DPP-7 Maths - IV Unit-2

Topic - One Dimensional Heat Equation

Q-1 : The temperature distribution in a bar of length π which is perfectly insulated at ends $x=0$ and $x=\pi$ is governed by partial differential equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

Assuming the initial temperature distribution as $u(x,0) = f(x) = \cos 2x$, find the temperature distribution at any instant of time.

Sol :- Length of rod $(l) = \pi$

Boundry conditions

At $x = 0$, $\frac{\partial u}{\partial x} = 0$ BC-1

At $x = \pi$, $\frac{\partial u}{\partial x} = 0$ BC-2

Initial condition :

At $t = 0$, $u = \cos 2x$

$$u(x,t) = u_s(x) + u_t(x,t)$$

$$u_s(x) = a + bx$$

$$\frac{\partial u}{\partial x} = b$$

Download Our App from Google PlayStore
Helpline Number : 7455961284
PDF Notes PYQs Solution Recorded Lectures

GATEWAY CLASSES
Empowering learners, transforming futures

Apply BC-1

$b = 0$

$\boxed{u_s = a}$

$u(x,t) = a + u_t(x,t)$

Most suitable solution :

$$u(x,t) = a + (C_1 \cos px + C_2 \sin px)\, C_3 e^{-p^2c^2t}$$

$$\frac{\delta u}{\delta x} = 0 + (-C_1 p \sin px + p C_2 \cos px)\, C_3 e^{-c^2p^2t}$$

Apply BC-1

$$0 = C_2 p \times C_3 e^{-c^2p^2t}$$

$\boxed{C_2 = 0}$

Updated solution :

$$u(x,t) = a + (C_1 \cos px)\, C_3 e^{-c^2p^2t}$$

$$\frac{\delta u}{\delta x} = 0 + (-C_1 p \sin px)\, C_3 e^{-c^2p^2t}$$

Apply BC-2;

$$0 = -C_1 p \sin p\pi \times C_3 e^{-p^2c^2t}$$

$\sin p\pi = 0$

$p\pi = n\pi$

Download Our App from Google PlayStore
Helpline Number : 7455961284
PDF Notes
PYQs Solution
Recorded Lectures

GATEWAY CLASSES
Empowering learners, transforming futures

$\boxed{p = n}$

Updated solution $\Rightarrow$ $u = a + (C_1 \cos nx)\, C_3\, e^{-c^2 n^2 t}$

Most general solution :-

$$u = a + \sum_{n=1}^{\infty} b_n \cos nx \times e^{-c^2 n^2 t} \quad - (11)$$

Apply IC

$$\cos 2x = a + b_1 \cos x + b_2 \cos 2x + \cdots\cdots$$

On comparison $\Rightarrow$ $a = 0$, $b_2 = 1$, $b_1 = 0$

From equation (11)

$$u(x,t) = a + \sum_{n=1}^{\infty} b_n \cos nx \times e^{-c^2 n^2 t}$$

$$u(x,t) = a + b_1 \cos x \times e^{-c^2 t} + b_2 \cos 2x \times e^{-c^2 4t} + \cdots\cdots$$

$$u(x,t) = 0 + 0 + \cos 2x \times e^{-4c^2 t}$$

$$u(x,t) = \cos 2x \times e^{-4c^2 t}$$

$$u(x,t) = e^{-4c^2 t} \times \cos 2x$$

* From given equation in question $\Rightarrow$ $c^2 = 1$

$$\boxed{u(x,t) = e^{-4t} \times \cos 2x}$$

Ans

Download Our App from Google PlayStore
Helpline Number : 7455961284
PDF Notes
PYQs Solution
Recorded Lectures