ENGINEERING MATHEMATICS

UNIT-3 : *Statistical Techniques – I*

- *Overview of Measures of central tendency*
- *Moments*
- *Skewness*
- *Kurtosis*
- *Curve Fitting*
  - *(i) Method of least square*
  - *(ii) Fitting of straight lines*
  - *(iii) Fitting of second degree parabola*
  - *(iv) Exponential curve*
- *Correlation and Rank correlation*
- *Regression Analysis : Regression lines of y on x and x on y.*

Gateway Classes : 7455 9612 84

ENGINEERING MATHEMATICS

UNIT-3 : *Statistical Techniques - I*

Today's Target

- ***Overview of Measures of central tendency***
- PYQ
- DPP

Gateway Classes : 7455 9612 84

Measure of central Tendency

(An average is a measure of central tendency of data)

There are five types of averages in common use

1. Arithmetic Mean OR Mean
2. Median
3. Mode
4. Geometric Mean
5. Harmonic Mean

$$3 \text{ (Median)} = \text{Mode} + 2 \text{ (Mean)}$$

Gateway Classes : 7455 961284

Data

Ungrouped Data

(i) Individual Series

(ii) Discrete Series

Grouped Data

(i) Continuous Series

(ii) Discontinuous series

MEAN

Methods to find Mean

(i) Direct Method

(ii) Shortcut Method or Assumed mean method

(iii) Step Deviation method

For Individual series (Where frequency are not given)

Let $x_1, x_2, x_3, \ldots\ldots\ x_n$ are n observation

Direct Method

$$\text{Mean} = \frac{\text{Sum of Observations}}{\text{Number of Observations}}$$

$$\bar{x} = \frac{x_1 + x_2 + x_3 + \ldots\ldots + x_n}{n}$$

$$\bar{x} = \frac{\Sigma x}{n}$$

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

Gateway Classes : 7455 9612 84

Q.1: *The weights (in Kilogram) of 5 persons are 67, 65, 71, 57 and 45. Find the mean of their weights.*

$x_1 = 67$

$x_2 = 65$

$x_3 = 71$

$x_4 = 57$

$x_5 = 45$

$$\bar{x} = \frac{\sum x}{n}$$

$$\bar{x} = \frac{67 + 65 + 71 + 57 + 45}{5}$$

$$\bar{x} = 61 \text{ kg}$$

Gateway Classes : 7455 9612 84

For Discrete Series (Where frequency is given)

Let $x_1, x_2, x_3, \ldots\ldots x_n$ are n observation with respective frequencies $f_1, f_2, f_3, \ldots\ldots f_n$.

Method used to find Mean

(i) Direct Method

$$\bar{x} = \frac{f_1 x_1 + f_2 x_2 + \ldots\ldots + f_n x_n}{f_1 + f_2 + f_3, + \ldots\ldots + f_n}$$

$$\bar{x} = \frac{\Sigma fx}{\Sigma f}$$

Also $\Sigma f = N$

$$\bar{x} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum f_i}$$

Gateway Classes : 7455 9612 84

(ii) Shortcut Method or Assumed mean method

$$\bar{x} = A + \frac{\Sigma fd}{\Sigma f}$$

Where A = Assume Mean

Deviation $(d) = x - A$

Also $\Sigma f = N$

(iii) Step Deviation method

$$\bar{x} = A + h \times \frac{\Sigma fu}{\Sigma f}$$

Where A = Assume mean

h = common factor

$$u = \frac{x - A}{h}$$

Also $\Sigma f = N$

Gateway Classes : 7455 961284

Q.2. *Find the mean of the following frequency distribution of marks in a test in mathematics.*

Marks	(x)	10	20	30	40	50	60	70	80
No. of students	(f)	3	6	10	12	9	6	2	2

MARKS(x)	Number of students (f)	fx
10	3	30
20	6	120
30	10	300
40	12	480
50	9	450
60	6	360
70	2	140
80	2	160
	$\Sigma f = 50$	$\Sigma fx = 2040$

By Direct Method

$$\bar{x} = \frac{\Sigma fx}{\Sigma f}$$

$$\bar{x} = \frac{2040}{50}$$

$$\boxed{\bar{x} = 40.8}$$

Gateway Classes: 7455 961284

MARKS (x)	Number of students (f)	$d = x - A$	fd
10	3	−30	−90
20	6	−20	−120
30	10	−10	−100
A → 40	12	0	0
50	9	10	90
60	6	20	120
70	2	30	60
80	2	40	80
	$\Sigma f = 50$		$\Sigma fd = 40$

By Assume Mean Method

$$\bar{x} = A + \frac{\Sigma fd}{\Sigma f}$$

$$\bar{x} = 40 + \frac{40}{50}$$

$$\bar{x} = 40 + 0.8$$

$$\boxed{\bar{x} = 40.8}$$

Gateway Classes 7455 961284

x	f	$x - A$	$u = \frac{x-A}{h}$	fu
10	3	−30	−3	−9
20	6	−20	−2	−12
30	10	−10	−1	−10
40 (A)	12	0	0	0
50	9	10	1	9
60	6	20	2	12
70	2	30	3	6
80	2	40	4	8
	$\Sigma f = 50$			$\Sigma fu = 4$

By Step Deviation Method

$$\bar{x} = A + h \times \frac{\Sigma fu}{\Sigma f}$$

$$\bar{x} = 40 + 10 \times \frac{4}{50}$$

$$\bar{x} = 40 + 0.8$$

$$\boxed{\bar{x} = 40.8}$$

Gateway Classes: 7455 961284

For Continuous series (Where frequency and class intervals are given)

Let $x_1, x_2, x_3, \ldots\ldots x_n$ are n observation with respective frequencies $f_1, f_2, f_3, \ldots\ldots f_n$.

Note :- $$x = \frac{\text{Lower limit of class interval} + \text{Upper limit of class interval}}{2}$$

x ↑ Mid value

Method used to find Mean

(i) Direct Method

$$\bar{x} = \frac{f_1 x_1 + f_2 x_2 + \ldots\ldots + f_n x_n}{f_1 + f_2 + f_3, + \ldots\ldots + f_n}$$

$$\bar{x} = \frac{\Sigma f x}{\Sigma f}$$

Also $\Sigma f = N$

Gateway Classes : 7455 961284

(i) Shortcut Method or Assumed mean method

$$\bar{x} = A + \frac{\Sigma fd}{\Sigma f}$$

Where A = Assume Mean

Also $\Sigma f = N$

(Deviation) $d = x - A$

(iii) Step Deviation method

$$\bar{x} = A + h \times \frac{\Sigma fu}{\Sigma f}$$

Where h = Width of class interval

Also $\Sigma f = N$

$$u = \frac{x - A}{h}$$

Gateway Classes : 7455 961284

Q.3. *Find the mean of the following frequency distribution*

Class	5-15	15-25	25-35	35-45	45-55	55-65
frequency	10	20	25	20	15	10

CLASS	Frequency (f)	Mid value (x)	fx
5-15	10	10	100
15-25	20	20	400
25-35	25	30	750
35-45	20	40	800
45-55	15	50	750
55-65	10	60	600
	$\Sigma f = 100$		$\Sigma fx = 3400$

By Direct Method

$$\bar{x} = \frac{\Sigma fx}{\Sigma f}$$

$$\bar{x} = \frac{3400}{100}$$

$$\bar{x} = 34$$

Gateway Classes : 7455 961284

CLASS	Frequency (f)	Mid value (x)	$d = x - A$	fd
5-15	10	10	-20	-200
15-25	20	20	-10	-200
25-35	25	30 (A)	0	0
35-45	20	40	10	200
45-55	15	50	20	300
55-65	10	60	30	300
	$\Sigma f = 100$			$\Sigma fd = 400$

By Assume Mean Method

$$\bar{x} = A + \frac{\Sigma fd}{\Sigma f}$$

$$\bar{x} = 30 + \frac{400}{100}$$

$$\bar{x} = 34$$

Gateway Classes : 7455 9612 84

CLASS	Frequency (f)	Mid value (x)	$u = \frac{x-A}{h}$	fu
5-15	10	10	-2	-20
15-25	20	20	-1	-20
25-35	25	30 (A)	0	0
35-45	20	40	1	20
45-55	15	50	2	30
55-65	10	60	3	30
	$\Sigma f = 100$			$\Sigma fu = 40$

By Step-Deviation Method

$$\bar{x} = A + h \times \frac{\Sigma fu}{\Sigma f}$$

$$\bar{x} = 30 + 10 \times \frac{40}{100}$$

$$\boxed{\bar{x} = 34}$$

Gateway Classes : 7455 961284

DPP- 1

Topic : *Overview of Measures of central tendency*

Q.1. Find the mean for the following individual series

x	3	6	8	10	18

Q. 2. *Find the mean of the following frequency distribution*

x	0	1	2	3	4	5	6	7
f	1	8	28	56	70	56	28	8

Q.3. *Find the mean of the following frequency distribution*

Marks	0-10	10-20	20-30	30-40	40-50
No. of Students	5	10	40	20	25

Gateway Classes : 7455 9612 84

ENGINEERING MATHEMATICS

UNIT-3 : *Statistical Techniques – I*

Lec-2

Today's Target

- ***Moments***
- PYQ
- DPP

Gateway Classes : 7455 9612 84

Moments

Moment of distribution are arithematic mean of various powers of deviation from some given numbers.



(i) For frequency distribution

(ii) For individual series

(i) For frequency distribution

(ii) For individual series

(i) For frequency distribution

(ii) For individual series

Gateway Classes: 7455 9612 84

Moment about mean (μ_r)

For frequency distribution

If $x_1, x_2, x_3 \ldots\ldots x_n$ are n observations with respective frequency $f_1, f_2, f_3 \ldots\ldots f_n$ Respectively.

Then the r^{th} moment μ_r about mean $\bar{x}$ is defined as

$$\mu_r = \frac{\sum f(x-\bar{x})^r}{\sum f}$$

Moment about any arbitrary number (μ'_r)

For frequency distribution

If $x_1, x_2, x_3 \ldots\ldots x_n$ are n observations with respective frequency $f_1, f_2, f_3 \ldots\ldots f_n$ Respectively.

Then the r^{th} moment μ'_r about any arbitrary number (A) is defined as

$$\mu'_r = \frac{\sum f(x-A)^r}{\sum f}$$

Moment about origin (ϑ_r)

For frequency distribution

If $x_1, x_2, x_3 \ldots\ldots x_n$ are n observations with respective frequency $f_1, f_2, f_3 \ldots\ldots f_n$ Respectively.

Then the r^{th} moment ϑ_r about origin (0,0) is defined as

$$\vartheta_r = \frac{\sum f x^r}{\sum f}$$

Gateway Classes 7455 96 9 84

Moment about mean (μ_r)

Where $r = 0, 1, 2, 3, 4 \ldots$

(i) $\mu_0 = 1$

(ii) $\mu_1 = 0$

(iii) $\mu_2 = \dfrac{\Sigma f(x-\bar{x})^2}{\Sigma f}$

Moment about any arbitrary number (μ'_r)

Where $r = 0, 1, 2, 3, 4 \ldots$

$\mu'_0 = 1$

$\mu'_1 = \bar{x} - A$

$\mu'_2 = \dfrac{\Sigma f(x-A)^2}{\Sigma f}$

Moment about origin (ϑ_r)

Where $r = 0, 1, 2, 3, 4 \ldots$

$\vartheta_0 = 1$

$\vartheta_1 = \bar{x}$

$\vartheta_2 = \dfrac{\Sigma f x^2}{\Sigma f}$

Gateway Classes 7455 9612 84

Moment about mean (μ_r)	Moment about any arbitrary number (μ'_r)	Moment about origin (ϑ_r)
(IV) $\mu_3 = \frac{\sum f(x-\bar{x})^3}{\sum f}$	$\mu'_3 = \frac{\sum f(x-A)^3}{\sum f}$	$\vartheta_3 = \frac{\sum f x^3}{\sum f}$
(V) $\mu_4 = \frac{\sum f(x-\bar{x})^4}{\sum f}$	$\mu'_4 = \frac{\sum f(x-A)^4}{\sum f}$	$\vartheta_4 = \frac{\sum f x^4}{\sum f}$

Gateway Classes : 7455 9612 84

Moment about mean (μ_r)

For individual series

If $x_1, x_2, x_3 \ldots\ldots x_n$ are n observations

Then the r^{th} moment μ_r about mean $\bar{x}$ is defined as

Remove $\longrightarrow$ f

$\Sigma f = n$(number of observation)

Moment about any arbitrary number (μ'_r)

For individual series

If $x_1, x_2, x_3 \ldots\ldots x_n$ are n observations

Then the r^{th} moment μ'_r about any Arbitrary number(A) is defined as

Remove $\longrightarrow$ f

$\Sigma f = n$

Moment about origin (ϑ_r)

For individual series

If $x_1, x_2, x_3 \ldots\ldots x_n$ are n observations

Then the r^{th} moment ϑ_r about origin(0,0) is defined as

Remove $\longrightarrow$ f

$\Sigma f = n$

Gateway Classes : 7455 06 12 84

Q.1 Find the first four moments for the following individual series :

x	3	6	8	10	18

Solution

S.NO	x	$(x-\bar{x})$	$(x-\bar{x})^2$	$(x-\bar{x})^3$	$(x-\bar{x})^4$
1	3	-6	36	-216	1296
2	6	-3	9	-27	81
3	8	-1	1	-1	1
4	10	1	1	1	1
5	18	9	81	729	6561
$n=5$	$\Sigma x=45$	$\Sigma(x-\bar{x})=0$	$\Sigma(x-\bar{x})^2=128$	$\Sigma(x-\bar{x})^3=486$	$\Sigma(x-\bar{x})^4=7940$

Gateway Classes : 7455 961 284

Mean ($\bar{x}$)

$$\bar{x} = \frac{\Sigma x}{n}$$

$$\bar{x} = \frac{45}{5}$$

$$\boxed{\bar{x} = 9}$$

Moments

$$\boxed{\mu_1 = 0}$$

$$\mu_2 = \frac{\Sigma (x-\bar{x})^2}{n}$$

$$\mu_2 = \frac{128}{5}$$

$$\boxed{\mu_2 = 25.6}$$

$$\mu_3 = \frac{\Sigma (x-\bar{x})^3}{n} = \frac{486}{5}$$

$$\boxed{\mu_3 = 97.2}$$

$$\mu_4 = \frac{\Sigma (x-\bar{x})^4}{n} = \frac{7940}{5}$$

$$\boxed{\mu_4 = 1588}$$

Gateway Classes : 7455 96 12 84

Q.2 The number of flowers on Sun-flower plants are given below: (AKTU)

No. of flowers (x)	3	6	12	16	25
No. of plants (f)	1	2	3	4	5

Calculate the first four moments about the mean

x	f	fx	$(x-\bar{x})$	$f(x-\bar{x})$	$f(x-\bar{x})^2$	$f(x-\bar{x})^3$	$f(x-\bar{x})^4$
3	1	3	−13	−13	169	−2197	28561
6	2	12	−10	−20	200	−2000	20000
12	3	36	−4	−12	48	−192	768
16	4	64	0	0	0	0	0
25	5	125	9	45	405	3645	32805
	$\Sigma f = 15$	$\Sigma fx = 240$		$\Sigma f(x-\bar{x}) = 0$	$\Sigma f(x-\bar{x})^2 = 822$	$\Sigma f(x-\bar{x})^3 = -744$	$\Sigma f(x-\bar{x})^4 = 82134$

Gateway Classes : 7455 961284

Mean ($\bar{x}$)

$$\bar{x} = \frac{\Sigma fx}{\Sigma f}$$

$$\bar{x} = \frac{240}{15}$$

$$\boxed{\bar{x} = 16}$$

Moments

$$\boxed{\mu_1 = 0}$$

$$\mu_2 = \frac{\Sigma f(x-\bar{x})^2}{\Sigma f}$$

$$\mu_2 = \frac{822}{15}$$

$$\boxed{\mu_2 = 54.8}$$

$$\mu_3 = \frac{\Sigma f(x-\bar{x})^3}{\Sigma f} = \frac{-744}{15}$$

$$\boxed{\mu_3 = -49.6}$$

$$\mu_4 = \frac{\Sigma f(x-\bar{x})^4}{\Sigma f} = \frac{82134}{15}$$

$$\boxed{\mu_4 = 5475.6}$$

Gateway Classes : 7455 961284

Q.3 :- *Calculate $\mu_1, \mu_2, \mu_3, \mu_4$, for the following frequency distribution :*

