Engineering Mathematics

UNIT : Partial Differential Equation

Today's Target

- Homogeneous Linear Partial Differential Equation with constant coefficient

  Particular Integral(PI) : Case-I(Type-2)

- PYQs
- DPP

Course Details(Paid) : All Subjects

1	Recorded Video Lectures (100 % Syllabus Coverage)
2	Pdf Notes
3	Lecture wise DPP with Answers
4	DPP Solutions
5	Unit wise set of PYQs with Answers
6	Course will be completed by 'Last week of Nov 2023'
	Tentative AKTU Semester Exam- Jan/Feb

Paid Courses are available in Gateway Classes Application
Link in Description

Homogeneous Linear PDE with constant coefficient
(All partial Derivatives are of same order)

Particular Integral

Case – 1

$$F(D, D')z = \phi(ax + by)$$

$$\text{PI} = \frac{1}{F(D, D')}\phi(ax + by)$$

Where $F(D, D')$ *is a Homogeneous function of degree n*

Replace

$$D = a\,,\ D' = b$$

TYPE-1

if $F(a, b) \neq 0$

put $ax + by = u,$

$$\text{PI} = \frac{1}{F(a,b)} \iiint \phi(u)du\,du \ \ldots\ldots n \text{ times}$$

Type - 2

If $F(a,b)=0$ case of failure

(i) When power of D $\geq$ power of D'	When power of D' $>$ power of D
(ii) $\mathbf{PI}=x\left[\frac{1\times\phi(ax+by)}{\frac{\partial}{\partial D}[F(D,D')]}\right]$	$\mathbf{PI}=y\left[\frac{1\times\phi(ax+by)}{\frac{\partial}{\partial D'}[F(D,D')]}\right]$
Again Repeat if $\frac{\partial}{\partial D}[F(D,D')]=0$	Again Repeat if $\frac{\partial}{\partial D'}[F(D,D')]=0$

Q.1 Solve the linear partial differential equation $(D - D')^2 z = \tan(y + x)$

$$(D-D')^2 z = \tan(y+x)$$

Put $D = m$, $D' = 1$

$$(m-1)^2 z = \tan(y+x)$$

Aux. Eqn

$$(m-1)^2 = 0$$

$$m = 1, 1$$

$$CF = f_1(y+x) + x f_2(y+x)$$

$$PI = \frac{1}{(D-D')^2} \tan(y+x) \quad \{\text{case of failure}\}$$

$$PI = x \times \frac{1}{2(D-D')} \tan(y+x) \quad \{\text{case of failure}\}$$

$$PI = x^2 \times \frac{1}{2 \times 1} \tan(y+x) = \frac{x^2}{2} \tan(y+x)$$

complete solution is

$$z = CF + PI = f_1(y+x) + x f_2(y+x) + \frac{x^2}{2} \tan(y+x)$$

Q.2 Solve $4r - 4s + t = 16\log(x + 2y)$

$$4r - 4s + t = 16\log(x+2y)$$

$$4\frac{\partial^2 z}{\partial x^2} - 4\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y} = 16\log(x+2y)$$

$$(4D^2 - 4DD' + D'^2)z = 16\log(x+2y)$$

$$(2D - D')^2 z = 16\log(x+2y)$$

Put $D = m$, $D' = 1$

$$(2m-1)^2 z = 16\log(x+2y)$$

Aux. Eqn

$$(2m-1)^2 = 0$$

$$m = \frac{1}{2}, \frac{1}{2}$$

$$CF = f_1\left(y + \frac{x}{2}\right) + x f_2\left(y + \frac{x}{2}\right)$$

$$PI = \frac{1}{(2D - D')^2} 16\log(x+2y)$$

{ case of failure }

$$PI = x \times \frac{1}{2(2D-D')\times 2} \times 16 \log(x+2y)$$

{Again case of failure}

$$PI = x^2 \times \frac{1}{4\times 2} \times \cancel{16}^{2} \log(x+2y)$$

$$\boxed{PI = 2x^2 \log(x+2y)}$$

Complete solution is

$$z = CF + PI$$

$$\boxed{z = f_1(y+\tfrac{x}{2}) + x f_2(y+\tfrac{x}{2}) + 2x^2 \log(x+2y)}$$

Q.3 Solve the differential equation : $\frac{\partial^3 z}{\partial x^2 \partial y} - 2\frac{\partial^3 z}{\partial x \partial y^2} + \frac{\partial^3 z}{\partial y^3} = \frac{1}{x^2}$.

