DPP - 8 Maths - IV Unit - 3

Topic - Curve Fitting

Q-1- Given the following experimental values:

| | | | | |
|---|---|---|---|---|
| $x$: | 0 | 1 | 2 | 3 |
| $y$: | 2 | 4 | 10 | 15 |

Find by the method of Least squares a parabola of the type $y = a + bx^2$.

Sol- $y = a + bx^2$ — (1)

Normal equations-

$$\Sigma y = na + b\Sigma x^2 \quad — (1)$$

$$\Sigma x^2 y = a\Sigma x^2 + b\Sigma x^4 \quad — (2)$$

| $x$ | $y$ | $x^2$ | $x^2y$ | $x^4$ |
|---|---|---|---|---|
| 0 | 2 | 0 | 0 | 0 |
| 1 | 4 | 1 | 4 | 1 |
| 2 | 10 | 4 | 40 | 16 |
| 3 | 15 | 9 | 135 | 81 |
| $\Sigma x = 6$ | $\Sigma y = 31$ | $\Sigma x^2 = 14$ | $\Sigma x^2 y = 179$ | $\Sigma x^4 = 98$ |

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$n = 4$

Put values of $\Sigma x^2$, $\Sigma y$, $\Sigma x^2 y$ and $\Sigma x^4$ in normal eq:-

$$31 = 4a + 14b \quad \text{— (4)}$$

$$179 = 14a + 98b \quad \text{— (5)}$$

On solving eq (4) and eq (5) we get,

$$a = 2.71, \quad b = 1.43$$

Hence the required curve is:-

$$\boxed{y = 2.71 + 1.43x^2}$$

Q-2:- Using method of least squares, derive the normal equations to fit the cure $y = ax^2 + bx$, Hence fit this curve to following data:

| $x$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| $y$ | 1 | 1.2 | 1.8 | 2.5 | 3.6 | 4.7 | 6.6 | 9.1 |

$$y = ax^2 + bx \quad \text{— (1)}$$

Normal equations:-

$$\Sigma xy = a\Sigma x^3 + b\Sigma x^2$$

$$\Sigma x^2 y = a\Sigma x^4 + b\Sigma x^3$$

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| $x$ | $y$ | $xy$ | $x^2$ | $x^2y$ | $x^3$ | $x^4$ |
|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | 1.2 | 2.4 | 4 | 4.8 | 8 | 16 |
| 3 | 1.8 | 5.4 | 9 | 16.2 | 27 | 81 |
| 4 | 2.5 | 10 | 16 | 40 | 64 | 256 |
| 5 | 3.6 | 18 | 25 | 90 | 125 | 625 |
| 6 | 4.7 | 28.2 | 36 | 169.2 | 216 | 1296 |
| 7 | 6.6 | 46.2 | 49 | 323.4 | 343 | 2401 |
| 8 | 9.1 | 72.8 | 64 | 582.4 | 512 | 4096 |
| | $\Sigma y = 30.5$ | $\Sigma xy = 184$ | $\Sigma x^2 = 204$ | $\Sigma x^2y = 1227$ | $\Sigma x^3 = 1296$ | $\Sigma x^4 = 8772$ |

Put values in normal equations -

$$184 = 1296a + 204b \quad - (4)$$

$$1227 = 8772a + 1296b \quad - (5)$$

On solving eq (4) and (5)

$$a = 0.1077 \qquad b = 0.2171$$

Required curve is

$$\boxed{y = 0.1077x^2 + 0.2171x}$$

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Q-3 : Fit the curve $pv^r = k$ to the following data

| $p$ (kg/cm²) | 0.5 | 1 | 1.5 | 2 | 2.5 | 3 |
|---|---|---|---|---|---|---|
| $v$ (liters) | 1620 | 1000 | 750 | 620 | 520 | 460 |

Sol -

$$pv^r = k \quad \text{— (1)}$$

$$p = \frac{k}{v^r}$$

$$p = kv^{-r}$$

Taking log both side

$$\log p = \log k - r \log v$$

$Y \quad A \quad BX$

$$\boxed{Y = A + BX}$$

Normal equations

$$\Sigma Y = nA + B\Sigma X \quad \text{— (2)}$$

$$\Sigma XY = A\Sigma X + B\Sigma X^2 \quad \text{— (3)}$$

$$n = 6$$

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| $v$ | $p$ | $X = \log v$ | $Y = \log p$ | $XY$ | $X^2$ |
|---|---|---|---|---|---|
| 1620 | 0.5 | 3.209 | −0.301 | −0.965 | 0.25 |
| 1000 | 1 | 3 | 0 | 0 | 9 |
| 750 | 1.5 | 2.87 | 0.176 | 0.505 | 8.23 |
| 620 | 2 | 2.79 | 0.301 | 0.839 | 7.78 |
| 520 | 2.5 | 2.71 | 0.397 | 1.075 | 7.34 |
| 460 | 3 | 2.66 | 0.477 | 1.268 | 7.07 |
| | | $\Sigma X = 17.239$ | $\Sigma Y = 1.05$ | $\Sigma XY = 2.722$ | $\Sigma X^2 = 39.67$ |

Put values in normal equation -

$1.05 = 6A + 17.23B$ — (4)

$2.722 = 17.23A + 39.67B$ — (5)

On solving eq (4) and (5)

$A = 0.089$ $\quad B = 0.029$

$\log k = 0.089$

$k = \text{antilog}(0.089)$

$k = 1.22$

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$-\gamma = 0.029$

$$\boxed{\gamma = -0.029}$$

Put $k$ and $\gamma$ in the curve.

$$\boxed{PV^{-0.029} = 1.22}$$

Ans

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