# ENGINEERING MATHEMATICS

## UNIT-5 : *Statistical Techniques – III*

***(Testing a Hypothesis and Statistical Quality Control)***

## *Lec-01*

**Today's Target**

- *Basic terms of Hypothesis testing*

  *(i) Population*

  *(ii) Sample*

  *(iii) Parameter and Statistics*

  *(iv) Hypothesis: Null Hypothesis and Alternative Hypothesis*

  *(v) Test of significance*

  *(vi) Level of significance*

Gateway Classes : 7455961284

# ENGINEERING MATHEMATICS

## UNIT-5 : Statistical Techniques – III

### Aktu Syllabus

**Testing a Hypothesis**

- Introduction of sampling theory, Hypothesis, Null hypothesis, Alternative Hypothesis, Testing a Hypothesis, Level of Significance, Confidence limit,
- Test of significance of difference of means : (i) t – test (ii) Z – test (iii) Chi-square test 14 Marks

**Statistical Quality Control** 7 Marks

- Control charts
- Control charts for variables (X and R charts)
- Control charts for variables (p, np and C charts)

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## Basic terms of Hypothesis testing

**(i) Population or Universe:**

*Any collection of individual | members |objects | set of all experimental data*




**(ii) SAMPLE : A Finite subset of population**

- *To study population, we select a random sample in such a way that each member of population has equal chance to include in sample*
- **Sample Size** : *The number of member | objects |individual in a sample*
- **Sampling** : *Process of selecting a sample*

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## (iii) Parameter and Statistics: Symbols for Population and Samples

| | Parameter | Statistic |
|---|---|---|
| S.NO. | Measures used for Population | Measures used for Sample |
| 1 | Population Size $(N)$ | Sample Size $(n)$ |
| 2 | Population Mean $(\mu)$ | Sample Mean $(\bar{x})$ |
| 3 | Population S. D $(\sigma)$ | Sample S. D $(s)$ |
| 4 | Population Variance $(\sigma^2)$ | Sample Variance $(s^2)$ |
| 5 | PopulationProportion $(P)$ | Sample Proportion $(p)$ |

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## (iv) Hypothesis / Statistical Hypotheses

**Hypotheses** → Assumptions required to make decisions about population on the basis of sample informatiion

**Test of Hypotheses** → Deciding a hypotheses is to be accepted or rejected

### Null Hypothesis ($H_o$)

- It is a definite statement about population parameter
- Tested under the assumption that it is true

### Alternative Hypotheses($H_1$)

It is complementary to the null hypothesis

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**Note:** (1) $(H_0)$ : *Accept* *then* $(H_1)$ : *Reject*

(2) $(H_0)$ : *Reject* *then* $(H_1)$ : *Accept*

***Test of significance*** *(or Null Hypothesis testing)*

*The tests which enable us, to decide whether the Null Hypothesis is eccepted or rejected on the basis of sample information*