Marks	5-15	15-25	25-35	35-45	45-55	55-65
No. of students (f)	10	20	25	20	15	10

MARKS	f	Mid value (x)	fx	$(x-\bar{x})$	$f(x-\bar{x})$	$f(x-\bar{x})^2$	$f(x-\bar{x})^3$	$f(x-\bar{x})^4$
5-15	10	10	100	-24	-240	5760	-138240	3317760
15-25	20	20	400	-14	-280	3920	-54880	768320
25-35	25	30	750	-4	-100	400	-1600	6400
35-45	20	40	800	6	120	720	4320	25920
45-55	15	50	750	16	240	3840	61440	983040
55-65	10	60	600	26	260	6760	175760	4569760
	$\Sigma f =$ 100		$\Sigma fx =$ 3400		$\Sigma f(x-\bar{x}) = 0$	$\Sigma f(x-\bar{x})^2 = 21400$	$\Sigma f(x-\bar{x})^3 = 46800$	$\Sigma f(x-\bar{x})^4 = 9671200$

Gateway Classes : 7455 961284

Mean ($\bar{x}$)

$$\bar{x} = \frac{\Sigma fx}{\Sigma f}$$

$$= \frac{3400}{100}$$

$$\bar{x} = 34$$

Moments

$$\mu_1 = 0$$

$$\mu_2 = \frac{\Sigma f(x-\bar{x})^2}{\Sigma f}$$

$$\mu_2 = \frac{21400}{100}$$

$$\mu_2 = 214$$

$$\mu_3 = \frac{\Sigma f(x-\bar{x})^3}{\Sigma f} = \frac{46800}{100}$$

$$\mu_3 = 468$$

$$\mu_4 = \frac{\Sigma f(x-\bar{x})^4}{\Sigma f} = \frac{9671200}{100}$$

$$\mu_4 = 96712$$

Gateway Classes : 7455 961284

DPP- 2

Topic : *Moments*

Q.1. $Calculate\ \mu_1, \mu_2, \mu_3, \mu_4, for\ the\ series : 4, 7, 10, 13, 16, 19, 22$

Q.2 *Calculate the first four moment about* $x = 15$ *and hence find the moments about the mean of the following distribution :*

x	10	11	12	13	14	15	16	17	18	19	20	21
f	9	36	75	105	116	107	88	66	45	30	18	5

Q.3 Find the first four moments about origin for the following frequency distribution:

$Marks$	0-10	10-20	20-30	30-40	40-50
$No.of\ Students$	5	10	40	20	25

Gateway Classes : 7455961284

ENGINEERING MATHEMATICS

UNIT-3 : *Statistical Techniques – I*

Lec-3

Today's Target

- ***Relation between μ_r and μ'_r***
- ***Relation between μ_r and ν_r***
- PYQ
- DPP

Gateway Classes : 7455 9612 84

Moments

Moment about mean (μ_r)

$\mu_0 = 1$

$\mu_0' = 1$

$\nu_0 = 1$

$\mu_1 = 0$

$\mu_1' = \bar{x} - A$

$\nu_1 = \bar{x}$

Moment about any arbitrary number (μ_r')

Moment about origin (ν_r)

$$\text{Variance} = \frac{\sum f_i (x_i - \bar{x})^2}{N} = \mu_2$$

$$\text{Variance} = \mu_2$$

$$\text{Variance} = (S.D)^2 = \sigma^2$$

$$S.D = \sqrt{\mu_2}$$

Gateway Classes : 7455 9612 84

Relation between μ_r and μ'_r

$$\mu_1 = 0$$

$$\mu_2 = \mu'_2 - \mu'^2_1$$

$$\mu_3 = \mu'_3 - 3\mu'_2\mu'_1 + 2\mu'^3_1$$

$$\mu_4 = \mu'_4 - 4\mu'_3\mu'_1 + 6\mu'_2\mu'^2_1 - 3\mu'^4_1$$

(1)

$\mu_1 = 0$

$\mu_2 = \mu_2' - \mu_1'$

$\mu_3 = \mu_3' - \mu_2' + \mu_1'$

$\mu_4 = \mu_4' - \mu_3' + \mu_2' - \mu_1'$

μ_1' (II) $\quad \mu_1'^2$ (III) $\quad \mu_1'^3$ (IV)

(3) $\mu_1 = 0$ ✓

$\mu_2 = \mu_2' - \mu_1'^2$ ✓

$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3$ (3 2)

$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$

(4 6 3)

Gateway Classes : 7455 961284

Relation between μ_r and ν_r

$$\nu_1 = \bar{x}$$

$$\nu_2 = \mu_2 + \bar{x}^2$$

$$\nu_3 = \mu_3 + 3\mu_2\,\bar{x} + \bar{x}^3$$

$$\nu_4 = \mu_4 + 4\mu_3\,\bar{x} + 6\mu_2\bar{x}^2 + \bar{x}^4.$$

(1)

$$\nu_1 = \bar{x}$$

$$\nu_2 = \mu_2 + \bar{x}$$

$$\nu_3 = \mu_3 + \mu_2 + \bar{x}$$

$$\nu_4 = \mu_4 + \mu_3 + \mu_2 + \bar{x}$$

$\bar{x}$ (II) $\quad \bar{x}^2$ (III) $\quad \bar{x}^3$ (IV)

(2)

$$\nu_1 = \bar{x} \quad \checkmark$$

$$\nu_2 = \mu_2 + \bar{x}^2 \quad \checkmark$$

$$\nu_3 = \mu_3 + 3\mu_2\bar{x} + \bar{x}^3$$

$$\nu_4 = \mu_4 + 4\mu_3\bar{x} + 6\mu_2\bar{x}^2 + \bar{x}^4$$

Gateway Classes : 7455 96 12 84

KARL PEARSON'S β AND γ COEFFICIENTS

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3}$$

$$\beta_2 = \frac{\mu_4}{\mu_2^2}$$

→ β-coefficient

$$\gamma_1 = +\sqrt{\beta_1}$$

$$\gamma_2 = \beta_2 - 3$$

— γ-coefficient

Gateway Classes : 7455 96 12 84

Q.1. *The first three moments of a distribution, about the value 2 of the variable are 1, 16 and −40.*

(i) Show that the mean is 3, variance is 15 and $\mu_3 = -86$.

(ii) Find the values of the first three moments about the origin.

Given

$A = 2$

$\mu_1' = 1$

$\mu_2' = 16$

$\mu_3' = -40$

(i) We know that

$\mu_1' = \bar{x} - A$

$\bar{x} = \mu_1' + A$

$\bar{x} = 1 + 2$

$\bar{x} = 3$

variance $= \mu_2$

$= \mu_2' - \mu_1'^2$

$= 16 - 1^2$

variance $= 15$

$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1^3$

$= -40 - 3 \times 16 \times 1 + 2 \times 1^3$

$\mu_3 = -86$

Gateway Classes : 7455 961284

(ii) We know that

$$\nu_1 = \bar{x}$$

$$\boxed{\nu_1 = 3}$$

$$\nu_2 = \mu_2 + \bar{x}^2$$

$$\nu_2 = 15 + 3^2$$

$$\boxed{\nu_2 = 24}$$

$$\nu_3 = \mu_3 + 3\mu_2\bar{x} + \bar{x}^3$$

$$\nu_3 = -86 + 3 \times 15 \times 3 + 3^3$$

$$\nu_3 = -86 + 135 + 27$$

$$\boxed{\nu_3 = 76}$$

Gateway Classes : 7455 961 284

Q. 2. For a distribution, the mean is 10, variance is 16, γ_1 is 1, and β_2 is 4. Find the first four moments about the origin.

Given

$\bar{x} = 10$

$\mu_2 = 16$

$\gamma_1 = 1$

$\beta_2 = 4$

We know that

$\gamma_1 = +\sqrt{\beta_1}$

$1 = \sqrt{\beta_1}$

$\boxed{\beta_1 = 1}$

$\beta_1 = \dfrac{\mu_3^2}{\mu_2^3}$

$1 = \dfrac{\mu_3^2}{(16)^3}$

$\mu_3^2 = 1 \times (16)^3$

$\mu_3 = \sqrt{16 \times 16 \times 16}$

$\mu_3 = 16 \times 4$

$\boxed{\mu_3 = 64}$

Also

$\beta_2 = \dfrac{\mu_4}{\mu_2^2}$

$4 = \dfrac{\mu_4}{16^2}$

$\mu_4 = 256 \times 4$

$\boxed{\mu_4 = 1024}$

$\boxed{\mu_2 = 16}$

$\boxed{\mu_1 = 0}$

Gateway Classes : 7455 961 284

Q.3. *The first four moments of a distribution about the value '4' of the variable are* $-1.5, 17, -30$ *and* 108. *Find the moments about mean, about origin,* β_1 *and* β_2. *Also, find the moments about the point* $x = 2$. **(UPTU-2017)**

Given

$A = 4$

$\mu_1' = -1.5$

$\mu_2' = 17$

$\mu_3' = -30$

$\mu_4' = 108$

Moments about Mean

$\mu_1 = 0$

$\mu_2 = \mu_2' - \mu_1'^2$

$\mu_2 = 17 - (-1.5)^2$

$\mu_2 = 17 - 2.25$

$\mu_2 = 14.75$

$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1^3$

$\mu_3 = -30 - 3 \times 17 \times (-1.5) + 2(-1.5)^3$

$\mu_3 = 39.75$

Gateway Classes : 7455961284

$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$$

$$\mu_4 = 108 - 4(-30)(-1.5) + 6(17)(-1.5)^2 - 3(-1.5)^4$$

$$\boxed{\mu_4 = 142.3125}$$

Moments about origin

$$\nu_1 = \bar{x}$$

$$\nu_1 = \mu_1' + A$$

$$\nu_1 = -1.5 + 4$$

$$\boxed{\nu_1 = 2.5}$$

$$\nu_2 = \mu_2 + \bar{x}^2$$

$$\nu_2 = 14.75 + (2.5)^2$$

$$\boxed{\nu_2 = 21}$$

$$\nu_3 = \mu_3 + 3\mu_2\bar{x} + \bar{x}^3$$

$$\nu_3 = 39.75 + 3 \times 14.75 \times 2.5 + (2.5)^3$$

Gateway Classes : 7455 961284

$$\nu_3 = 166$$

$$\nu_4 = \mu_4 + 4\mu_3\bar{x} + 6\mu_2\bar{x}^2 + \bar{x}^4$$

$$\nu_4 = 142.3125 + 4 \times 39.75 \times 2.5 + 6 \times 14.75 \times (2.5)^2 + (2.5)^4$$

$$\nu_4 = 1132$$

Karl pearson's coefficient

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3} = \frac{(39.75)^2}{(14.75)^3} = 0.4923775$$

Gateway Classes : 7455 9612 84

$$\beta_2 = \frac{\mu_4}{\mu_2^2}$$

$$\beta_2 = \frac{142.3125}{(14.75)^2}$$

$$\beta_2 = 0.65412$$

Moment About $A = 2$

$A = 2$

$\mu_1' = \bar{x} - A$

$\mu_1' = 2.5 - 2$

$\boxed{\mu_1' = 0.5}$

$\mu_2 = \mu_2' - \mu_1'^2$

$\mu_2' = \mu_2 + \mu_1'^2$

$\mu_2' = 14.75 + (0.5)^2$

$\boxed{\mu_2' = 15}$

$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3$

$\mu_3' = \mu_3 + 3\mu_2'\mu_1' + 2\mu_1'^3$

$\mu_3 = 39.75 + 3 \times 15 \times 0.5 + 2 \times (0.5)^3$

$\boxed{\mu_3 = 62}$

Gateway Classes 7455 961284

$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$$

$$\mu_4' = \mu_4 + 4\mu_3'\mu_1' - 6\mu_2'\mu_1'^2 + 3\mu_1'^4$$

$$\mu_4' =$$

$$\boxed{\mu_4' = 244}$$

Gateway Classes : 7455 9612 84

Q.4. The first four mements of a distribution about the value 0 are $-0.20, 1.76, -2.36$ and 10.88. Find the moments about the mean. (UPTU-2009)

Given

$A = 0$

$\mu_1' = -0.20$

$\mu_2' = 1.76$

$\mu_3' = -2.36$

$\mu_4' = 10.88$

Gateway Classes : 7455 9612 84

DPP- 3

Topic : Relation between μ_r and μ'_r
Relation between μ_r and ν_r

Q.1. The first four moments of a distribution about $x = 2$ are $1, 2.5, 5.5$ and 16. Calcutate the first four moments about the mean and about origin.

Q.2 In a certain distribution, the first four mements about the point $x = 4$ are $-1.5, 17, -30$ and 308. Find the moments about mean and about origin. Also, calculate β_1 and β_2. (UPTU-2014)

Q.3. The first four moments of a distribution about the value 5 of the variable are 2, 20, 40 and 50. Calculate mean, variance, μ_3 and μ_4.

Gateway Classes : 7455 961284

ENGINEERING MATHEMATICS

UNIT-3 : *Statistical Techniques – I*

Lec-4

Today's Target

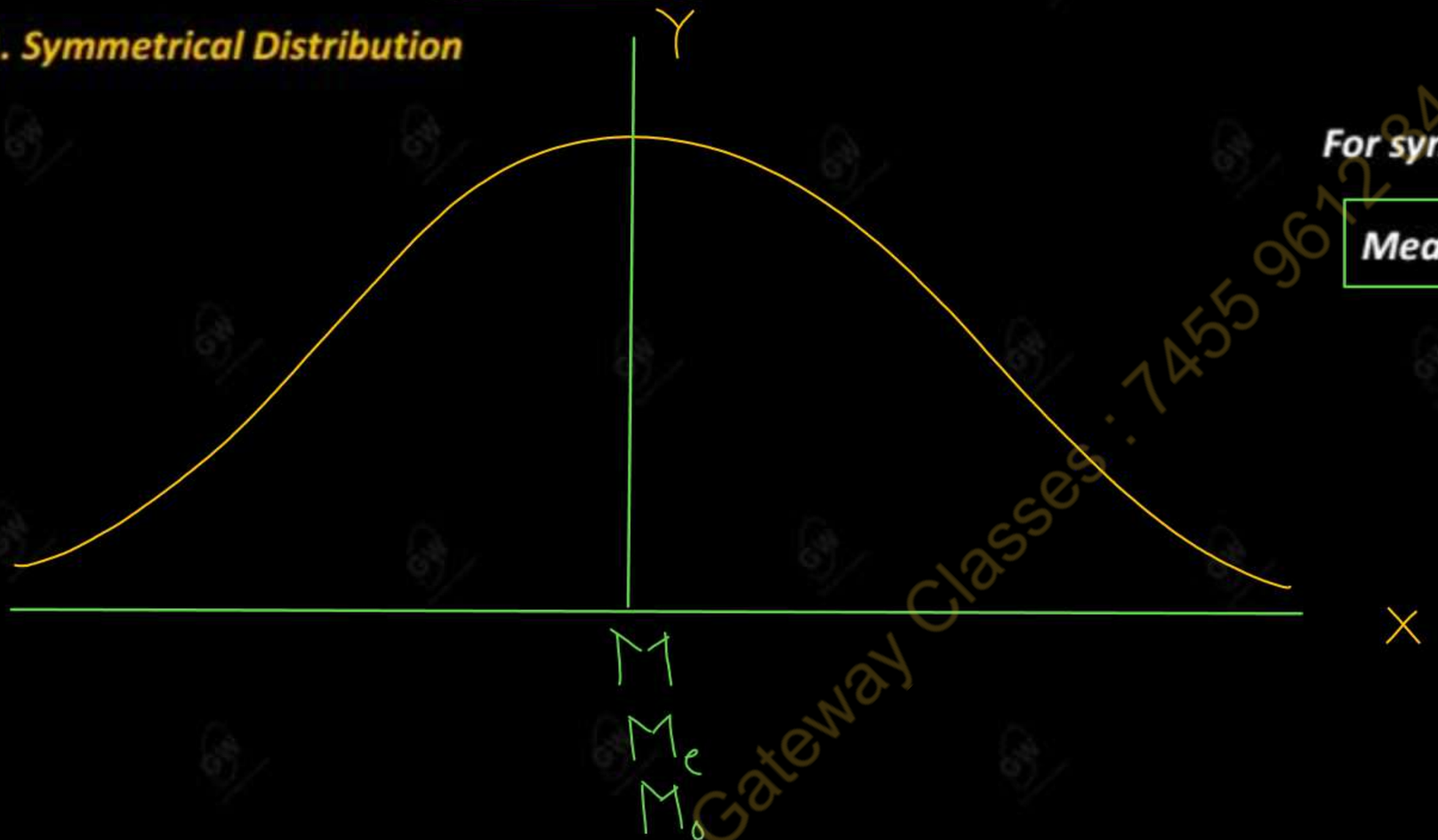
- ***Skewness***
- PYQ
- DPP

Gateway Classes : 7455 9612 84

Skewness

Skewness means lack of symmetry in frequency distribution.

