac{2x}{x^2-a}dx$$

$$x^2 - a = t$$
$$2x\,dx = dt$$

$$\log(z-ay) = \int \frac{1}{t}dt$$

$$\log(z-ay) = \log t + \log b$$

$$\log(z-ay) = \log(x^2-a) + \log b$$

$$z - ay = (x^2-a)b$$

$$\boxed{z = ay + b(x^2-a)}$$

Ans

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Q-2 :- Solve the following equation by charpit's method
$z^2 = pqxy$

Sol :- $z^2 - pqxy = 0$

Let $f(x, y, z, p, q) = z^2 - pqxy = 0$ — (1)

$\frac{\delta f}{\delta x} = -pqy$, $\frac{\delta f}{\delta y} = -pqx$, $\frac{\delta f}{\delta z} = 2z$

$\frac{\delta f}{\delta p} = -qxy$, $\frac{\delta f}{\delta q} = -pxy$

Charpit's Auxiliary Equation :-

$$\frac{dx}{\frac{-\delta f}{\delta p}} = \frac{dy}{\frac{-\delta f}{\delta q}} = \frac{dz}{-p\frac{\delta f}{\delta p} - q\frac{\delta f}{\delta q}} = \frac{dp}{\frac{\delta f}{\delta x} + p\frac{\delta f}{\delta z}} = \frac{dq}{\frac{\delta f}{\delta y} + q\frac{\delta f}{\delta z}}$$

$$\frac{dx}{qxy} = \frac{dy}{pxy} = \frac{dz}{pqxy + pqxy} = \frac{dp}{-pqy + 2pz} = \frac{dq}{-pqx + 2qz}$$

No fraction pair formed, so taking x and p as a multipliers in first and fourth fraction. Similiary taking q and y as a multipliers in second and last fraction then equating :-

$$\frac{x\,dp + p\,dx}{-pqxy + 2pxz + pqxy} = \frac{y\,dq + q\,dy}{-pqxy + 2qyz + pqxy}$$

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$$\frac{x\,dp + p\,dx}{\cancel{p}x\cancel{z}} = \frac{y\,dq + q\,dy}{\cancel{q}y\cancel{z}}$$

$$\frac{x\,dp + p\,dx}{px} = \frac{y\,dq + q\,dy}{qy}$$

Now Integerate Both Side

$$\log xp = \log yq + \log c$$

$$\log xp - \log yq = \log c$$

$$\frac{xp}{yq} = c$$

$$\boxed{p = \frac{cyq}{x}} \quad -(2)$$

Put value of p in equation (1)

$$z^2 = pqxy$$

$$z^2 = cy^2q^2$$

$$q^2 = \frac{z}{cy^2}$$

$$q = \frac{z}{\sqrt{c}y}$$

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Again put value of q in eq ② -

$$p = \frac{cy}{x} \cdot \frac{z}{\sqrt{c}\,y}$$

$$p = \frac{z\sqrt{c}}{x}$$

Let $\sqrt{c} = b$

$$\therefore \boxed{p = \frac{zb}{x}, \qquad q = \frac{z}{by}}$$

We know that,

$$dz = p\,dx + q\,dy$$

$$dz = \frac{zb}{x}\,dx + \frac{z}{by}\,dy$$

$$\frac{dz}{z} = b\frac{dx}{x} + \frac{1}{b}\frac{dy}{y}$$

On Integration :-

$$\log z = b \log x + \frac{1}{b}\log y + \log a$$

$$\log z = \log x^b + \log y^{1/b} + \log a$$

$$\log z = \log\left(a x^b y^{1/b}\right)$$

$$\boxed{z = a x^b y^{1/b}}$$

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Q-3 : Solve : $z = p^2x + q^2y$

$$z - p^2x - q^2y = 0 \quad \text{— (1)}$$

Let $f(x, y, z, p, q) = z - p^2x - q^2 = 0$

$$\frac{\delta f}{\delta x} = -p^2 \;,\; \frac{\delta f}{\delta y} = -q^2 \;,\; \frac{\delta f}{\delta z} = 1$$

$$\frac{\delta f}{\delta p} = -2px \;,\; \frac{\delta f}{\delta q} = -2qy$$

Charpit Auxiliary Equation :

$$\frac{dx}{-\frac{\delta f}{\delta p}} = \frac{dy}{-\frac{\delta f}{\delta q}} = \frac{dz}{-p\frac{\delta f}{\delta p} - q\frac{\delta f}{\delta q}} = \frac{dp}{\frac{\delta f}{\delta x} + p\frac{\delta f}{\delta z}} = \frac{dq}{\frac{\delta f}{\delta y} + q\frac{\delta f}{\delta z}}$$

$$\frac{dx}{2px} = \frac{dy}{2qy} = \frac{dz}{2p^2x + 2q^2y} = \frac{dp}{-p^2 + p} = \frac{dq}{q - q^2}$$

Multiply by p^2 and $2px$ as multipliers in first and fourth fraction, Similiarly q^2 and $2qx$ in second and last fraction then equating them -

$$\frac{p^2dx + 2px\,dp}{2p^3x - 2p^3x + 2p^2x} = \frac{q^2dy + 2qy\,dq}{2q^3y + 2q^2y - 2q^3y}$$

$$\frac{p^2dx + 2px\,dp}{2p^2x} = \frac{q^2dy + 2qy\,dq}{2q^2y}$$

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$$\frac{p^2dx + 2px\,dp}{p^2x} = \frac{q^2dy + 2qy\,dq}{q^2y}$$

Now Integrate Both Side

$$\log(p^2x) = \log(q^2y) + \log a$$

$$\boxed{p^2x = aq^2y} \quad -(2)$$

Putting value of p^2x in eq (1)

$$aq^2y + q^2y = z$$

$$\boxed{q = \sqrt{\frac{z}{(1+a)y}}}$$

Now, again put value of q in eq (2)

$$p^2x = \frac{a\,z\,y}{(1+a)\,y}$$

$$p^2x = \frac{az}{1+a}$$

$$\boxed{p = \sqrt{\frac{az}{x(1+a)}}}$$

We know that, $dz = p\,dx + q\,dy$

$$dz = \left[\frac{az}{x(1+a)}\right]^{1/2} dx + \left[\frac{z}{(1+a)y}\right]^{1/2} dy$$

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$$\sqrt{1+a}\,\frac{dz}{z^{1/2}} = a^{1/2}\frac{dx}{x^{1/2}} + \frac{dy}{y^{1/2}}$$

Integerate both side

$$\sqrt{1+a}\,\frac{z^{1/2}}{1/2} = \sqrt{a}\,\frac{x^{1/2}}{1/2} + \frac{y^{1/2}}{1/2} + b$$

$$2\sqrt{(1+a)z} = 2\left[\sqrt{ax} + \sqrt{y} + b\right]$$

$$\boxed{\sqrt{(1+a)z} = \sqrt{ax} + \sqrt{y} + b}$$

Ans

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