(*i*) $t$ − *test*

(*ii*) $Z$ − *test*

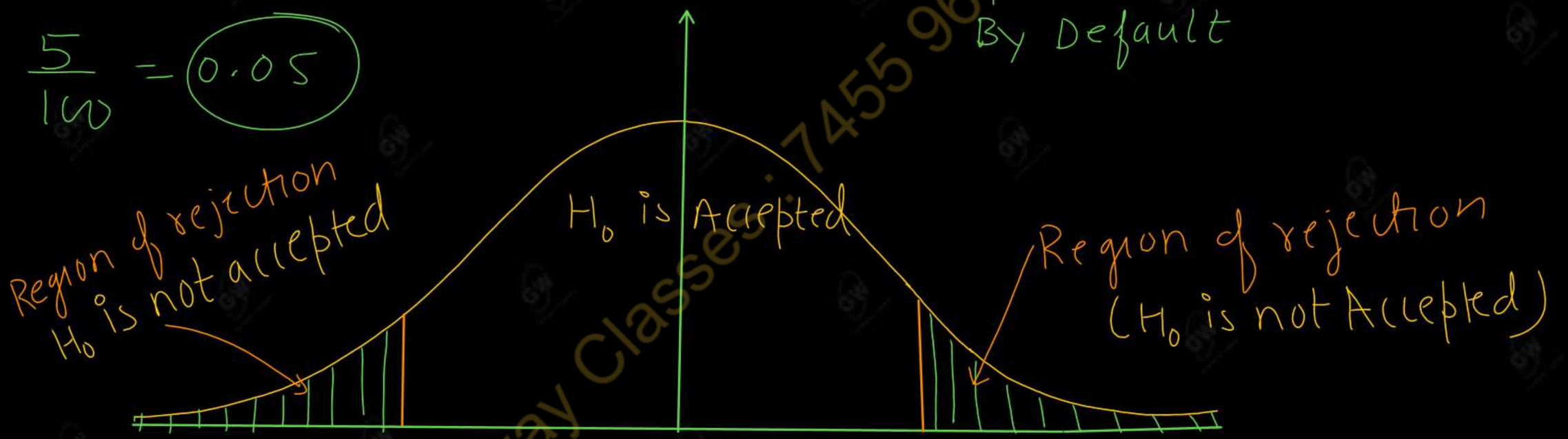
(*iii*) $F$ − *test*

(*iv*) *chi,* −*square test*

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## Level of significance ($\alpha$)

- The value of probability above which we do not reject the Null Hypothesis.
- Usualiy, level of significance in testing of hypothesis are 5 % , 1%




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# ENGINEERING MATHEMATICS

## UNIT-5 : *Statistical Techniques – III*

## (*Testing a Hypothesis and Statistical Quality Control)*

### *Lec-02*

**Today's Target**

- $t - test\ for\ one\ sample$
- PYQ
- DPP

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## *Parameter and Statistics:* Symbols for Population and Samples

| | Parameter | Statistic |
|---|---|---|
| S.NO. | Measures used for Population known as Parameter | Measures used for Sample known as statistic |
| 1 | Population Size $(N)$ | Sample Size $(n)$ |
| 2 | Population Mean $(\mu)$ | Sample Mean $(\bar{x})$ |
| 3 | Population S. D $(\sigma)$ | Sample S. D $(s)$ |
| 4 | Population Variance $(\sigma^2)$ | Sample Variance $(s^2)$ |
| 5 | PopulationProportion $(P)$ | Sample Proportion $(p)$ |

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# Hypothesis Testing

## Test of significance

(i) $t$ – test

(ii) $z$ – test

(iii) $F$ – test

(iv) chi – square test

7 M

7 M

**$t$ – test**

**OR**

**Students $t$ – distribution**

- It is a parametric test of hypothesis testing based on student's $T$ distribution.
- To test the significance of the mean of a sample.
- $t$ – test is applicable when

(i) sample size is small ($n \leq 30$)

(ii) Population standard deviation ($\sigma$) is not known

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$t-test$

- For one sample ✓
- For two sample — Next Lecture

## $t-test$ for one sample

- To test the significant difference between the sample mean ($\bar{x}$) and population mean ($\mu$).

**Steps to solve Questions**

**STEP – 1 : Set Null Hypothesis ($H_0$)**

$H_0$ : There is no significant difference between sample mean and population mean

$H_0 : \mu = \mu_0$ (given)

**STEP – 2 : Alternative Hypothesis ($H_1$)**

$H_1 : \mu \neq \mu_0$ (Two tailed test)

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## Step – 3 : Calculate $|t|$

### (i) When S.D is not given

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

$$t = \left(\frac{\bar{x} - \mu}{s}\right)\sqrt{n}$$

$\bar{x}$ = Sample mean

$\mu$ = Population mean

$n$ = Sample size

$$S = \text{Sample S.D} = \sqrt{\frac{\Sigma(x - \bar{x})^2}{n-1}}$$

### (ii) When S.D is given

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n-1}}}$$

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**Step – 4 :** Write degree of freedom (n-1) and level of significance $\propto$ ($\propto = 5\%$ $OR$ $\propto = 1\%$)

By default

**Step – 5:** From the $t-$ test table

Find the value of $t_{\propto}$ at the level of significance $\propto$ for the degree of freedom $(n-1)$

**Step – 6: Conclusion**

**(i)** If $|t| < t_{\propto}$ then $H_0$ is accepted

- There is no significant difference between sample mean and population mean
- Sample have been taken from correct population.

**(ii)** If $|t| > t_{\propto}$ then $H_0$ is rejected

- There is significant difference between sample mean and population mean
- Sample could not have come from this population.