1. Symmetrical Distribution



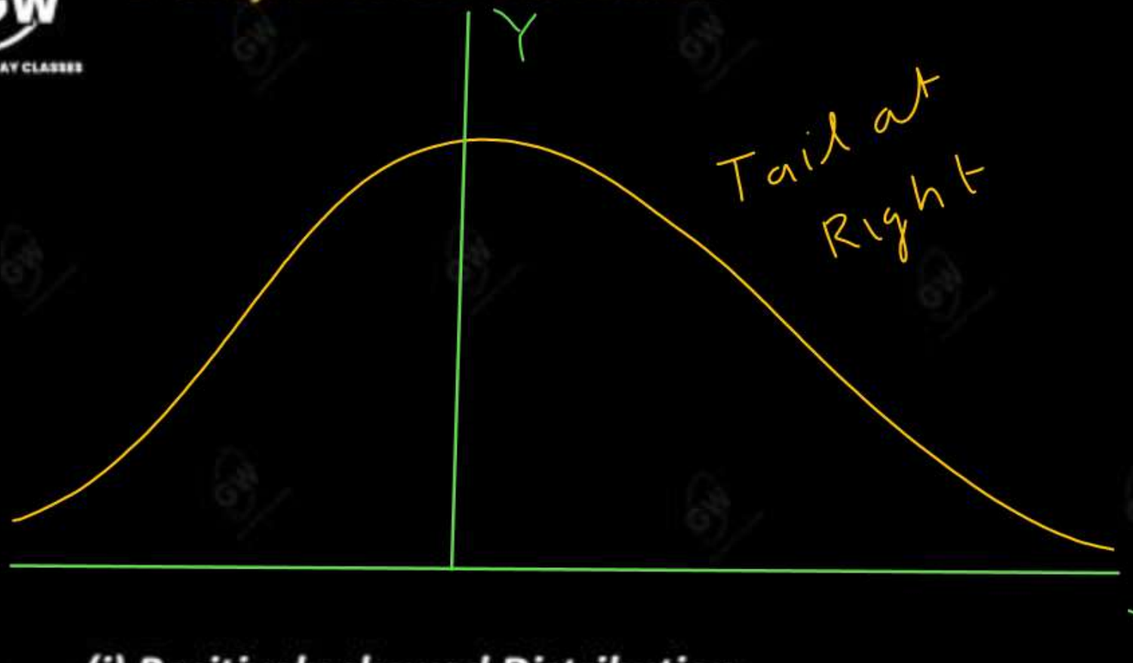
For symmetrical Distribution

Mean = Median = Mode

Gateway Classes : 7455 961284

Note:- If Mean = Median = Mode, then there is no skewness in the distribution

2. Unsymmetrical Distribution



(i) Positively skewed Distribution

Mean > Mode



(ii) Negatively skewed Distribution

Mean < Mode

Methods of Measuring Skewness

(i) Karl Pearson's Method

V.Imp (ii) Method of Moments

Note:- Relative measures of skewness are called coefficient of skewness

Gateway Classes : 7455 9612 84

Karl Pearson's Method

Karl Pearson's coefficient of skewness (SK_p)

(i) $$SK_p = \frac{Mean - Mode}{S.D}$$

Mode = 3 Median – 2 Mean

(ii) $$SK_p = \frac{3(Mean - Median)}{S.D}$$

S.D = Standard Deviation (σ)

Note:- (i) If Skp = 0

Mean = Mode

Distribution is symmetrical

(ii) If Skp > 0

Mean > Mode

Distribution is positively skewed

(iii) If Skp < 0

Mean < Mode

Distribution is negatively skewed

Gateway Classes : 7455 9612 84

Standard Deviation

$$S.D = \sqrt{variance}$$

$$variance = \mu_2$$

$$\boxed{S.D = \sqrt{\mu_2}}$$

Relation between μ_r and μ'_r

$$\mu_1 = 0$$

$$\mu_2 = \mu'_2 - \mu'^2_1$$

$$\mu_3 = \mu'_3 - 3\mu'_2\mu'_1 + 2\mu'^3_1$$

$$\mu_4 = \mu'_4 - 4\mu'_3\mu'_1 + 6\mu'_2\mu'^2_1 - 3\mu'^4_1$$

Relation between μ_r and ν_r

$$\nu_1 = \bar{x}$$

$$\nu_2 = \mu_2 + \bar{x}^2$$

$$\nu_3 = \mu_3 + 3\mu_2\,\bar{x} + \bar{x}^3$$

$$\nu_4 = \mu_4 + 4\mu_3\,\bar{x} + 6\mu_2\bar{x}^2 + \bar{x}^4$$

Gateway Classes : 7455 9612 84

Q.1 *For a moderately skewed data, the arithmetic mean is 100 , the variance is 35 and Karl Pearson's coefficient of skewness is 0.2. Find its mode and median.*

Given data

Mean($\bar{x}$) = 100

Variance = 35

SK_p = 0.2

We know that

$S.D = \sqrt{\text{Variance}}$

$S.D = \sqrt{35}$

Karl pearson's coefficient of skewness

$$SK_p = \frac{\text{Mean} - \text{Mode}}{SD}$$

$$0.2 = \frac{100 - \text{Mode}}{\sqrt{35}}$$

$$\text{Mode} = 100 - 0.2 \times \sqrt{35}$$

$$\boxed{\text{Mode} = 98.816}$$

Mode = 3Median − 2Mean

98.816 = 3Median − 2 × 100

$$\boxed{\text{Median} = 99.61}$$

Gateway Classes 7455 962 284

Q.2 *The sum of 20 observations is 300 and sum of their squares is 5000. The median is 15. Find the Karl Pearson's coefficient of skewness.*

Given Data

$n = 20$

$\sum x = 300$

$\sum x^2 = 5000$

Median = 15

We know that

$$\text{Mean} = \frac{\sum x}{n}$$

$$\text{Mean} = \frac{300}{20}$$

Mean = 15

$$\mu_2 = \nu_2 - \bar{x}^2$$

$$\mu_2 = \frac{\sum x^2}{n} - \bar{x}^2$$

$$= \frac{5000}{20} - (15)^2$$

$$= 250 - 225$$

$$\mu_2 = 25$$

$$S.D = \sqrt{\mu_2} = \sqrt{25}$$

SD = 5

$$Sk_p = \frac{3(\text{Mean} - \text{Median})}{S.D}$$

$$Sk_p = \frac{3(15-15)}{5}$$

$Sk_p = 0$ Symmetrical Distribution

Gateway Classes : 7455 961284

2. Method of Moments

μ_2 and μ_3

In this Method, second and third central moments of the distribution are used.

This measure of skewness is called the moment coefficient of skewness

Moment coefficient of skewness (SK_M) OR γ_1

$$SK_M = \frac{\mu_3}{\sqrt{\mu_2^3}}$$

Note:- *We know that*

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3}$$

$$\sqrt{\beta_1} = \frac{\mu_3}{\sqrt{\mu_2^3}}$$

$$SK_M = \sqrt{\beta_1}$$

$$SK_M = \sqrt{\beta_1} = \gamma_1$$

Gateway Classes : 7455 9612 84

Q.3 *The first three central moments of a distribution are 0, 15, - 31. Find the moment coefficient of skewness.*

Given Data

$\mu_1 = 0$

$\mu_2 = 15$

$\mu_3 = -31$

$$Sk_M = \frac{\mu_3}{\sqrt{\mu_2^3}}$$

$$= \frac{-31}{\sqrt{(15)^3}}$$

$$Sk_M = -0.53$$

Negatively skewed Distribution

Gateway Classes. 7455 9612 84

Q.4 *The first four moments of a distribution about the value 5 of the variable are 2, 20, 40 and 50. calculate the moment coefficient of skewness.*

Given Data

$A = 5$

$\mu_1' = 2$

$\mu_2' = 20$

$\mu_3' = 40$

$\mu_4' = 50$

We know that

$$\mu_2 = \mu_2' - \mu_1'^2$$

$$= 20 - (2)^2$$

$$\boxed{\mu_2 = 16}$$

$$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3$$

$$= 40 - 3 \times 20 \times 2 + 2(2)^3$$

$$\mu_3 = 40 - 120 + 16$$

$$\boxed{\mu_3 = -64}$$

$$SK_M = \frac{\mu_3}{\sqrt{\mu_2^3}}$$

$$= \frac{-64}{\sqrt{16^3}} = -\frac{64}{16 \times 4}$$

$$\boxed{SK_M = -1}$$

Negatively skewed Distribution

Gateway Classes : 7455 961284

DPP- 4

Topic : *Skewness*

Q.1 (i) Define skewness of a distribution. (UPTU-2015)

(ii) Define the coefficients of skewness. (MTU 2012, 2013)

Q.2. Karl Pearson's coefficient of skewness of a distribution is 0.32, its standard deviation is 6.5 and mean is 29.6. Find the mode of the distribution.

Q.3. The first-three central moments of a distribution are 0, 2.5, 0.7. Find the value of the moment coefficient of skewness. (UPTU-2015)

Q.4 In a certain distribution, the first four moments about the point 4 are – 1.5, 17, - 30 and 308. Calculate the moment coefficient of skewness. (UPTU-2015)

Gateway Classes : 7455 9612 84

ENGINEERING MATHEMATICS

UNIT-3 : *Statistical Techniques – I*

Lec-5

Today's Target

- **KURTOSIS**
- ***Problems based on Moments, Skewness and Kurtosis***
- PYQ
- DPP

Gateway Classes : 7455 9612 84

Lec-1

Measure of central Tendency

1. Arithmetic Mean OR Mean
   - (i) Direct Method
   - (ii) Assume Mean Method
   - (iii) Step Deviation Method
2. Median
3. Mode
4. Geometric Mean
5. Harmonic Mean

Note : 3(Median) = Mode + 2 (Mean)

Gateway Classes : 7455 9612 84

Lec-2

Moment For frequency distribution

A

Moment About Mean (μ_r)	Moment About Number (μ'_r)	Moment About Origin (ν_r)
$\mu_0 = 1$	$\mu'_0 = 1$	$\nu_0 = 1$
$\mu_1 = 0$	$\mu'_1 = \bar{x} - A$	$\nu_1 = \bar{x}$
$\mu_2 = \frac{\sum f(x-\bar{x})^2}{\sum f}$	$\mu'_2 = \frac{\sum f(x-A)^2}{\sum f}$	$\nu_2 = \frac{\sum fx^2}{\sum f}$
$\mu_3 = \frac{\sum f(x-\bar{x})^3}{\sum f}$	$\mu'_3 = \frac{\sum f(x-A)^3}{\sum f}$	$\nu_3 = \frac{\sum fx^3}{\sum f}$
$\mu_4 = \frac{\sum f(x-\bar{x})^4}{\sum f}$	$\mu'_4 = \frac{\sum f(x-A)^4}{\sum f}$	$\nu_4 = \frac{\sum fx^4}{\sum f}$

Gateway Classes 7455 9612 84

Moment Individual series

Moment About Mean (μ_r)	Moment About Number (μ'_r)	Moment About Origin (ν_r)
$\mu_0 = 1$	$\mu'_0 = 1$	$\nu_0 = 1$
$\mu_1 = 0$	$\mu'_1 = \bar{x} - A$	$\nu_1 = \bar{x}$
$\mu_2 = \frac{\sum(x-\bar{x})^2}{n}$	$\mu'_2 = \frac{\sum(x-A)^2}{n}$	$\nu_2 = \frac{\sum x^2}{n}$
$\mu_3 = \frac{\sum(x-\bar{x})^3}{n}$	$\mu'_3 = \frac{\sum(x-A)^3}{n}$	$\nu_3 = \frac{\sum x^3}{n}$
$\mu_4 = \frac{\sum f(x-\bar{x})^4}{n}$	$\mu'_4 = \frac{\sum(x-A)^4}{n}$	$\nu_4 = \frac{\sum x^4}{n}$

Gateway Classes 7455 9612 84

Lec-3

Relation between μ_r and μ'_r

$$\mu_1 = 0$$

$$\mu_2 = \mu'_2 - \mu'^2_1$$

$$\mu_3 = \mu'_3 - 3\mu'_2\mu'_1 + 2\mu'^3_1$$

$$\mu_4 = \mu'_4 - 4\mu'_3\mu'_1 + 6\mu'_2\mu'^2_1 - 3\mu'^4_1$$

Relation between μ_r and ν_r

$$\nu_1 = \bar{x}$$

$$\nu_2 = \mu_2 + \bar{x}^2$$

$$\nu_3 = \mu_3 + 3\mu_2\,\bar{x} + \bar{x}^3$$

$$\nu_4 = \mu_4 + 4\mu_3\,\bar{x} + 6\mu_2\bar{x}^2 + \bar{x}^4$$

KARL PEARSON'S β and γ Coefficients

(i) β − Coefficients

$$\beta_1 = \frac{{\mu_3}^2}{{\mu_2}^3}$$

$$\beta_2 = \frac{\mu_4}{{\mu_2}^2}$$

(ii) γ Coefficients

$$\gamma_1 = +\sqrt{\beta_1}$$

$$\gamma_2 = \beta_2 - 3$$

Gateway Classes : 7455 9612 84

Lec-4

SKEWNESS

Karl Pearson's Method

Karl Pearson's coefficient of skewness (SK_p)

(i) $SK_p = \frac{Mean - Mode}{S.D}$

Mode = 3 Median – 2 Mean

(ii) $SK_p = \frac{3(Mean - Mode)}{S.D}$

Standard Deviation

$$S.D = \sqrt{variance}$$

$$S.D = \sqrt{\mu_2}$$

V.Imp

Method of Moments

Moment coefficient of skewness (SK_M)

$$\gamma_1 = SK_M = \frac{\mu_3}{\sqrt{\mu_2^3}}$$

Note:- We know that

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3}$$

$$\sqrt{\beta_1} = \frac{\mu_3}{\sqrt{\mu_2^3}}$$

$$SK_M = \gamma_1 = \sqrt{\beta_1}$$

Gateway Classes : 7455 9612 84

Note:- (i) If Skp = 0

Mean = Mode

Distribution is symmetrical

(ii) If Skp > 0

Mean > Mode

Distribution is positively skewed

(iii) If Skp < 0

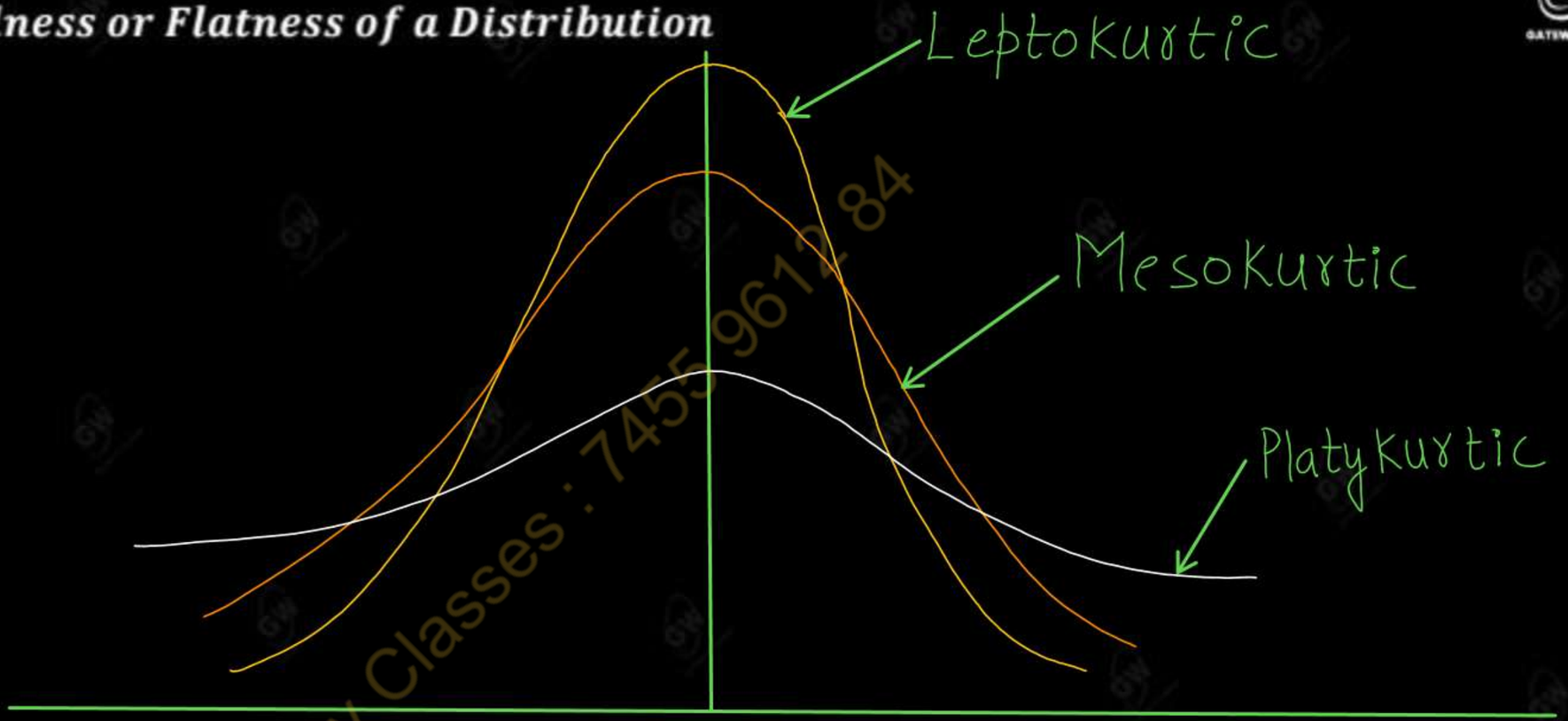
Mean < Mode

Distribution is negatively skewed

Gateway Classes : 7455 9612 84

KURTOSIS

Measure of peakedness or Flatness of a Distribution



Measure of Kurtosis

$$\beta_2 = \frac{\mu_4}{{\mu_2}^2}$$

Kurtosis of Distribution is also difined as

$$\gamma_2 = \beta_2 - 3$$

Where and μ_2 are μ_4 the second and fourth moment about mean of the distribution.

