Maths - IV

DPP - 6

Topic - Non Linear Partial Differential Equation (Some Standard Form)

Q-1 : Solve $p+q = pq$

Sol : $p+q = pq$ — (1)

Replace p by a and q by b

$a+b = ab$ — (2)

$a = ab-b$

$a = b(a-1)$

$b = \frac{a}{a-1}$

Complete solution is

$z = ax+by+c$

$$\boxed{z = ax + \frac{a}{a-1}y + c}$$

Ans

Q-2 : Solve the partial differential equation : $p^2+q^2 = 2$

Sol : $p^2+q^2 = 2$

Replace p by a and q by b

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$a^2 + b^2 = 2$ — (2)

$b^2 = 2 - a^2$

$b = \sqrt{2-a^2}$

Complete solution is - $z = ax + by + c$

$$\boxed{z = ax + \sqrt{2-a^2}\, y + c}$$

Ans

Q-3 : Solve the partial differential eq $z = px + qy + \sin(p+q)$

$z = px + qy + \sin(p+q)$ — (1)

Replace p by a and q by b

$$\boxed{z = ax + by + \sin(a+b)}$$

Ans

Q-4 : Solve: $9(p^2z + q^2) = 4$

$9(p^2z + q^2) = 4$ — (1)

Let $u = x + ay$ so that

$p = \frac{dz}{du}$ and $q = a\frac{dz}{du}$

$$9\left[\left(\frac{dz}{du}\right)^2 z + a^2\left(\frac{dz}{du}\right)^2\right] = 4$$

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$$9\left(\frac{dz}{du}\right)^2\left[z+a^2\right]=4$$

$$\frac{9}{4}\left(\frac{dz}{du}\right)^2=\frac{1}{z+a^2}$$

Square Root Both Side

$$\frac{3}{2}\left(\frac{dz}{du}\right)=\frac{1}{\sqrt{z+a^2}}$$

$$\frac{3}{2}\left(z+a^2\right)^{1/2}dz=du$$

Integrate both side

$$\frac{3}{2}\int\left(z+a^2\right)^{1/2}dz=\int du$$

$$\frac{3}{2}\frac{\left(z+a^2\right)^{3/2}}{3/2}=u+c$$

$$\frac{\cancel{3}}{\cancel{2}}\times\frac{\cancel{2}}{\cancel{3}}\left(z+a^2\right)^{3/2}=u+c$$

$$\left(z+a^2\right)^{3/2}=u+c$$

Squaring both side and put value of $u=x+ay$

$$\boxed{\left(z+a^2\right)^3=\left(x+ay+c\right)^2}$$

Ans

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Q-5: Solve : $p - 3x^2 = q^2 - y$

Let $p - 3x^2 = q^2 - y = a$ — (1)

$p - 3x^2 = a$

$p = a + 3x^2$

$f_1(x) = a + 3x^2$

$q^2 - y = a$

$q^2 = a + y$

$q = \sqrt{a+y}$

$f_2(y) = \sqrt{y+a}$

We know that

$dz = p\,dx + q\,dy$

Put p and q in above equation

$dz = (a+3x^2)\,dx + \sqrt{y+a}\,dy$

Integrate both side

$z = \int (a+3x^2)\,dx + \int (y+a)^{1/2}\,dy$

$$\boxed{z = ax + x^3 + \frac{2}{3}(y+a)^{3/2} + c}$$

Ans

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