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## *Fiducial limits or confidence limits*

$$\bar{x} - t_{\propto}\frac{s}{n} < \mu < \bar{x} + t_{\propto}\frac{s}{n}$$

*(i)* **95%** *confidence limits (level of significance is* **5%***) are* $\bar{x} \pm t_{0.05}\frac{s}{n}$

*(i)* **99%** *confidence limits (level of significance is* **1%***) are* $\bar{x} \pm t_{0.01}\frac{s}{n}$

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**Q.1** *A sample of 18 items has a mean 24 units and standard deviation 3 units. Test the hypothesis that it is a random sample from a normal population with mean 27 units.*

## Given

$n = 18$

$\bar{x} = 24$

$s = 3$

$\mu = 27$

## Null Hypothesis ($H_0$)

$H_0$ : ——

$H_0 : \mu = 27$

## Alternative Hypothesis ($H_1$)

$H_1 : \mu \neq 27$ (two tailed test

## Calculation of |t|

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n-1}}}$$

$$t = \frac{24 - 27}{\frac{3}{\sqrt{18-1}}}$$

$$t = -\frac{3}{3} \times \sqrt{17}$$

$$t = -\sqrt{17} = -4.123$$

$$|t| = 4.123$$

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Degree of freedom = $n-1$

$n-1 = 18-1$

$= 17$

Level of significance

$\alpha = 5\% = \frac{5}{100}$

$\alpha = 0.05$

From the t-test table

$$t_{0.05} = 2.11$$

## conclusion

$|t| = 4.123$ and $t_{0.05} = 2.11$

$\because |t| > t_{0.05}$

$\therefore H_0$ Hypothesis is rejected

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## Table 2 : SIGNIFICANT VALUES $t_\nu(\alpha)$ OF t-DISTRIBUTION (TWO TAIL AREAS) $[|t| > t_\nu(\alpha)] = \alpha$

| d.f. (ν) | Probability (Level of Significance) | | | | | |
|---|---|---|---|---|---|---|
| | 0.50 | 0.10 | 0.05 | 0.02 | 0.01 | 0.001 |
| 1 | 1.00 | 6.31 | 12.71 | 31.82 | 63.66 | 636.62 |
| 2 | 0.82 | 0.92 | 4.30 | 6.97 | 6.93 | 31.60 |
| 3 | 0.77 | 2.32 | 3.18 | 4.54 | 5.84 | 12.94 |
| 4 | 0.74 | 2.13 | 2.78 | 3.75 | 4.60 | 8.61 |
| 5 | 0.73 | 2.02 | 2.57 | 3.37 | 4.03 | 6.86 |
| 6 | 0.72 | 1.94 | 2.45 | 3.14 | 3.71 | 5.96 |
| 7 | 0.71 | 1.90 | 2.37 | 3.00 | 3.50 | 5.41 |
| 8 | 0.71 | 1.80 | 2.31 | 2.90 | 3.36 | 5.04 |
| 9 | 0.70 | 1.83 | 2.26 | 2.82 | 3.25 | 4.78 |
| 10 | 0.70 | 1.81 | 2.23 | 2.76 | 3.17 | 4.59 |
| 11 | 0.70 | 1.80 | 2.20 | 2.72 | 3.11 | 4.44 |
| 12 | 0.70 | 1.78 | 2.18 | 2.68 | 3.06 | 4.32 |
| 13 | 0.69 | 1.77 | 2.16 | 2.05 | 3.01 | 4.22 |
| 14 | 0.69 | 1.76 | 2.15 | 2.62 | 2.98 | 4.14 |
| 15 | 0.69 | 1.75 | 2.13 | 2.60 | 2.95 | 4.07 |
| 16 | 0.69 | 1.75 | 2.12 | 2.58 | 2.92 | 4.02 |
| 17 | 0.69 | 1.74 | 2.11 | 2.57 | 2.90 | 3.97 |
| 18 | 0.69 | 1.73 | 2.10 | 2.55 | 2.88 | 3.92 |
| 19 | 0.69 | 1.73 | 2.09 | 2.54 | 2.86 | 3.88 |
| 20 | 0.69 | 1.73 | 2.09 | 2.53 | 2.85 | 3.85 |
| 21 | 0.69 | 1.72 | 2.08 | 2.52 | 2.83 | 3.83 |
| 22 | 0.69 | 1.72 | 2.07 | 2.51 | 2.42 | 3.79 |
| 23 | 0.69 | 1.71 | 2.07 | 2.50 | 2.81 | 3.77 |
| 24 | 0.69 | 1.71 | 2.06 | 2.49 | 2.80 | 3.75 |
| 25 | 0.68 | 1.71 | 2.06 | 2.49 | 2.79 | 3.73 |
| 26 | 0.68 | 1.71 | 2.06 | 2.48 | 2.78 | 3.71 |
| 27 | 0.68 | 1.70 | 2.05 | 2.47 | 2.77 | 3.69 |
| 28 | 0.68 | 1.70 | 2.05 | 2.47 | 2.76 | 3.67 |
| 29 | 0.68 | 1.70 | 2.05 | 2.46 | 2.76 | 3.66 |
| 30 | 0.68 | 1.70 | 2.04 | 2.46 | 2.75 | 3.65 |
| ∞ | 0.67 | 1.65 | 1.96 | 2.33 | 2.58 | 3.29 |