Gateway Classes : 7455 9612 84

Note :-

(i) The distribution is Leptokurtic if

$$\beta_2 > 3$$

$$\gamma_2 > 0$$

(ii) The distribution is Mesokurtic if

$$\beta_2 = 3$$

$$\gamma_2 = 0$$

(iii) The distribution is Platykurtic if

$$\beta_2 < 3$$

$$\gamma_2 < 0$$

Gateway Classes : 7455 9612 84

Q.1 The fourth central moment is 48. What must be its standard deviation in order that the distribution be Mesokurtic. (U.P.T.U.2023)

Given

$\mu_4 = 48$

For Mesokurtic distribution

$\beta_2 = 3$

We know that

$\beta_2 = \frac{\mu_4}{\mu_2^2}$

$3 = \frac{48}{\mu_2^2}$

$\mu_2^2 = \frac{48}{3} = 16$

$\mu_2 = \sqrt{16}$

$\mu_2 = 4$

$S.D = \sqrt{\mu_2}$

$S.D = \sqrt{4}$

$S.D = 2$

Gateway Classes : 7455 9612 84

Q.2 *The first four moments of a distribution are 0, 2.5, 0.7 and 18.71. Find the coefficient of skewness and kurtosis.* **(U.P.T.U.2015)**

Given

$\mu_1 = 0$

$\mu_2 = 2.5$

$\mu_3 = 0.7$

$\mu_4 = 18.71$

coefficient of skewness

$$\gamma_1 = Sk_M = \frac{\mu_3}{\sqrt{\mu_2^3}}$$

$$Sk_M = \frac{0.7}{\sqrt{(2.5)^3}}$$

$$\boxed{Sk_M = 0.17708}$$

Positively skewed

kurtosis

$$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{18.71}{(2.5)^2}$$

$$\boxed{\beta_2 = 2.9936}$$

$\beta_2 < 3$

Platykurtic

Gateway Classes : 7455 95512 84

Q.3 *The first four moments of a distribution about x = 4 are 1, 4, 10 and 45. Obtain the various characteristic of the distribution on the basic of the given information. Comment upon the nature of the distribution.*

(A.K.T.U.2016, 2018)

Given

$A = 4$

$\mu_1' = 1$

$\mu_2' = 4$

$\mu_3' = 10$

$\mu_4' = 45$

Moments About Mean

$\mu_2 = \mu_2' - \mu_1'^2$

$= 4 - 1^2$

$\boxed{\mu_2 = 3}$

$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3$

$= 10 - 3 \times 4 \times 1 + 2(1)^3$

$= 10 - 12 + 2$

$= 0$

$\boxed{\mu_3 = 0}$

$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$

$= 45 - 4 \times 10 \times 1 + 6 \times 4 \times (1)^2 - 3 \times (1)^4$

$= 45 - 40 + 24 - 3$

$= 26$

$\boxed{\mu_4 = 26}$

Gateway Classes : 7455 961284

Skewness

$$Sk_M = \frac{\mu_3}{\sqrt{\mu_2^3}}$$

$$= \frac{0}{\sqrt{3^3}}$$

$$\boxed{Sk_M = 0}$$

Symmetrical Distribution

Kurtosis

$$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{26}{9}$$

$$\boxed{\beta_2 = 2.89}$$

$$\beta_2 < 3$$

Platykurtic

Gateway Classes : 7455 961284

Q.4 The first four moments about the working mean 28.5 of a distribution are 0.294, 7.144, 42.409 and 454.98. Calculate the moments about the mean. Also evaluate β_1, β_2 and comment upon the skewness and kurtosis of the distribution. (A.K.T.U.2016)

Given

$A = 28.5$

$\mu_1' = 0.294$

$\mu_2' = 7.144$

$\mu_3' = 42.409$

$\mu_4' = 454.98$

Moments About Mean

$$\mu_2 = \mu_2' - \mu_1'^2$$

$$= 7.144 - (0.294)^2$$

$$\boxed{\mu_2 = 7.0576}$$

$$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3$$

$$= 42.409 - 3 \times 7.144 \times 0.294 + 2(.294)^3$$

$$\boxed{\mu_3 = 36.1588}$$

$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$$

$$= 454.98 - 4 \times 42.409 \times 0.294 + 6 \times 7.144 \times (.294)^2 - 3(0.294)^4$$

$$\boxed{\mu_4 = 408.7896}$$

Gateway Classes : 7455 9612 84

Now

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3}$$

$$\beta_1 = \frac{(36.1588)^2}{(7.0576)^3}$$

$$\boxed{\beta_1 = 3.7193}$$

$$\beta_2 = \frac{\mu_4}{\mu_2^2}$$

$$= \frac{408.7896}{(7.0576)^2}$$

$$\boxed{\beta_2 = 8.2070}$$

Skewness

$$SK_M = \gamma_1 = \frac{\mu_3}{\sqrt{\mu_2^3}}$$

$$SK_M = \frac{36.1588}{\sqrt{(7.0576)^3}}$$

$$\boxed{SK_M = 1.9285}$$ positively skewed Distribution

KURTOSIS

$$\boxed{\beta_2 = 8.2070}$$

$\beta_2 > 3 \Rightarrow$ Leptokurtic

Gateway Classes, 7455 9612 84

Q.5 *The following table represents the heights of a batch of 100 students. Calculate Skewness and kurtosis.*

(A.K.T.U.2016)

Height (in cm)	59	61	63	65	67	69	71	73	75
No. Of students	0	2	6	20	40	20	8	2	2

Height (in cm) (x)	No. of Students (f)	$d = x - 67$	$u = \frac{x-67}{2} = h$	fu	fu^2	fu^3	fu^4
59	0	−8	−4	0	0	0	0
61	2	−6	−3	−6	18	−54	162
63	6	−4	−2	−12	24	−48	96
65	20	−2	−1	−20	20	−20	20
A 67	40	0	0	0	0	0	0
69	20	2	1	20	20	20	20
71	8	4	2	16	32	64	128
73	2	6	3	6	18	54	162
75	2	8	4	8	32	128	512
	$\Sigma f = N = 100$			$\Sigma fu =$ 12	$\Sigma fu^2 =$ 164	$\Sigma fu^3 =$ 144	$\Sigma fu^4 =$ 1100

Gateway Classes : 7455 961284

Moments About 67

$$\mu_1' = \frac{\Sigma f u}{N} \times h$$

$$\mu_1' = \frac{12}{100} \times 2$$

$$\boxed{\mu_1' = 0.24}$$

$$\mu_2' = \frac{\Sigma f u^2}{N} \times h^2$$

$$= \frac{164}{100} \times 4$$

$$\boxed{\mu_2' = 6.56}$$

$$\mu_3' = \frac{\Sigma f u^3}{N} \times h^3$$

$$\mu_3' = \frac{144}{100} \times 8$$

$$\boxed{\mu_3' = 11.52}$$

$$\mu_4' = \frac{\Sigma f u^4}{N} \times h^4$$

$$= \frac{1100}{100} \times 16$$

$$\boxed{\mu_4' = 176}$$

Moments About Mean

$$\mu_1 = 0$$

$$\mu_2 = \mu_2' - \mu_1'^2 = 6.56 - (0.24)^2$$

$$\boxed{\mu_2 = 6.5024}$$

$$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3$$

$$= 11.52 - 3 \times 6.56 \times 0.24 + 2(0.24)^3$$

$$\boxed{\mu_3 = 6.824}$$

Gateway Classes : 7455 9612 84

$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$$

$$\mu_4 = 176 - 4 \times 11.52 \times 0.24 + 6 \times 6.56 (0.24)^2 - 3 \times (0.24)^4$$

$$\boxed{\mu_4 = 167.19798}$$

Skewness

$$SK_M = \frac{\mu_3}{\sqrt{\mu_2^3}}$$

$$SK_M = \frac{6.824}{\sqrt{(6.5024)^3}}$$

$$\boxed{SK_M = 0.411}$$

Positively skewed Distribution

KURTOSIS

$$\beta_2 = \frac{\mu_4}{\mu_2^2}$$

$$= \frac{167.19798}{(6.5024)^2}$$

$$\boxed{\beta_2 = 3.9544}$$

$\beta_2 > 3$

Leptokurtic

Gateway Classes 7459 961284

Q.6 *Find the measure of skewness and kurtosis on the basis of moments for the following distribution and draw your conclusion :* **(A.K.T.U.2018)**

Marks	5-15	15-25	25-35	35-45	45-55
No. of Students	1	3	5	7	4

MARKS	No. of Students (f)	Mid value (x)	$d = x - 30$	$u = \frac{x-30}{10} = h$	fu	fu^2	fu^3	fu^4
5-15	1	10	−20	−2	−2	4	−8	16
15-25	3	20	−10	−1	−3	3	−3	3
25-35	5	30 (A)	0	0	0	0	0	0
35-45	7	40	10	1	7	7	7	7
45-55	4	50	20	2	8	16	32	64
	$\Sigma f = N = 20$				$\Sigma fu = 10$	$\Sigma fu^2 = 30$	$\Sigma fu^3 = 28$	$\Sigma fu^4 = 90$

Gateway Classes : 7455 961284

Moments About 30

$$\mu_1' = \frac{\Sigma fu}{N} \times h$$

$$= \frac{10}{20} \times 10$$

$$\boxed{\mu_1' = 5}$$

$$\mu_2' = \frac{\Sigma fu^2}{N} \times h^2$$

$$= \frac{30}{\cancel{20}} \times \cancel{100}^{5}$$

$$\boxed{\mu_2' = 150}$$

$$\mu_3' = \frac{\Sigma fu^3}{N} \times h^3$$

$$= \frac{28}{\cancel{20}} \times \cancel{1000}^{50}$$

$$\boxed{\mu_3' = 1400}$$

$$\mu_4' = \frac{\Sigma fu^4}{N} \times h^4$$

$$\mu_4' = \frac{90}{\cancel{2\phi}}^{45} \times 1000\phi$$

$$\boxed{\mu_4' = 45000}$$

Moments About Mean

$$\mu_1 = 0$$

$$\mu_2 = \mu_2' - \mu_1'^2$$

$$= 150 - (5)^2$$

$$\boxed{\mu_2 = 125}$$

Gateway Classes : 7455 961284

$$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3$$

$$= 1400 - 3 \times 150 \times 5 + 2(5)^3$$

$$\boxed{\mu_3 = -600}$$

$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$$

$$= 45000 - 4 \times 1400 \times 5 + 6 \times 150 \times (5)^2 - 3 \times (5)^4$$

$$\boxed{\mu_4 = 37625}$$

Skewness

$$SK_M = \frac{\mu_3}{\sqrt{\mu_2^3}}$$

$$= \frac{-600}{\sqrt{(125)^3}}$$

$$\boxed{SK_M = -0.4293}$$

Negatively skewed Distribution

KURTOSIS

$$\beta_2 = \frac{\mu_4}{\mu_2^2}$$

$$= \frac{37625}{(125)^2}$$

$$\boxed{\beta_2 = 2.408}$$

$\beta_2 < 3$

Platykurtic

Gateway Classes : 7455 0612 84

Q.1 *The first four moments of a distribution about the value '4' of the variable are -1.5, 17, -30 and 108. State whether the distribution is leptokurtic or platykurtic.* *(U.P.T.U.2014)*

Q.2 *The first four moments of a distribution are 0, 2.5, 0.7 and 18.75. Test the kurtosis of the distribution.* *(M.T.U.2013)*

Q.3 *The first four moments of a distribution about the value 5 of the variable are 2, 20, 40 and 50. Calculate the moments about the mean and comment upon the skewness and kurtosis of the distribution.* *(G.B.T.U.2011)*

Q.4 *Find the measure of skewness and kurtosis on the basis of moments for the following distribution :*

x	1	3	5	7	9
f	1	4	6	4	1

Q.5 *Calculate the first four moments about the mean for the following frequency distribution and hence find the coefficient of skewness and kurtosis and comment upon the nature of the distribution.* *(U.P.T.U.2014)*

Class – Interval	5-10	10-15	15-20	20-25	25-30	30-35	35-40
Frequency	6	8	17	21	15	11	2

Gateway Classes : 7455 96 12 84

ENGINEERING MATHEMATICS

UNIT-3 : *Statistical Techniques – I*

Lec-6

Today's Target

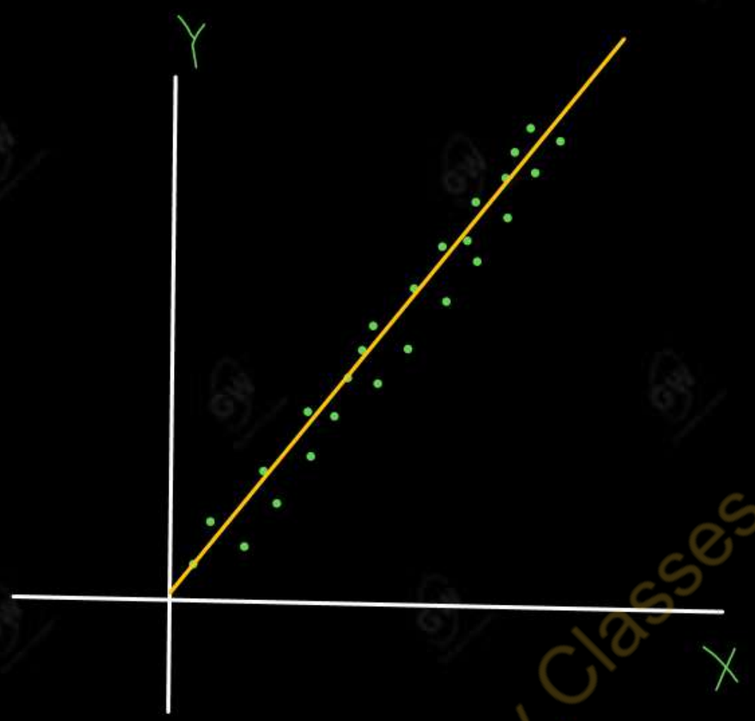
- Curve Fitting by the Method of Least Square : Fitting of Straight line
- PYQ
- DPP