$$\frac{\partial^3 z}{\partial x^2 \partial y} - 2\frac{\partial^3 z}{\partial x \partial y^2} + \frac{\partial^3 z}{\partial y^3} = \frac{1}{x^2}$$

$$(D^2D' - 2DD'^2 + D'^3)z = \frac{1}{x^2}$$

$$D'(D^2 - 2DD' + D'^2)z = \frac{1}{x^2}$$

Aux. Eqn

$D' = 0$

$CF = f_1(x)$

$D^2 - 2DD' + D'^2 = 0$

$m^2 - 2m + 1 = 0$

$(m-1)^2 = 0$

$m = 1, 1$

$CF = f_2(y+x) + x f_3(y+x)$

$$CF = f_1(x) + f_2(y+x) + x f_3(y+x)$$

$$PI = \frac{1}{(D^2D' - 2DD'^2 + D'^3)} \times \frac{1}{(x+oy)^2}$$

{ case of failure }

$$PI = y \times \frac{1}{(D^2 - 4DD' + 3D'^2)} \times \frac{1}{(x+0y)^2}$$

Put $D = 1,\ D' = 0,\ (x+0y) = u$

$$PI = y \times \frac{1}{1} \times \iint \frac{1}{u^2}\, du\, du$$

$$PI = y \times \int -\frac{1}{u}\, du$$

$$PI = -y \log u$$

$$\boxed{PI = -y \log x}$$

Complete solution is

$$z = CF + PI$$

$$\boxed{z = f_1(x) + f_2(y+x) + x f_3(y+x) - y \log x}$$

Q.4 Solve $(D^3 - 4D^2D' + 5DD'^2 - 2D'^3)z = e^{y+2x} + (y+x)^{1/2}$ **(AKTU-2017)**

$$(D^3 - 4D^2D' + 5DD'^2 - 2D'^3)z = e^{y+2x} + (y+x)^{1/2}$$

Put $D = m$, $D' = 1$

$$(m^3 - 4m^2 + 5m - 2)z = e^{y+2x} + (y+x)^{1/2}$$

Aux. Eqn

$$m^3 - 4m^2 + 5m - 2 = 0$$

$$m = 1, 1, 2$$

$$CF = f_1(y+x) + x f_2(y+x) + f_3(y+2x)$$

$$PI = \frac{1}{D^3 - 4D^2D' + 5DD'^2 - 2D'^3}\left[e^{y+2x} + (y+x)^{1/2}\right]$$

$$PI = \underbrace{\frac{1}{(D^3 - 4D^2D' + 5DD'^2 - 2D'^3)} e^{y+2x}}_{P_1} + \underbrace{\frac{1}{(D^3 - 4D^2D' + 5DD'^2 - 2D'^3)} (y+x)^{1/2}}_{P_2}$$