$t_{0.05} = 2.11$

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**Q. 2** *The 9 items of a sample have the following values:* $x$: 45, 47, 50, 52, 48, 47, 49, 53, 51.

*Does the mean of these values differ significantly from the assumed mean 47.5 ?*

**AKTU – 2021**

Given

$n = 9$

$\mu = 47.5$

| $x$ | $(x-\bar{x})$ | $(x-\bar{x})^2$ |
|---|---|---|
| 45 | −4.1 | 16.81 |
| 47 | −2.1 | 4.41 |
| 50 | 0.9 | 0.81 |
| 52 | 2.9 | 8.41 |
| 48 | −1.1 | 1.21 |
| 47 | −2.1 | 4.41 |
| 49 | −0.1 | 0.001 |
| 53 | 3.9 | 15.21 |
| 51 | 1.9 | 3.61 |
| $\Sigma x = 442$ | | $\Sigma(x-\bar{x})^2 =$ | 54.89 |

$$\bar{x} = \frac{\Sigma x}{n}$$

$$\bar{x} = \frac{442}{9}$$

$$\boxed{\bar{x} = 49.1}$$

$$\Sigma(x-\bar{x})^2 = 54.89$$

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$$S = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n-1}}$$

$$S = \sqrt{\frac{54.89}{8}}$$

$$\boxed{S = 2.619}$$

Null Hypothesis ($H_0$)

$H_0$ : _____

$H_0 : \mu = 47.5$

Alternative Hypothesis ($H_1$)

$H_1 : \mu \neq 47.5$ (Two tailed test)