Gateway Classes : 7455 9612 84

Curve Fitting

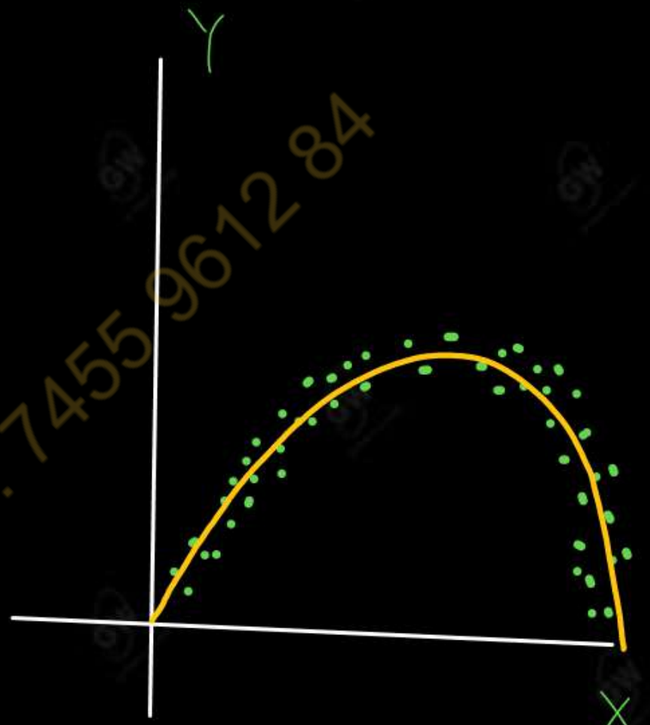
Curve Fitting means to form an equation of the curve from the given data

x	y
x_1	y_1
x_2	y_2
x_3	y_3
x_4	y_4
x_5	y_5
x_6	y_5
Σx	Σy



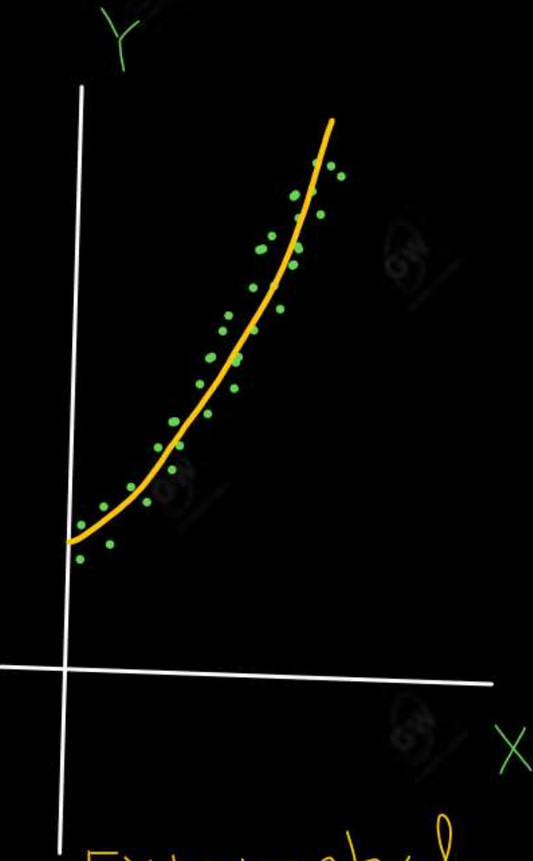
Straight line

$y = a + bx$



Parabola

$y = a + bx + cx^2$



Exponential curve

$y = ae^{bx}$

Gateway Classes : 7455 9612 84

Curve Fitting by the Method of Least Square

1. Fitting of Straight line
2. Fitting of second degree parabola
3. Fitting of Exponential curve

$$\Sigma y = \Sigma a + \Sigma bx$$

$$\Sigma y = a\Sigma 1 + b\Sigma x$$

$$\Sigma 1 = 1 + 1 + 1 + 1 + \dots + 1 \ (n \text{ times})$$

Fitting of Straight line

Equation of line

$$y = a + bx \quad \text{— (1)}$$

Normal Equations

$$\Sigma y = na + b\Sigma x \quad \text{— (2)}$$

$$\Sigma xy = a\Sigma x + b\Sigma x^2 \quad \text{— (3)}$$

Gateway Classes: 7455 961284

Q.1 Fit a straight line to the following data by least square method: (U.K.T.U.2011)

x:	0	1	2	3	4
y:	1	1.8	3.3	4.5	6.3

Equation of line

$$y = a + bx \quad \text{—} (1)$$

Normal Equations

$$\Sigma y = na + b\Sigma x \quad \text{—} (2)$$

$$\Sigma xy = a\Sigma x + b\Sigma x^2 \quad \text{—} (3)$$

$$n = 5$$

x	y	xy	x^2
0	1	0	0
1	1.8	1.8	1
2	3.3	6.6	4
3	4.5	13.5	9
4	6.3	25.2	16
$\Sigma x = 10$	$\Sigma y = 16.9$	$\Sigma xy = 47.1$	$\Sigma x^2 = 30$

Gateway Classes : 7455 961284

Put Σx, Σy and n in (2)

$$16.9 = 5a + 10b$$

$$5a + 10b = 16.9 \quad \text{—— (4)}$$

Put Σxy, Σx, and Σx^2 in (3)

$$47.1 = 10a + 30b$$

$$10a + 30b = 47.1 \quad \text{—— (5)}$$

Solve eqn (4) and (5)

$$a = 0.72$$

$$b = 1.33$$

Put a and b in (1)

$$\boxed{y = 0.72 + 1.33x}$$

Required straight line

Gateway Classes : 7455 9612 84

Q.2 *What straight line best fits the following data ?*

x	1	2	3	4
y	3	7	13	21

Equation of line

$$y = a + bx \quad \text{—} \ (1)$$

Normal Equations

$$\Sigma y = na + b\Sigma x \quad \text{—} \ (2)$$

$$\Sigma xy = a\Sigma x + b\Sigma x^2 \quad \text{—} \ (3)$$

$$n = 4$$

x	y	xy	x^2
1	3	3	1
2	7	14	4
3	13	39	9
4	21	84	16
$\Sigma x = 10$	$\Sigma y = 44$	$\Sigma xy = 140$	$\Sigma x^2 = 30$

Gateway Classes : 7455 9612 84

Put Σx, Σy and n in (2)

$$44 = 4a + 10b$$

$$4a + 10b = 44 \quad \text{— (4)}$$

Put Σxy, Σx and Σx^2 in (3)

$$140 = 10a + 30b$$

$$10a + 30b = 140 \quad \text{— (5)}$$

solve eqn (4) and (5)

$$a = -4$$

$$b = 6$$

Put a and b in (1)

$$\boxed{y = -4 + 6x}$$

Required straight line

Gateway Classes: 7455 961284

Q.1 *By the method of least squares, find the straight line that best fits the following data:*

x:	1	2	3	4	5
y:	14	27	40	55	68

Q.2 *Using the method of least square, fit a straight line to the following data:*

x	1	2	3	4	5
y	2	4	6	8	10

Gateway Classes : 7455 9612 84

ENGINEERING MATHEMATICS

UNIT-3 : *Statistical Techniques – I*

Lec-7

Today's Target

- **Curve Fitting by the Method of Least Square :** (i) Fitting of Parabola
  (ii) Fitting of Exponential Curve
- PYQ
- DPP

Gateway Classes : 7455 9612 84

Curve Fitting by the Method of Least Square

1. Fitting of Straight line
2. Fitting of second degree parabola
3. Fitting of an Exponential curve

Fitting of second degree Parabola

Equation of parabola

$$y = a + bx + cx^2$$

Normal equations

$$\Sigma y = na + b\Sigma x + c\Sigma x^2 \quad -(1)$$

$$\Sigma xy = a\Sigma x + b\Sigma x^2 + c\Sigma x^3 \quad -(2)$$

$$\Sigma x^2 y = a\Sigma x^2 + b\Sigma x^3 + c\Sigma x^4 \quad -(3)$$

Gateway Classes : 7455 962 84

Q.1 *Fit a second degree parabola in the following data ?*

x	0	1	2	3	4
y	1	4	10	17	30

Equation of parabola

$$y = a + bx + cx^2 \quad \text{—— } ①$$

Normal Equations

$$\Sigma y = na + b\Sigma x + c\Sigma x^2 \quad \text{— } ②$$

$$\Sigma xy = a\Sigma x + b\Sigma x^2 + c\Sigma x^3 \quad \text{— } ③$$

$$\Sigma x^2 y = a\Sigma x^2 + b\Sigma x^3 + c\Sigma x^4 \quad \text{— } ④$$

$$n = 5$$

x	y	xy	x^2	x^3	x^4	x^2y
0	1	0	0	0	0	0
1	4	4	1	1	1	4
2	10	20	4	8	16	40
3	17	51	9	27	81	153
4	30	120	16	64	256	480
$\Sigma x =$ 10	$\Sigma y =$ 62	Σxy =195	$\Sigma x^2 =$ 30	$\Sigma x^3 =$ 100	$\Sigma x^4 =$ 354	$\Sigma x^2 y =$ 677

Gateway Classes : 7455 961284

Put these values in eqn ②, ③ and ④

$5a + 10b + 30c = 62$ — ⑤

$10a + 30b + 100c = 195$ — ⑥

$30a + 100b + 354c = 677$ — ⑦

Solve eqn ⑤, ⑥ and ⑦

$a = 1.2$

$b = 1.1$

$c = 1.5$

Put a, b, c in ①

$y = a + bn + cn^2$

$y = 1.2 + 1.1n + 1.5n^2$

Gateway Classes : 7405 961284

Fitting of an Exponential Curve

Equation of Exponential curve

$$y = ae^{bx}$$

Taking log both side

$$\log_{10} y = \log_{10}(ae^{bx})$$

$$\log_{10} y = \log_{10} a + \log_{10} e^{bx}$$

$$\log_{10} y = \log_{10} a + bx \log_{10} e$$

$$\log mn = \log m + \log n$$

$$\log m^n = n \log m$$

$$\log_{10} y = \log_{10} a + (b \log_{10} e)x$$

Put

$$\log_{10} y = Y$$

$$\log_{10} a = A$$

$$b \log_{10} e = B$$

$$Y = A + Bx$$

Gateway Classes : 7455 9612 84

Q. 2:- ***Find the curve of best fit of the type $y = ae^{bx}$ to the following data by the method of Least square :***

x:	1	5	7	9	12
y:	10	15	12	15	21

Equation of curve

$$y = a e^{bx} \quad \text{—— (1)}$$

Taking $\log_{10}$ both side

$$\log_{10} y = \log_{10} a + \log_{10} e^{bx}$$

$$\log_{10} y = \log_{10} a + (b \log_{10} e) x$$

Put

$\log_{10} y = Y$

$\log_{10} a = A$

$b \log_{10} e = B$

$$\boxed{Y = A + Bx} \quad \text{—— (2)}$$

Normal Equations

$$\Sigma Y = nA + B \Sigma x \quad \text{—— (3)}$$

$$\Sigma xY = A \Sigma x + B \Sigma x^2 \quad \text{—— (4)}$$

$n = 5$

GW GATEWAY CLASSES

Gateway Classes : 7455 961284

x	y	$Y=\log_{10} y$	x^2	xY
1	10	1	1	1
5	15	1.1761	25	5.8805
7	12	1.0792	49	7.5544
9	15	1.1761	81	10.5849
12	21	1.3222	144	15.8664
$\Sigma x=34$		$\Sigma Y=$ 5.7536	$\Sigma x^2=$ 300	$\Sigma xY=$ 40.8862

Put these values in (3) and (4)

$5A + 34B = 5.7536$ — (5)

$34A + 300B = 40.8862$ — (6)

Solve (5) and (6)

$A = 0.9766$

$B = 0.0256$

Gateway Classes : 7455 961284

Now

$\log_{10} a = A$

$\log_{10} a = 0.9766$

$a = \text{Antilog}_{10}(0.9766)$

$\boxed{a = 9.4754}$

$b \log_{10} e = B$

$b = \frac{B}{\log_{10} e} = \frac{0.0256}{0.4342}$

$\boxed{b = 0.059}$

Put a and b in (1)

$\boxed{y = 9.4754\, e^{0.059x}}$

Gateway Classes : 7455 961284

Engineering Mathematics by Gulshan Sir

DPP- 7

Topic : :
1. Fitting of Parabola
2. Fitting of Exponential Curve

Q.1 *Using the method of least squares, fit second degree parabola $y = a + bx + cx^2$ for the following data:*

x:	2	3	4	5	6
y:	14	17	20	24	29

Q.2 *Using the method of least square, fit a parabolic curve of second degree to the following data:*

x	0	1	2	3	4
y	1	1.8	1.3	2.5	6.3

Gateway Classes : 7455 9612 84

ENGINEERING MATHEMATICS

UNIT-3 : *Statistical Techniques – I*

Lec-8

Today's Target

- **Important Questions based on Curve Fitting**
- PYQs
- DPP

Gateway Classes : 7455 9612 84

1. Fitting a Straight Line $y = a + bx$

Normal Equations

$$\sum y = na + b\sum x$$

$$\sum xy = a\sum x + b\sum x^2$$

2. Fitting of second degree parabola $y = a + bx + cx^2$

Normal Equations

$$\sum y = na + b\sum x + c\sum x^2$$

$$\sum xy = a\sum x + b\sum x^2 + c\sum x^3$$

$$\sum x^2y = a\sum x^2 + b\sum x^3 + c\sum x^4$$

Gateway Classes : 7455 9612 84

3. Fitting of an Exponential Curve $y = ae^{bx}$

$Y = A + Bx$

Normal Equations

$$\sum Y = nA + B\sum x$$

$$\sum xY = A\sum x + B\sum x^2$$

$$y = ae^{bx}$$

Taking log both side

$$\log_{10} y = \log_{10} ae^{bx}$$

$$\log_{10} y = \log_{10} a + \log_{10} e^{bx}$$

$$\log_{10} y = \log_{10} a + (b \log_{10} e)x$$

$$Y = A + Bx$$

$$\log mn = \log m + \log n$$

$$\log m^n = n \log m$$

Gateway Classes: 7455 9612 84

4. Fitting of The Curve $y = ax + bx^2$

Normal Equations

$$\sum xy = a\sum x^2 + b\sum x^3$$

$$\sum x^2y = a\sum x^3 + b\sum x^4$$

Gateway Classes : 7455 9612 84

Q. 1 :- By the method of least squares, find the curve $y = ax + bx^2$ that best fits the following data: **(UPTU – 2014)**

x	1	2	3	4	5
y	1.8	5.1	8.9	14.1	19.8

Equation of curve

$$y = ax + bx^2 \quad \text{—— (1)}$$

Normal Equations

$$\Sigma xy = a\Sigma x^2 + b\Sigma x^3 \quad \text{— (2)}$$

$$\Sigma x^2y = a\Sigma x^3 + b\Sigma x^4 \quad \text{— (3)}$$

x	y	x^2	x^3	x^4	xy	x^2y
1	1.8	1	1	1	1.8	1.8
2	5.1	4	8	16	10.2	20.4
3	8.9	9	27	81	26.7	80.1
4	14.1	16	64	256	56.4	225.6
5	19.8	25	125	625	99	495
		$\Sigma x^2 =$ 55	$\Sigma x^3 =$ 225	$\Sigma x^4 =$ 979	$\Sigma xy =$ 194.1	$\Sigma x^2y =$ 822.9

Gateway Classes : 7455961284

Put these valus in ② and ③

$$55a + 225b = 194.1 \quad -④$$

$$225a + 979b = 822.9 \quad -⑤$$

solve eqn ④ and ⑤

$$a = 1.51$$

$$b = 0.49$$

Put a and b in ①

$$y = an + bn^2$$

$$\boxed{y = 1.51n + 0.49n^2}$$

Required parabola

Gateway Classes : 7455 961284

5. Fitting of The Curve $y = \frac{c_0}{x} + c_1\sqrt{x}$

$$\frac{y}{x} = \frac{c_0}{x^2} + c_1\frac{\sqrt{x}}{x}$$

Normal Equations

$$\Sigma\frac{y}{x} = c_0\Sigma\frac{1}{x^2} + c_1\Sigma\frac{1}{\sqrt{x}}$$

$$\Sigma y\sqrt{x} = c_0\Sigma\frac{1}{\sqrt{x}} + c_1\Sigma x$$

$$\frac{y}{x} = \frac{c_0}{x^2} + \frac{c_1}{\sqrt{x}}$$

$$y\sqrt{x} = \frac{c_0\sqrt{x}}{x} + c_1\sqrt{x}\times\sqrt{x}$$

$$y\sqrt{x} = \frac{c_0}{\sqrt{x}} + c_1 x$$

Gateway Classes 7455 961284

Q. 2 :- ***Use the method of Least squares to fit the curve:*** $y = \frac{c_0}{x} + c_1\sqrt{x}$ ***to the following table of values:***

(AKTU – 2016 & 2018)

x:	0.1	0.2	0.4	0.5	1	2
y:	21	11	7	6	5	6

Equation of curve

$$y = \frac{c_0}{x} + c_1\sqrt{x} \quad \text{—— (1)}$$

Normal Equations

$$\Sigma \frac{y}{x} = c_0 \Sigma \frac{1}{x^2} + c_1 \Sigma \frac{1}{\sqrt{x}} \quad \text{— (2)}$$

$$\Sigma y\sqrt{x} = c_0 \Sigma \frac{1}{\sqrt{x}} + c_1 \Sigma x \quad \text{— (3)}$$