$$PI = P_1 + P_2$$

$$P_1 = \frac{1}{(D^3 - 4D^2D' + 5DD'^2 - 2D'^3)} e^{y+2x}$$

Put $D = 2, \quad D' = 1$

$$P_1 = \frac{1}{(8 - 16 + 10 - 2)} e^{y+2x}$$

{case of failure}

$$P_1 = x \times \frac{1}{(3D^2 - 8DD' + 5D'^2)} e^{y+2x}$$

Put $D = 2, \; D'_1 = 1, \; y + 2x = u$

$$P_1 = \frac{x}{12 - 16 + 5} \int\int e^u \, du \, du$$

$$P_1 = x \, e^u$$

$$\boxed{P_1 = x \, e^{y+2x}}$$

$$P_2 = \frac{1}{D^3 - 4D^2D' + 5DD'^2 - 2D'^3)} (y+x)^{1/2}$$

{case of failure}

$$P_2 = x \times \frac{1}{(3D^2 - 8DD' + 5D'^2)} (y+x)^{1/2}$$

{ case of failur }

$$P_2 = x^2 \times \frac{1}{(6D - 8D')} (y + x)^{1/2}$$

Put $D = 1,\ D' = 1,\ (y+x) = u$

$$P_2 = x^2 \times \frac{1}{(6-8)} \int u^{1/2}\, du$$

$$P_2 = \frac{x^2}{-2} \times \frac{u^{3/2}}{\frac{3}{2}}$$

$$P_2 = -\frac{x^3}{3} u^{3/2}$$

$$\boxed{P_2 = -\frac{x^3}{3} (y+x)^{3/2}}$$

$$PI = x e^{y+2x} - \frac{x^3}{3} (y+x)^{3/2}$$

complete solution is

$$Z = CF + PI$$

$$\boxed{Z = f_1(y+x) + x f_2(y+x) + f_3(y+2x) + x e^{y+2x} - \frac{x^3}{3} (y+x)^{3/2}}$$

Q.5 Solve the linear partial differential equation : $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} = \sin x \cos y$

$$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} = \sin x \cos y$$

$$(D^2 - DD')z = \sin x \cos y$$

$$D(D - D')z = \sin x \cos y$$

Aux. Eqn

$D = 0$

$CF = f_1(y)$

$D - D' = 0$

$m - 1 = 0$

$m = 1$

$CF = f_2(y + x)$

$$PI = \frac{1}{(D^2 - DD')} \sin x \cos y$$

$$PI = \frac{1}{2(D^2 - DD')} (2 \sin x \cos y)$$

$$PI = \frac{1}{2(D^2 - DD')} \{ \sin(x+y) + \sin(x-y) \}$$

$$PI = \frac{1}{2(D^2 - DD')} \sin(x+y) + \frac{1}{2(D^2 - DD')} \sin(x-y)$$

$$PI = P_1 + P_2$$

$$P_1 = \frac{1}{2(D^2 - DD')} \sin(x+y)$$

{case of failur}

$$P_1 = x \times \frac{1}{2(2D - D)} \sin(x+y)$$

Put $D = 1,\ D' = 1,\ x+y = u$

$$P_1 = x \times \frac{1}{2(2-1)} \int \sin u \, du$$

$$P_1 = \frac{x}{2}(-\cos u)$$

$$\boxed{P_1 = \frac{-x}{2}\cos(x+y)}$$

$$P_2 = \frac{1}{2(D^2 - DD')} \sin(x-y)$$

Put $D = 1,\ D' = -1,\ x-y = u$

$$P_2 = \frac{1}{2(1+1)} \int\int \sin u \, du \, du$$

$$P_2 = \frac{1}{4}(-\sin u)$$

$$P_2 = -\frac{1}{4}\sin(x-y)$$

$$PI = -\frac{x}{2}\cos(x+y) - \frac{1}{4}\sin(x-y)$$

complete solution

$$z = CF + PI$$

$$\boxed{z = f_1(y) + f_2(y+x) - \frac{x}{2}\cos(x+y) - \frac{1}{4}\sin(x-y)}$$

PDE : DPP-10

Topic : Homogeneous Linear Partial Differential Equation with constant coefficient

Particular Integral(PI) : Case-I (Type-2)

Q .1 *Solve the linear partial differential equation* $\frac{\partial^3 z}{\partial x^3} - 4\frac{\partial^3 z}{\partial x^2 \partial y} + 4\frac{\partial^3 z}{\partial x \partial y^2} = 4\sin(2x+y)$.

Ans $f_1(y) + f_2(y+2x) + x f_3(y+2x) - x^2 \cos(2x+y)$

Q .2 *Solve :* $2r - s - 3t = 5\frac{e^x}{e^y}$

Ans $z = f_1(y-x) + f_2\left(y+\frac{3}{2}x\right) + x e^{x-y}$

Q .3 *Solve :* $\frac{\partial^3 z}{\partial x^3} - 7\frac{\partial^3 z}{\partial x \partial y^2} - 6\frac{\partial^3 z}{\partial y^3} = \sin(x+2y) + e^{3x+y}$. *(UPTU-2014)*

Ans $z = f_1(y-x) + f_2(y-2x) + f_3(y+3x) - \frac{1}{75}\cos(x+2y) + \frac{x}{20}e^{3x+y}$

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Thank You

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