calculation of $|t|$

$$t = \frac{\bar{x} - \mu}{\frac{S}{\sqrt{n}}}$$

$$= \frac{49.1 - 57.5}{\frac{2.619}{\sqrt{9}}}$$

$$t = -1.8327$$

$$\boxed{|t| = 1.8327}$$

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Degree of freedom

$n-1 = 9-1$

$= 8$

Level of significance

$\alpha = 5\% = \frac{5}{100}$

$\alpha = 0.05$

From t-test table

$t_{0.05} = 2.31$

Conclusion

$|t| = 1.8327$ and $t_{0.05} = 2.31$

$\because |t| < t_{0.05}$

$\therefore H_0$ Hypothesis accepted

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$t_{0.05} = 2.31$

## Table 2 : SIGNIFICANT VALUES $t_\nu(\alpha)$ OF t-DISTRIBUTION (TWO TAIL AREAS) $[|t| > t_\nu(\alpha)] = \alpha$

| d.f. (ν) | Probability (Level of Significance) | | | | | |
|---|---|---|---|---|---|---|
| | 0.50 | 0.10 | 0.05 | 0.02 | 0.01 | 0.001 |
| 1 | 1.00 | 6.31 | 12.71 | 31.82 | 63.66 | 636.62 |
| 2 | 0.82 | 0.92 | 4.30 | 6.97 | 6.93 | 31.60 |
| 3 | 0.77 | 2.32 | 3.18 | 4.54 | 5.84 | 12.94 |
| 4 | 0.74 | 2.13 | 2.78 | 3.75 | 4.60 | 8.61 |
| 5 | 0.73 | 2.02 | 2.57 | 3.37 | 4.03 | 6.86 |
| 6 | 0.72 | 1.94 | 2.45 | 3.14 | 3.71 | 5.96 |
| 7 | 0.71 | 1.90 | 2.37 | 3.00 | 3.50 | 5.41 |
| 8 | 0.71 | 1.80 | 2.31 | 2.90 | 3.36 | 5.04 |
| 9 | 0.70 | 1.83 | 2.26 | 2.82 | 3.25 | 4.78 |
| 10 | 0.70 | 1.81 | 2.23 | 2.76 | 3.17 | 4.59 |
| 11 | 0.70 | 1.80 | 2.20 | 2.72 | 3.11 | 4.44 |
| 12 | 0.70 | 1.78 | 2.18 | 2.68 | 3.06 | 4.32 |
| 13 | 0.69 | 1.77 | 2.16 | 2.05 | 3.01 | 4.22 |
| 14 | 0.69 | 1.76 | 2.15 | 2.62 | 2.98 | 4.14 |
| 15 | 0.69 | 1.75 | 2.13 | 2.60 | 2.95 | 4.07 |
| 16 | 0.69 | 1.75 | 2.12 | 2.58 | 2.92 | 4.02 |
| 17 | 0.69 | 1.74 | 2.11 | 2.57 | 2.90 | 3.97 |
| 18 | 0.69 | 1.73 | 2.10 | 2.55 | 2.88 | 3.92 |
| 19 | 0.69 | 1.73 | 2.09 | 2.54 | 2.86 | 3.88 |
| 20 | 0.69 | 1.73 | 2.09 | 2.53 | 2.85 | 3.85 |
| 21 | 0.69 | 1.72 | 2.08 | 2.52 | 2.83 | 3.83 |
| 22 | 0.69 | 1.72 | 2.07 | 2.51 | 2.42 | 3.79 |
| 23 | 0.69 | 1.71 | 2.07 | 2.50 | 2.81 | 3.77 |
| 24 | 0.69 | 1.71 | 2.06 | 2.49 | 2.80 | 3.75 |
| 25 | 0.68 | 1.71 | 2.06 | 2.49 | 2.79 | 3.73 |
| 26 | 0.68 | 1.71 | 2.06 | 2.48 | 2.78 | 3.71 |
| 27 | 0.68 | 1.70 | 2.05 | 2.47 | 2.77 | 3.69 |
| 28 | 0.68 | 1.70 | 2.05 | 2.47 | 2.76 | 3.67 |
| 29 | 0.68 | 1.70 | 2.05 | 2.46 | 2.76 | 3.66 |
| 30 | 0.68 | 1.70 | 2.04 | 2.46 | 2.75 | 3.65 |
| ∞ | 0.67 | 1.65 | 1.96 | 2.33 | 2.58 | 3.29 |

Gateway Classes, 7415 961 284

**Q.3** *The lifetime of electric bulbs for a random sample of 10 from a large consignment gave the following data:*

| *Item* | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| *Life in 000hrs.* | 4.2 | 4.6 | 3.9 | 4.1 | 5.2 | 3.8 | 3.9 | 4.3 | 4.4 | 5.6 |

*Can we accept the hypothesis that the average lifetime of bulb is 4000 hrs? a*

Given

$n = 10$

$\mu = 4000$

$\mu = 4$

| $x$ | $(x-\bar{x})$ | $(x-\bar{x})^2$ |
|---|---|---|
| 4.2 | −0.2 | 0.04 |
| 4.6 | 0.2 | 0.04 |
| 3.9 | −0.5 | 0.25 |
| 4.1 | −0.3 | 0.09 |
| 5.2 | 0.8 | 0.64 |
| 3.8 | −0.6 | 0.36 |
| 3.9 | −0.5 | 0.25 |
| 4.3 | −0.1 | 0.01 |
| 4.4 | 0 | 0 |
| 5.6 | 1.2 | 1.44 |
| $\Sigma x = 44$ | | $\Sigma(x-\bar{x})^2 = 3.12$ |

$$\bar{x} = \frac{\Sigma x}{n} = \frac{44}{10}$$

$$\boxed{\bar{x} = 4.4}$$

$$s = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n-1}}$$

$$= \sqrt{\frac{3.12}{9}}$$

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$s = 0.589$

Null Hypothesis ($H_0$)

$H_0$ : ——

$H_0 : \mu = 4000$

Alternative Hypothesis ($H_1$)

$H_1 : \mu \neq 4000$ (Two tailed test)

calculation of $|t|$

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{4.4 - 4}{\frac{0.589}{10}}$$