Put these values in (2) an (3)

$$136.5\, c_0 + 10.100816\, c_1 = 302.5 \quad \text{— (4)}$$

$$10.100816\, c_0 + 4.2\, c_1 = 33.71524 \quad \text{— (5)}$$

Solve eqn (4) and (5)

$$\boxed{c_0 = 1.973267}$$

Gateway Classes : 7455 961284

x	y	$\frac{y}{x}$	$y\sqrt{x}$	$\frac{1}{\sqrt{x}}$	$\frac{1}{x^2}$
0.1	21	210	6.64078	3.16228	100
0.2	11	55	4.91935	2.23607	25
0.4	7	17.5	4.42719	1.58114	6.25
0.5	6	12	4.24264	1.41421	4
1	5	5	5	1	1
2	6	3	8.48528	0.70711	0.25
$\Sigma x = 4.2$		$\Sigma y/x =$ 302.5	$\Sigma y\sqrt{x} =$ 33.71524	$\Sigma 1/\sqrt{x} =$ 10.10081	$\Sigma 1/x^2 =$ 136.5

$C_0 = 1.973267$

$C_1 = 3.281819$

Put C_0 and C_1 in (1)

$$y = \frac{1.9733}{x} + 3.2818\sqrt{x}$$

Gateway Classes : 7455 961284

6. Fitting of The Curve $y = ax + \frac{b}{x}$

$$xy = ax^2 + b$$

Normal Equations

$$\sum xy = a\sum x^2 + nb$$

$$\sum \frac{y}{x} = na + b\sum \frac{1}{x^2}$$

$$\frac{y}{x} = a + \frac{b}{x^2}$$

Gateway Classes : 7455 961284

Q. 3:- *Fit a relation* $y = ax + \frac{b}{x}$ *which satisfies the following data, using method of least squares:*

x	1	2	3	4	5	6	7	8
y	5.4	6.2	8.2	10.3	12.6	14.8	17.2	19.5

Equation of the curve

$$y = ax + \frac{b}{x} \quad \text{— (1)}$$

Normal Equations

$$\Sigma xy = a\Sigma x^2 + nb \quad \text{— (2)}$$

$$\Sigma \frac{y}{x} = na + b\Sigma \frac{1}{x^2} \quad \text{— (3)}$$

$$n = 8$$

Put these values in eqn (2) and (3)

$$204a + 8b = 511.8 \quad \text{— (4)}$$

$$8a + 1.5274 = 23.705 \quad \text{— (5)}$$

Solve eqn (4) and (5)

$$a = 2.39139$$

$$b = 2.99455$$

GatewayClasses : 7455 961284

x	y	xy	x^2	y/x	$1/x^2$
1	5.4	5.4	1	5.4	1
2	6.2	12.4	4	3.1	0.25
3	8.2	24.6	9	2.74	0.1111
4	10.3	41.2	16	2.575	0.0625
5	12.6	63	25	2.52	0.04
6	14.8	88.8	36	2.47	0.0278
7	17.2	120.4	49	2.46	0.0204
8	19.5	156	64	2.44	0.0156
		$\Sigma xy =$ 511.8	$\Sigma x^2 =$ 204	$\Sigma y/x =$ 23.705	$\Sigma 1/x^2 =$ 1.5274

Put a and b in (1)

$$y = 2.39139x + \frac{2.99455}{x}$$

Gateway Classes : 7455 961284

7. Fitting of The Curve $y = ab^x$

$$Y = A + Bx$$

Normal Equations

$$\sum Y = nA + B\sum x$$

$$\sum xY = A\sum x + B\sum x^2$$

$$y = ab^x$$

Taking log both side

$$\log_{10} y = \log_{10} ab^x$$

$$\log_{10} y = \log_{10} a + \log_{10} b^x$$

$$\log_{10} y = \log_{10} a + (\log_{10} b)x$$

$$\boxed{Y = A + Bx}$$

Gateway Classes : 7455 961284

Q. 4:- Obtain a relation of the form $y = ab^x$ for the following data by the method of least squares: **(GBTU – 2019)**

x	2	3	4	5	6
y	8.3	15.4	33.1	65.2	127.4

Equation of curve

$$y = ab^x \quad \text{— (1)}$$

Taking log both side

$$\log_{10} y = \log_{10} ab^x$$

$$\log_{10} y = \log_{10} a + \log_{10} b^x$$

$$\log_{10} y = \log_{10} a + (\log_{10} b)x$$

Put

$$\log_{10} y = Y \quad \text{— (2)}$$

$$\log_{10} a = A \quad \text{— (3)}$$

$$\log_{10} b = B \quad \text{— (4)}$$

$$Y = A + Bx$$

Normal Equation

$$\Sigma Y = nA + B\Sigma x \quad \text{— (5)}$$

$$\Sigma xY = A\Sigma x + B\Sigma x^2 \quad \text{— (6)}$$

Gateway Classes : 7455 961284

$n = 5$

x	y	$Y = \log_{10} y$	x^2	xY
2	8.3	0.9191	4	1.8382
3	15.4	1.1872	9	3.5616
4	33.1	1.5198	16	6.0792
5	65.2	1.8142	25	9.0710
6	127.4	2.1052	36	12.6312
$\Sigma x = 20$		$\Sigma Y =$ 7.5455	$\Sigma x^2 =$ 90	$\Sigma xY =$ 33.1812

Put these values in ⑤ and ⑥

$5A + 20B = 7.5455$

$20A + 90B = 33.1812$

$A = 0.3094$

$B = 0.2999$

Gateway Classes : 7455 961284

Put A in (3)

$\log_{10} a = 0.3094$

$a = \text{Antilog}(0.3094)$

$\boxed{a = 2.0389}$

Put B in (4)

$\log_{10} b = 0.2999$

$b = \text{Antilog}(0.2999)$

$\boxed{b = 1.9948}$

Put a and b in (1)

$\boxed{y = 2.0389\,(1.9948)^{x}}$

Gateway Classes : 7455 8612 84

8. Fitting of The Curve $pv^{\gamma} = c$

$$Y = A + BX$$

Normal Equations

$$\sum Y = nA + B\sum X$$

$$\sum XY = A\sum X + B\sum X^2$$

$$PV^{\gamma} = c$$

$$P = \frac{c}{V^{\gamma}}$$

$$P = cV^{-\gamma}$$

Taking log both side

$$\log P = \log cV^{-\gamma}$$

$$\log P = \log c + \log V^{-\gamma}$$

$$\log P = \log c - \gamma \log V$$

Put

$$\log P = Y$$

$$\log c = A$$

$$\log V = X$$

$$-\gamma = B$$

$$Y = A + BX$$

Gateway Classes : 7455 961284

Q. 5:- ***The pressure of the gas correspomding to various volumes V is measured, given by the following data:***

$V\ (cm^3)$:	50	60	70	90	100
$P\ (kg/cm^2)$	64.7	51.3	40.5	25.9	78

Fit the data to the equation $\boxed{PV^\gamma = C}$

Equation of the curve

$$PV^\gamma = C \quad \text{—— (1)}$$

$$P = \frac{C}{V^\gamma}$$

$$P = CV^{-\gamma}$$

Taking log both side

$$\log_{10} P = \log_{10} CV^{-\gamma}$$

$$\log_{10} P = \log_{10} C - \gamma \log_{10} V$$

Put

$$\log_{10} P = Y \quad \text{— (2)}$$

$$\log_{10} C = A \quad \text{— (3)}$$

$$\log_{10} V = X \quad \text{— (4)}$$

$$-\gamma = B \quad \text{— (5)}$$

$$\boxed{Y = A + BX}$$

Normal Equations

$$\Sigma Y = nA + B\Sigma X \quad \text{— (6)}$$

$$\Sigma XY = A\Sigma X + B\Sigma X^2 \quad \text{— (7)}$$

Gateway Classes: 7455 961284

$n = 5$

V	P	$X = \log_{10} V$	$Y = \log_{10} P$	XY	X^2
50	64.7	1.69897	1.81090	3.07666	2.88655
60	51.3	1.77815	1.71012	3.04085	3.16182
70	40.5	1.84510	1.60746	2.96592	3.40439
90	25.9	1.95424	1.41330	2.76193	3.81905
100	78	2	1.89209	3.78418	4
		$\Sigma X =$ 9.27646	$\Sigma Y =$ 8.43387	$\Sigma XY =$ 15.62954	$\Sigma X^2 =$ 17.27176

Put these values in ⑥ and ⑦

$5A + 9.27646B = 8.43387$ ⑧

$9.27646A + 17.27176 =$

15.62954 ⑨

Solve ⑧ and ⑨

Gateway Classes: 7455961284

$A = 2.2248$

$B = -0.2899$

Put A in ③

$\log_{10} c = 2.2248$

$c = \text{Antilog}(2.2248)$

$\boxed{c = 167.8031}$

Put B in ⑤

$-\gamma = -0.2899$

$\boxed{\gamma = 0.2899}$

Put c and γ in ①

$\boxed{PV^{0.2899} = 167.8031}$

Gateway Classes: 7455 961284

9. Fitting of The Curve $xy = b + ax$

$$y = a + \frac{b}{x}$$

Normal Equations

$$\sum y = na + b\sum \frac{1}{x}$$

$$\sum \frac{y}{x} = a\sum \frac{1}{x} + b\sum \frac{1}{x^2}$$

Gateway Classes : 7455 9612 84

10. Fitting of The Curve $y = ax^2 + \frac{b}{x}$

$$x^2 y = a x^4 + b x$$

Normal Equations

$$\sum x^2 y = a \sum x^4 + b \sum x$$

$$\sum \frac{y}{x} = a \sum x + b \sum \frac{1}{x^2}$$

$$\frac{y}{x} = ax + \frac{b}{x^2}$$

Gateway Classes : 7455 96 12 84

11. Fitting of The Curve $y = a + \frac{b}{x} + \frac{c}{x^2}$

Normal Equations

$$\sum y = na + b\sum \frac{1}{x} + c\sum \frac{1}{x^2}$$

$$\sum \frac{y}{x} = a\sum \frac{1}{x} + b\sum \frac{1}{x^2} + c\sum \frac{1}{x^3}$$

$$\sum \frac{y}{x^2} = a\sum \frac{1}{x^2} + b\sum \frac{1}{x^3} + c\sum \frac{1}{x^4}$$

(12) $y = a x^b$

(13) $y = a + bx^2$

Gateway Classes : 7455 812 84

Q.1 *Given the following experimental values:*

x:	0	1	2	3
y:	2	4	10	15

Q.2 *Using method of least squares, derive the normal equations to fit the curve* $y = ax^2 + bx$ *. Hence fit this curve to the following data:*

x	1	2	3	4	5	6	7	8
y	1	1.2	1.8	2.5	3.6	4.7	6.6	9.1

Q.3 *Fit the curve* $pv^Y = k$ *to the following data:*

$p\ (kg/cm^2)$	0.5	1	1.5	2	2.5	3
$v\ (Liters)$	1620	1000	750	620	520	460

Gateway Classes : 7455 9612 84

ENGINEERING MATHEMATICS

UNIT-3 : *Statistical Techniques – I*

Lec-9

Today's Target

- Correlation and Co-efficient of Correlation
- PYQ
- DPP

Gateway Classes : 7455 9612 84

Correlation

- ***Correlation is a statistical tool which deal with two or more variables(or two or more series).***
- **The degree of relationship between the variables under consideration is measured through the corelation analysis**

x	y
1	10
2	11
3	12
4	13
5	12

x	y
5	15
4	14
3	13
2	12
1	11

x	y
1	10
2	9
4	8
5	7
6	6

Gateway Classes : 7455 9612 82

Types of Correlation

(i) *Positive corelation* :

If two variables move in same direction then corelation is said to be positive corelation.

(i) $x \uparrow$ $y \uparrow$ (ii) $x \downarrow$ $y \downarrow$

(ii) *Negative corelation* :

If two variables move in opposite direction then corelation is said to be negative corelation.

(i) $x \uparrow$ $y \downarrow$ (ii) $x \downarrow$ $y \uparrow$

Gateway Classes : 7455 9612 82

Method to measure Correlation

Co – efficient of corelation (r_{xy})

OR

Karl pearson's Co – efficient of corelation

OR

Product moment Co – efficient of corelation

$$r_{xy} = \frac{cov(x,y)}{\sigma_x \, \sigma_y}$$

$$cov(x,y) = \frac{\Sigma (x-\bar{x})(y-\bar{y})}{n}$$

$$\sigma_x = \sqrt{\frac{\Sigma (x-\bar{x})^2}{n}}$$

$$\sigma_y = \sqrt{\frac{\Sigma (y-\bar{y})^2}{n}}$$

$$r_{xy} = \frac{\Sigma (x-\bar{x})(y-\bar{y})}{\sqrt{\Sigma (x-\bar{x})^2}\sqrt{\Sigma (y-\bar{y})^2}}$$

Direct Method

Where

$\bar{x}$ = Mean of x

$\bar{y}$ = Mean of y

Gateway Classes : +91 9612 34

$(x - \bar{x})$ = Deviation from mean

$(y - \bar{y})$ = Deviation from mean

Another Form

$$r_{xy} = \frac{n\Sigma xy - \Sigma x \Sigma y}{\sqrt{n\Sigma x^2 - (\Sigma x)^2}\sqrt{n\Sigma y^2 - (\Sigma y)^2}}$$

Direct Method

Shortcut Method

$$d_x = x - A$$

$$d_y = y - A$$

$$r_{xy} = \frac{n\Sigma d_x d_y - \Sigma d_x \Sigma d_y}{\sqrt{n\Sigma d_x^2 - (\Sigma d_x)^2}\sqrt{n\Sigma d_y^2 - (\Sigma d_y)^2}}$$

Gateway Classes: 7455 9612 84

Step deviation Method

$$u = \frac{x - A}{h}$$

$$v = \frac{y - A}{h}$$

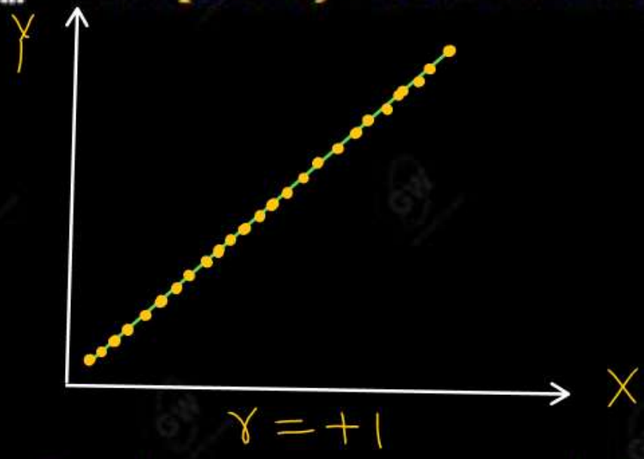
$$r_{xy} = \frac{n\Sigma uv - \Sigma u \Sigma v}{\sqrt{n\Sigma u^2 - (\Sigma u)^2}\sqrt{n\Sigma v^2 - (\Sigma v)^2}}$$

Important points

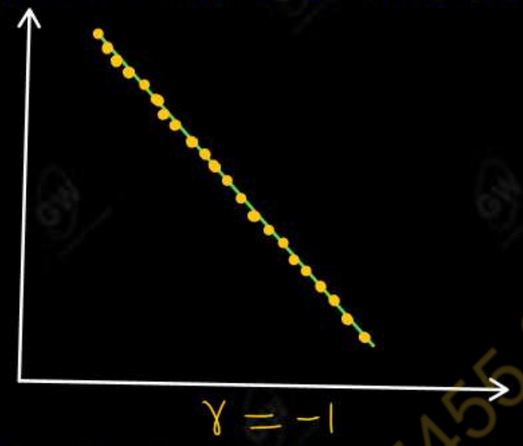
1. $-1 \leq r \leq 1$
2. Positive correlation
   $0 < r < 1$
3. Negative correlation
   $-1 < r < 0$
4. Perfect +ve correction
   $r = +1$
5. Perfect Negative correlation
   $r = -1$
6. No correlation ($r = 0$)

Gateway Classes 8455 96 12 84

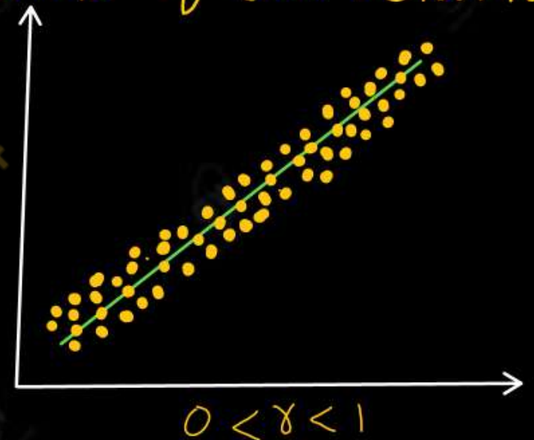
Graphs of correlation / scattered or dotted diagrams of correlation



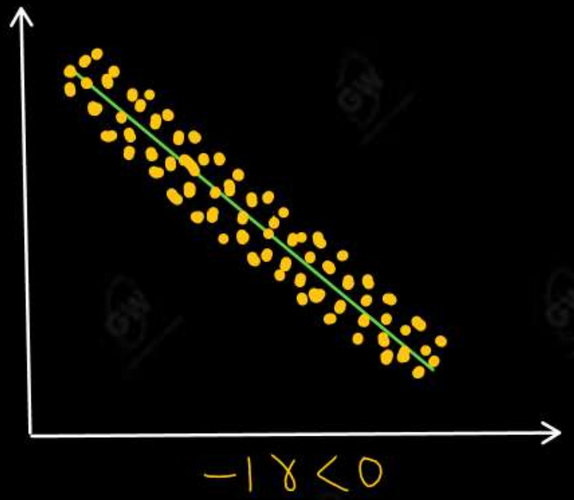
Perfect +ve correlation



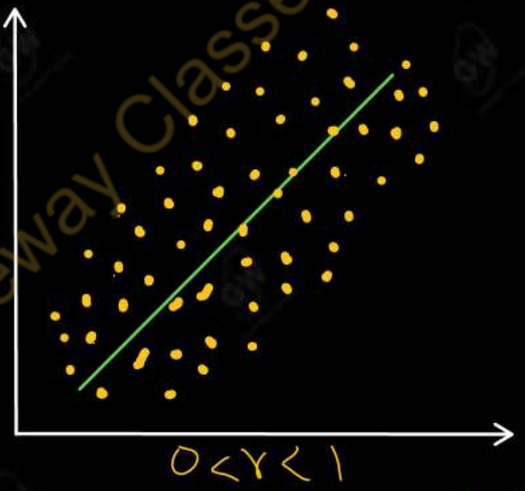
Perfect -ve correlation



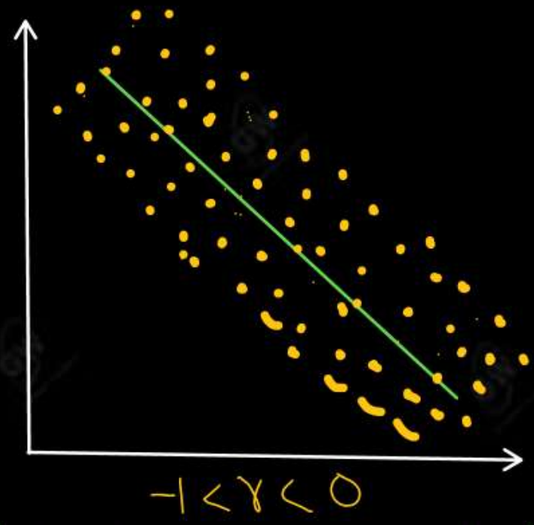
Strong +ve correlation



Strong -ve correlation

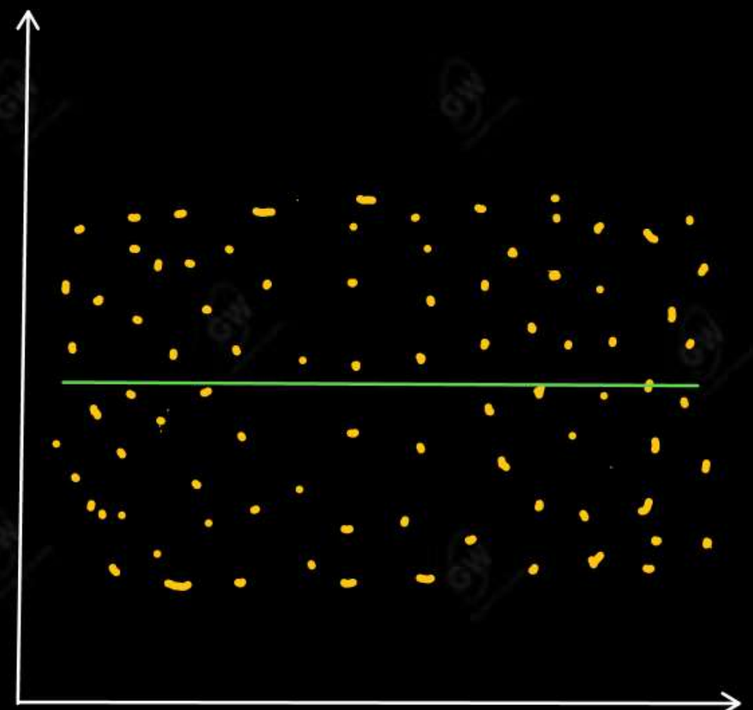


Weak +ve correlation



Weak -ve correlation

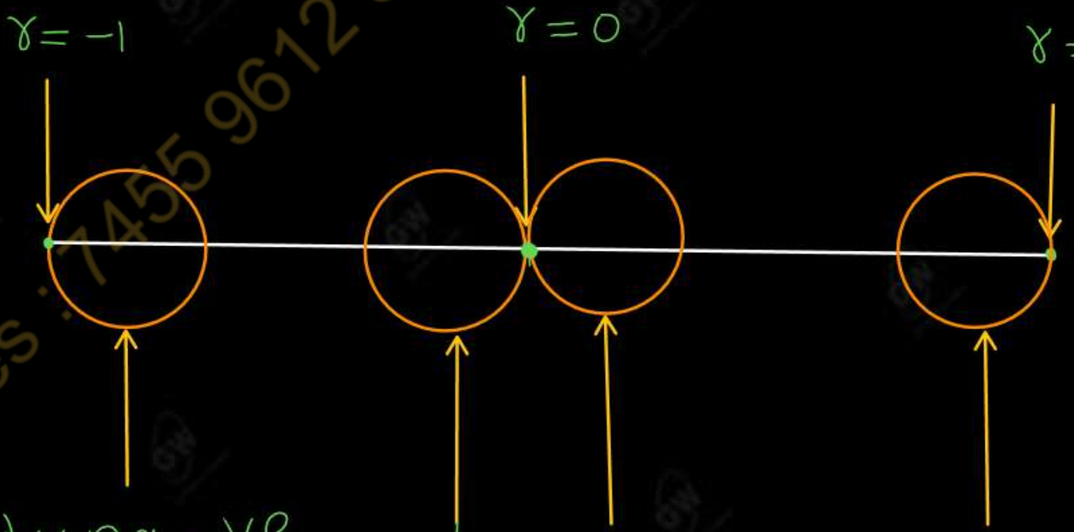
Gateway Classes : 7455 9612 84



$\gamma = 0$

No correlation

Gateway Classes : 7455 9612 84



Strong −ve correlation

Weak −ve correlation

Weak +ve correlation

Strong +ve correlation

Q. 1:– Form the data given below, find the number of items n:

$r_{xy} = 0.5,\ \sum XY = 120,\ \sum X^2 = 90, \sigma_y = 8$ *where X and Y are deviations from the arithmetic mean.*

(AKTU-2016)

Given

$r_{xy} = 0.5$

$\sum XY = 120$

$\sum X^2 = 90$

$\sigma_y = 8$

$X = x - \bar{x}$

$Y = y - \bar{y}$

$\sum (x-\bar{x})(y-\bar{y}) = 120$

$\sum (x-\bar{x})^2 = 90$

$\sigma_y = 8$

$\sqrt{\frac{\sum (y-\bar{y})^2}{n}} = 8$

$\frac{\sum (y-\bar{y})^2}{n} = 64$

$\sum (y-\bar{y})^2 = 64n$

Gateway Classes 7455 9612 84

$$r_{xy} = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\sqrt{\Sigma(x-\bar{x})^2}\sqrt{\Sigma(y-\bar{y})^2}}$$

$$0.5 = \frac{120}{\sqrt{90}\ \sqrt{64n}}$$

$$\frac{0.5}{\frac{10}{2}} = \frac{120}{\sqrt{90} \times 8\sqrt{n}}$$

$$\sqrt{n} = \frac{120 \times 2}{8 \times \sqrt{90}}$$

$$\sqrt{n} = \frac{30}{\sqrt{90}}$$

$$n = \frac{900}{90}$$

$$n = 10$$

Gateway Classes 7455 961284

Q. 2:- *Ten students got the following percentage of marks in Principles of Economics and Statistics:*

Roll Nos.:	1	2	3	4	5	6	7	8	9	10
(x) Marks in Econmics	78	36	98	25	75	82	90	62	65	39
(y) Marks in Statistics	84	51	91	60	68	62	86	58	53	47

Calculate the co – efficient of correlation

Mean

$$\bar{x} = \frac{\Sigma x}{n}$$

$$= \frac{650}{10}$$

$$\boxed{\bar{x} = 65}$$

$$\bar{y} = \frac{\Sigma y}{n}$$

$$= \frac{660}{10}$$

$$\boxed{\bar{y} = 66}$$

$$r_{xy} = \frac{\Sigma (x-\bar{x})(y-\bar{y})}{\sqrt{\Sigma (x-\bar{x})^2}\sqrt{\Sigma (y-\bar{y})^2}}$$

$$r_{xy} = \frac{2734}{\sqrt{5398}\sqrt{2224}}$$

$$\boxed{r_{xy} = 0.789}$$

Gateway Classes : 7455 961284

$n = 10$ $\quad \bar{x} = 65$ $\quad \bar{y} = 66$

x	y	$x-\bar{x}$	$y-\bar{y}$	$(x-\bar{x})^2$	$(y-\bar{y})^2$	$(x-\bar{x})(y-\bar{y})$
78	84	13	18	169	324	234
36	51	-29	-15	841	225	435
98	91	33	25	1089	625	825
25	60	-40	-6	1600	36	240
75	68	10	2	100	4	20
82	62	17	-4	289	16	-68
90	86	25	20	625	400	500
62	58	-3	-8	9	64	24
65	53	0	-13	0	169	0
39	47	-26	-19	676	361	494
$\Sigma x =$ 650	$\Sigma y =$ 660			$\Sigma(x-\bar{x})^2 =$ 5398	$\Sigma(y-\bar{y})^2 =$ 2224	$\Sigma(x-\bar{x})(y-\bar{y}) =$ 2734

GW GATEWAY CLASSES
GatewayClasses : 7455 961284

Q. 3:- Find the coefficient of correlation between the values of x and y

x	1	3	5	7	8	10
y	8	12	15	17	18	20

$n = 6$

x	y	x^2	y^2	xy
1	8	1	64	8
3	12	9	144	36
5	15	25	225	75
7	17	49	289	119
8	18	64	324	144
10	20	100	400	200
$\Sigma x = 34$	$\Sigma y = 90$	$\Sigma x^2 = 248$	$\Sigma y^2 = 1446$	$\Sigma xy = 582$

Gateway Classes : 7455960612 84

$$r_{xy} = \frac{n\Sigma xy - \Sigma x \Sigma y}{\sqrt{n\Sigma x^2 - (\Sigma x)^2}\sqrt{n\Sigma y^2 - (\Sigma y)^2}}$$

$$r_{xy} = \frac{6 \times 582 - 34 \times 90}{\sqrt{6 \times 248 - (34)^2}\sqrt{6 \times 1446 - (90)^2}}$$

$$r_{xy} = 0.9879$$

Gateway Classes : 7455 9612 84

Q. 4:- *A computer while calculating correlation co – efficient between two varibales X and Y from 25 pairs of observations obtained the following results :*

$n = 25, \quad \sum X = 125, \quad \sum X^2 = 650$

$\sum Y = 100, \quad \sum Y^2 = 460, \quad \sum XY = 508.$

It was, however, later discoveredе at the time of checking that he had copied down two pairs as

X	Y
6	14
8	6

while the correct values were

X	Y
8	12
6	8

Obtain the correct value of correlation co – efficient

$n = 25$

Corrected values

$\Sigma X = 125 - 6 - 8 + 8 + 6 = 125$

$\Sigma Y = 100 - 14 - 6 + 12 + 8 = 100$

$\Sigma x^2 = 650 - 36 - 64 + 64 + 36$

$\Sigma x^2 = 650$

$\Sigma y^2 = 460 - 196 - 36 + 144 + 64$

$\Sigma y^2 = 436$

Gateway Classes 7455 961284

$$\Sigma XY = 508 - 6\times14 - 8\times6 + 8\times12 + 6\times8$$

$$\Sigma XY = 520$$

$$r_{xy} = \frac{n\Sigma XY - \Sigma X \Sigma Y}{\sqrt{n\Sigma X^2 - (\Sigma X)^2}\sqrt{n\Sigma Y^2 - (\Sigma Y)^2}}$$

$$r_{xy} = \frac{25\times520 - 125\times100}{\sqrt{25\times650 - (125)^2}\sqrt{25\times436 - (100)^2}}$$

$$\boxed{r_{xy} = 0.67}$$

Gateway Classes : 7455 961284

Q. 5:- Find the coefficient of correlation for the following table: **(UPTU – 2011)**

x:	10	14	18	22	26	30
y:	18	12	24	6	30	36

x	y	$d_x = x-22$	$d_y = y-24$	$u = \frac{x-22}{4}$	$v = \frac{y-24}{6}$	u^2	v^2	uv
10	18	−12	−6	−3	−1	9	1	3
14	12	−8	−12	−2	−2	4	4	4
18	24	−4	0	−1	0	1	0	0
22	6	0	−18	0	−3	0	9	0
26	30	4	6	1	1	1	1	1
30	36	8	12	2	2	4	4	4
$\Sigma x =$ 120	$\Sigma y =$ 126			$\Sigma u = -3$	$\Sigma v = -3$	$\Sigma u^2 = 19$	$\Sigma v^2 =$ 19	$\Sigma uv =$ 12

Gateway Classes : 7455 9612 84

$n = 6$

$$r_{xy} = \frac{n\Sigma uv - \Sigma u \Sigma v}{\sqrt{n\Sigma u^2 - (\Sigma u)^2}\sqrt{n\Sigma v^2 - (\Sigma v)^2}}$$

$$r_{xy} = \frac{6 \times 12 - (-3)(-3)}{\sqrt{6 \times 19 - (-3)^2}\sqrt{6 \times 19 - (-3)^2}}$$

$$r_{xy} = \frac{72 - 9}{\sqrt{105} \times \sqrt{105}} = \frac{63}{105}$$

$$r_{xy} = 0.6$$

Gateway Classes : 7455 961284

Topic : Correlation and Coefficient of Correlation

Q.1 *Find the correlation coefficient between x and y for the following data:*

x	60	34	40	50	45	41	22	43
y	75	32	34	40	45	33	12	30

Q.2 *Calculate the coefficient of correlation between the following ages of husband* (x) *and wife* (y) *by taking* 30 *and* 28 *as assumed mean in case of x and y respectively:*

x:	24	27	28	28	29	30	32	33	35	35	40
y:	18	20	22	25	22	28	28	30	27	30	32

Gateway Classes: 7455 95 12 84

ENGINEERING MATHEMATICS

UNIT-3 : *Statistical Techniques – I*

Lec-10

Today's Target

- Rank Correlation
- PYQ
- DPP

Gateway Classes : 7455 9612 84

Rank Correlation

- Non Repeated Rank
- Repeated Rank

Rank Correlation (Non Repeated Rank)

$$r = 1 - \frac{6\sum D^2}{n(n^2-1)}$$

Spearman's Formula

Where n = number of observation

$$D = R_1 - R_2$$

r = Rank Correlation coefficient

Gateway Classes : 7455 9612 84

Rank Correlation (Repeated Rank)

$$r = 1 - \frac{6[\sum D^2 + F]}{n(n^2-1)}$$

$$Where\ F = \frac{m_1(m_1^2-1)}{12} + \frac{m_2(m_2^2-1)}{12} + \cdots \ldots$$

Gateway Classes : 7455 9612 84

Type – 1 : Where Rank is given

Q. 1 Compute the rank correlation coefficient for the following data:

Person:	A	B	C	D	E	F	G	H	I	J
Rank in Maths: (R_1)	9	10	6	5	7	2	4	8	1	3
Rank in Physics: (R_2)	1	2	3	4	5	6	7	8	9	10

Person	R_1	R_2	$D = R_1 - R_2$	D^2
A	9	1	8	64
B	10	2	8	64
C	6	3	3	9
D	5	4	1	1
E	7	5	2	4
F	2	6	-4	16
G	4	7	-3	9
H	8	8	0	0
I	1	9	-8	64
J	3	10	-7	49
				$\Sigma D^2 = 280$

Coefficient of Rank correlation

$$\gamma = 1 - \frac{6\Sigma D^2}{n(n^2-1)}$$

$$= 1 - \frac{6 \times 280}{10 \times 99}$$

$$\gamma = -0.6969$$

Gateway Classes : 7455 961284

Q. 2 *Ten competitors in a beauty contest were ranked by three judges in the following orders:*

	A	B	C	D	E	F	G	H	I	J
First Judge:	1	6	5	10	3	2	4	9	7	8
Second Judge:	3	5	8	4	7	10	2	1	6	9
Third Judge:	6	4	9	8	1	2	3	10	5	7

Use the method of rank correlation to determine which pair of judges has the nearest approach to common taste in beauty ?

$$r_{12} = 1 - \frac{6\sum D_{12}^2}{n(n^2-1)}$$

$$r_{12} = 1 - \frac{6 \times 20}{10 \times 99}$$

$$r_{12} = -0.212$$

$$r_{13} = 1 - \frac{6\sum D_{13}^2}{n(n^2-1)}$$

$$= 1 - \frac{6 \times 60}{10 \times 99}$$

$$r_{13} = 0.636$$

$$r_{23} = 1 - \frac{6\sum D_{23}^2}{n(n^2-1)}$$

$$= 1 - \frac{6 \times 214}{10 \times 99}$$

$$r_{23} = -0.296$$

Hence, First and third judges has the nearest approach.

Gateway Classes 7255 96128 84

competitors	R_1	R_2	R_3	$D_{12} = R_1 - R_2$	$D_{13} = R_1 - R_3$	$D_{23} = R_2 - R_3$	D_{12}^2	D_{13}^2	D_{23}^2
A	1	3	6	−2	−5	−3	4	25	9
B	6	5	4	1	2	1	1	4	1
C	5	8	9	−3	−4	−1	9	16	1
D	10	4	8	6	2	−4	36	4	16
E	3	7	1	−4	2	6	16	4	36
F	2	10	2	−8	0	8	64	0	64
G	4	2	3	2	1	−1	4	1	1
H	9	1	10	8	−1	−9	64	1	81
I	7	6	5	1	2	1	1	4	1
J	8	9	7	−1	1	2	1	1	4
							$\Sigma D_{12}^2 = 200$	$\Sigma D_{13}^2 = 60$	$\Sigma D_{23}^2 = 214$

Gateway Classes : 7455 961284

Type – 2 : Where Rank is not given

Q. 3 Ten students got the following perentage of marks in Chemistry and Physics :

Students:	1	2	3	4	5	6	7	8	9	10
Marks in Chemistry	78	36	98	25	75	82	90	62	65	39
Marks in Physics	84	51	91	60	68	62	86	58	63	47

Calculate the rank correlation co-efficient.

$n = 10$

$\Sigma D^2 = 26$

$$r = 1 - \frac{6\Sigma D^2}{n(n^2-1)}$$

$$r = 1 - \frac{6 \times 26}{10 \times 99}$$

$$r = 0.84$$

Gateway Classes : 7455 961284

Students	MARKS (chem)	R_1	MARKS (Phy)	R_2	$D = R_1 - R_2$	D^2
1	78	4	84	3	1	1
2	36	9	51	9	0	0
3	98	1	91	1	0	0
4	25	10	60	7	3	9
5	75	5	68	4	1	1
6	82	3	62	6	-3	9
7	90	2	86	2	0	0
8	62	7	58	8	-1	1
9	65	6	63	5	1	1
10	39	8	47	10	-2	4
						$\Sigma D^2 = 26$

Gateway Classes . 7455 9612 84

Q.4 ***Find the rank correlation coefficient of marks of A and B from the following data:*** **(AKTU-2016)**

Marks A:	15	20	27	13	45	60	20	75
Marks B:	50	30	55	30	25	10	30	70

Students	MARKS A	R_1	MARKS B	R_2	$D = R_1 - R_2$	D^2
1	15	7	50	3	4	16
2	20(5)	5.5	30(4)	5	0.5	0.25
3	27	4	55	2	2	4
4	13	8	30(5)	5	3	9
5	45	3	25	7	−4	16
6	60	2	10	8	−6	36
7	20(6)	5.5	30(6)	5	0.5	0.25
8	75	1	70	1	0	0
						$\Sigma D^2 = 81.5$

$$\frac{5+6}{2} = \frac{11}{2} = 5.5$$

$$\frac{4+5+6}{3} = \frac{15}{3} = 5$$

$20 \longrightarrow 2$

$m_1 = 2$

$30 \longrightarrow 3$

$m_2 = 3$

Gateway Classes : 7455 961284

coefficient of rank correlation

$$\gamma = 1 - \frac{6(\Sigma D^2 + F)}{n(n^2-1)} \quad \text{---- (1)}$$

$$F = \frac{m_1(m_1^2-1)}{12} + \frac{m_2(m_2^2-1)}{12}$$

$$F = \frac{2(4-1)}{12} + \frac{3(9-1)}{12}$$

$$F = \frac{\cancel{6}}{\cancel{12}_2} + \frac{\cancel{24}^2}{\cancel{12}}$$

$$F = 2.5$$

From (1)

$$\gamma = 1 - \frac{6(81.5+2.5)}{8(64-1)}$$

$$\gamma = 1 - \frac{6 \times 84}{8 \times 63}$$

$$\gamma = 1 - 1 = 0$$

$$\boxed{\gamma = 0}$$

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Q.1 *Obtain the rank correlation co – efficient for the following data*

X	68	64	75	50	64	80	75	40	55	64
Y	62	58	68	45	81	60	68	48	50	70

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ENGINEERING MATHEMATICS

UNIT-3 : *Statistical Techniques – I*

Lec-11

Today's Target

- ***Regression Lines***
- ***(i) Line of Regression of y on x***
- ***(ii) Line of Regression of x on y***
- PYQ
- DPP

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Regression Analysis

- Regression means functional relationship between two or more related variables.
- If two varibales are correlated then there exists a relation between them.
- Regression Analysis is a process of estimating relationship between dependent variable and independent variables.

Linear Regression

When points in scatter diagram are concentrated around a straight line then it is Linear Regression otherwise non − linear regression.

There are two lines of Regression

(i) Line of Regression of y on x

(ii) Line of Regression of x on y

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(1) Line of Regression of y on x

$$y - \bar{y} = b_{yx}(x - \bar{x})$$

$$b_{yx} = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - (\sum x)^2}$$

OR

$$b_{yx} = r\frac{\sigma y}{\sigma x}$$

Where

b_{yx} = Regression coefficient of y on x

Note: Regression lines are always passes through the point $(\bar{x}, \bar{y})$

Equation of a line passing through a given point (x_1, y_1) and given slope (m)

$$y - y_1 = m(x - x_1)$$

$$2x + 4y = 11$$

$$4y = -2x + 11$$

$$y = -\frac{1}{2}x + \frac{11}{4}$$

$$m = -\frac{1}{2}$$

$$b_{yx} = -\frac{1}{2}$$

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2) Line of Regression of x on y

$$x - \bar{x} = b_{xy}(y - \bar{y})$$

$$b_{xy} = \frac{n\sum xy - \sum x \sum y}{n\sum y^2 - (\sum y)^2}$$

OR

$$b_{xy} = r\frac{\sigma x}{\sigma y}$$

Where

b_{xy} = Regression coefficient of x on y

Note: Regression lines are always passes through the point $(\bar{x}, \bar{y})$

Regression line

$2x + 4y = 11$

$2x = -4y + 11$

$x = -2y + \frac{11}{2}$

$b_{xy} = -2$

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Relation between regression Coefficient and corelation Coefficient

We know that

$$b_{yx} = \frac{r\,\sigma_y}{\sigma_x} \quad \text{---} \quad (1)$$

$$b_{xy} = \frac{r\,\sigma_x}{\sigma_y} \quad \text{---} \quad (2)$$

Multiply (1) and (2)

$$b_{yx} \times b_{xy} = r\frac{\sigma_y}{\sigma_x} \times r\frac{\sigma_x}{\sigma_y}$$

$$r^2 = b_{yx} \times b_{xy}$$

$$\boxed{r = \sqrt{b_{yx} \times b_{xy}}}$$

r is the Geometric mean of b_{yx} and b_{xy}

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Q.1 *If the regression coefficients are 0.8 and 0.2, what would be the value of coefficient of correlation?*

AKTU-2021

Given

$b_{yx} = 0.8$

$b_{xy} = 0.2$

We know that

$$r = \sqrt{b_{yx} \times b_{xy}}$$

$$r = \sqrt{0.8 \times 0.2}$$

$$r = \sqrt{0.16}$$

$$r = 0.4$$

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Q. 2 *Find the coefficient of correlation and obtain the equation to the lines of regression for the following data:*

AKTU-2017

x:	6	2	10	4	8
y:	9	11	5	8	7

x	y	x^2	y^2	xy
6	9	36	81	54
2	11	4	121	22
10	5	100	25	50
4	8	16	64	32
8	7	64	49	56
$\Sigma x = 30$	$\Sigma y = 40$	$\Sigma x^2 = 220$	$\Sigma y^2 = 340$	$\Sigma xy = 214$

Regression coefficient

$$b_{yx} = \frac{n\Sigma xy - \Sigma x \Sigma y}{n\Sigma x^2 - (\Sigma x)^2}$$

$$b_{yx} = \frac{5 \times 214 - 30 \times 40}{5 \times 220 - (30)^2}$$

$$b_{yx} = \frac{-130}{200}$$

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$$b_{yx} = -0.65$$

$$b_{xy} = \frac{n\Sigma xy - \Sigma x \Sigma y}{n\Sigma y^2 - (\Sigma y)^2}$$

$$b_{xy} = \frac{-130}{100}$$

$$b_{xy} = -1.3$$

We know that

$$r = \sqrt{b_{yx} \times b_{xy}}$$

$$r = \sqrt{0.65 \times 1.3}$$

$$r = -0.9192$$

coefficient of correlation

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Mean

$$\bar{x} = \frac{\Sigma x}{n}$$

$$= \frac{30}{5} \quad 6$$

$$\boxed{\bar{x} = 6}$$

$$\bar{y} = \frac{\Sigma y}{n}$$

$$\bar{y} = \frac{40}{5} \quad 8$$

$$\boxed{\bar{y} = 8}$$

Line of regression of y on x

$$(y - \bar{y}) = b_{yx}(x - \bar{x})$$

$$y - 8 = -0.65(x - 6)$$

$$y - 8 = -0.65x + 3.90$$

$$\boxed{y = -0.65x + 11.9}$$

Line of regression of x on y

$$(x - \bar{x}) = b_{xy}(y - \bar{y})$$

$$x - 6 = -1.3(y - 8)$$

$$x - 6 = -1.3y + 10.4$$

$$\boxed{x = -1.3y + 16.4}$$

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Q.3 *The following table gives age (x) in years of cars and annual maintenance cost (y) in hundred rupes:*

AKTU-2021

X:	1	~~2~~ 3	5	7	9
Y:	15	18	21	23	22

Estimate the maintenance cost for a 4 year old car after finding the regression equation.

x	y	x^2	xy
1	15	1	15
3	18	9	54
5	21	25	105
7	23	49	161
9	22	81	198
$\Sigma x = 25$	$\Sigma y = 99$	$\Sigma x^2 = 165$	$\Sigma xy = 533$

Regression coefficient

$$b_{yx} = \frac{n\Sigma xy - \Sigma x \Sigma y}{n\Sigma x^2 - (\Sigma x)^2}$$

$$b_{yx} = \frac{5 \times 533 - 99 \times 25}{5 \times 165 - (25)^2}$$

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$$b_{yx} = 0.95$$

Mean

$$\bar{x} = \frac{\Sigma x}{n}$$

$$= \frac{25}{5}$$

$$= 5$$

$$\bar{x} = 5$$

$$\bar{y} = \frac{\Sigma y}{n}$$

$$\bar{y} = \frac{99}{5}$$

$$\bar{y} = 19.8$$

Regression line of y on x

$$(y - \bar{y}) = b_{yx}(x - \bar{x})$$

$$y - 19.8 = 0.95(x - 5)$$

$$y - 19.8 = 0.95x - 4.75$$

$$y = 0.95x + 15.05$$

Maintenance cost

$$x = 4$$

$$y = 0.95 \times 4 + 15.05$$

$$y = 18.85$$

$$y = 1885 \text{ Rs}$$

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Q.4 *The regression equations calculated from a given set of observations for two random variables are*

AKTU-2016

$$x = -0.4y + 6.4 \text{ and } y = -0.6x + 4.6.$$

Calculate (i) $\bar{x}$ (ii) $\bar{y}$ (ii) r

Regression lines

$$x = -0.4y + 6.4 \quad \text{—— (1)}$$

$$y = -0.6x + 4.6 \quad \text{—— (2)}$$

From (1)

$$b_{xy} = -0.4$$

From (2)

$$b_{yx} = -0.6$$

We know that

$$r = \sqrt{0.6 \times 0.4}$$

$$\boxed{r = 0.48989}$$

coeffiuent of correlation

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Since Regression lines always passing Through $(\bar{x}, \bar{y})$

From ①

$$\bar{x} = -0.4\bar{y} + 6.4$$

$$\bar{x} + 0.4\bar{y} = 6.4 \quad \text{—③}$$

From ②

$$\bar{y} = -0.6\bar{x} + 4.6$$

$$0.6\bar{x} + \bar{y} = 4.6 \quad \text{—④}$$

Solve ③ and ④

$$\bar{x} = 6$$

$$\bar{y} = 1$$

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Q.5:– *In a partially destroyed laboratory record of an analysis of a correlation data, the following results only are legible:*

AKTU-2018

Variance of $x = 9$

Regression equations: $8x - 10y + 66 = 0, 40x - 18y = 214.$

What were (a) *the mean values of* x *and* y (b) *the standard deviation of* y *and the ceofficient of correlation between* x *and* y *?*

Given

$\sigma_x^2 = 9$

$\sigma_x = 3$

Regression lines

$8x - 10y = -66$ — (1)

$40x - 18y = 214$ — (2)

Regression lines always passing through $(\bar{x}, \bar{y})$

From (1) and (2)

$8\bar{x} - 10\bar{y} = -66$ — (3)

$40\bar{x} - 18\bar{y} = 214$ — (4)

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Solve ③ and ④

$$\bar{x} = 13$$

$$\bar{y} = 17$$

From ① and ②

$$8x - 10y = -66$$

$$-10y = -8x - 66$$

$$y = 0.8x + 6.6$$

$$b_{yx} = 0.8$$

$$40x - 18y = 214$$

$$40x = 18y + 214$$

$$x = \frac{18}{40}y + 214$$

$$x = 0.45y + 214$$

$$b_{xy} = 0.45$$

We know that

$$r = \sqrt{b_{yx} \times b_{xy}}$$

$$r = \sqrt{0.8 \times 0.45}$$

$$r = 0.6$$

Also

$$b_{yx} = r\frac{\sigma_y}{\sigma_x}$$

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$$0.8 = \frac{0.6\,\sigma_y}{3}$$

$$\sigma_y = \frac{0.8 \times 3}{0.6_2}$$

$$\sigma_y = \frac{8}{2}$$

$$\boxed{\sigma_y = 4}$$

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DPP- 11

Topic : *Regression Lines*

Q. 1 *From the data given, find the equation of lines of regression of x on y and y on x. Also, calculate the correlation co − efficient.*

x:	2	4	6	8	10
y:	5	7	9	8	11

Q. 2 *For two random variables, x and y with the same mean, the two regression equations are* $y = ax + b$ *and* $x = \alpha y + \beta$. *Show that* $\frac{b}{\beta} = \frac{1-a}{1-\alpha}$.

Find also the common mean. (AKTU – 2017)

$\bar{x} = \bar{y} = m$

Q. 3 *The following data regarding the heights* (y) *and weights* (x) *of 100 college students are given:* (AKTU – 2018)

$$\sum x = 15000, \quad \sum x^2 = 2272500, \quad \sum y = 6800, \quad \sum y^2 = 463025 \text{ and } \sum xy = 1022250$$

Find the equation of regression line of height on weight.

Q. 4 *The equations of two regression lines, obtained in a correlation analysis of 60 observations are:*

$$5x = 6y + 24 \text{ and } 1000y = 768x - 3608.$$

What is the correlation coefficient? Show that the ratio of coefficient of variablity of x to that of y is $\frac{5}{24}$. *What is the ratio of x and y.* (AKTU-2019)

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