$t = 2.123$

$|t| = 2.123$

Remaning step → HW

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$t_{0.05} = 2.26$

## Table 2 : SIGNIFICANT VALUES $t_\nu(\alpha)$ OF t-DISTRIBUTION (TWO TAIL AREAS) $[|t| > t_\nu(\alpha)] = \alpha$

| d.f. (ν) | Probability (Level of Significance) | | | | | |
|---|---|---|---|---|---|---|
| | 0.50 | 0.10 | 0.05 | 0.02 | 0.01 | 0.001 |
| 1 | 1.00 | 6.31 | 12.71 | 31.82 | 63.66 | 636.62 |
| 2 | 0.82 | 0.92 | 4.30 | 6.97 | 6.93 | 31.60 |
| 3 | 0.77 | 2.32 | 3.18 | 4.54 | 5.84 | 12.94 |
| 4 | 0.74 | 2.13 | 2.78 | 3.75 | 4.60 | 8.61 |
| 5 | 0.73 | 2.02 | 2.57 | 3.37 | 4.03 | 6.86 |
| 6 | 0.72 | 1.94 | 2.45 | 3.14 | 3.71 | 5.96 |
| 7 | 0.71 | 1.90 | 2.37 | 3.00 | 3.50 | 5.41 |
| 8 | 0.71 | 1.80 | 2.31 | 2.90 | 3.36 | 5.04 |
| 9 | 0.70 | 1.83 | 2.26 | 2.82 | 3.25 | 4.78 |
| 10 | 0.70 | 1.81 | 2.23 | 2.76 | 3.17 | 4.59 |
| 11 | 0.70 | 1.80 | 2.20 | 2.72 | 3.11 | 4.44 |
| 12 | 0.70 | 1.78 | 2.18 | 2.68 | 3.06 | 4.32 |
| 13 | 0.69 | 1.77 | 2.16 | 2.05 | 3.01 | 4.22 |
| 14 | 0.69 | 1.76 | 2.15 | 2.62 | 2.98 | 4.14 |
| 15 | 0.69 | 1.75 | 2.13 | 2.60 | 2.95 | 4.07 |
| 16 | 0.69 | 1.75 | 2.12 | 2.58 | 2.92 | 4.02 |
| 17 | 0.69 | 1.74 | 2.11 | 2.57 | 2.90 | 3.97 |
| 18 | 0.69 | 1.73 | 2.10 | 2.55 | 2.88 | 3.92 |
| 19 | 0.69 | 1.73 | 2.09 | 2.54 | 2.86 | 3.88 |
| 20 | 0.69 | 1.73 | 2.09 | 2.53 | 2.85 | 3.85 |
| 21 | 0.69 | 1.72 | 2.08 | 2.52 | 2.83 | 3.83 |
| 22 | 0.69 | 1.72 | 2.07 | 2.51 | 2.42 | 3.79 |
| 23 | 0.69 | 1.71 | 2.07 | 2.50 | 2.81 | 3.77 |
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| ∞ | 0.67 | 1.65 | 1.96 | 2.33 | 2.58 | 3.29 |

Gateway Classes, 7415 961 284

**Q.4** *A random sample of size 16 has 53 as mean. The sum of squares of the deviation from mean is 135. Can this sample be regarded as taken from the population having 56 as mean? Obtain 95% and 99% confidence limits of the mean of the population.*

Given

$n = 16$

$\bar{x} = 53$

$\Sigma(x-\bar{x})^2 = 135$

$\mu = 56$

$$s = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n-1}}$$

$$s = \sqrt{\frac{135}{16-1}}$$

$$s = \sqrt{9}$$

$$\boxed{s = 3}$$

Null Hypothesis ($H_0$)

$H_0$ :

$H_0 : \mu = 56$

Alternative Hypothesis ($H_1$)

$H_1 : \mu \neq 56$ (Two tailed test)

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## calculation of |t|

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

$$t = \frac{53 - 56}{\frac{3}{\sqrt{16}}}$$

$$t = \frac{-3}{\frac{3}{\sqrt{16}}}$$

$$t = -4$$

$$\boxed{|t| = 4}$$

## Degree of freedom

$$n - 1 = 16 - 1$$

$$= 15$$

## Level of significance

$$\alpha = 5\% = \frac{5}{100}$$

$$\alpha = 0.05$$

From t-test table

$$\boxed{t_{0.05} = 2.13}$$

## conclusion

$|t| = 4$ and $t_{0.05} = 2.13$

$\therefore |t| > t_{0.05}$

$\therefore H_0$ Hypothesis is rejected

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When level of significance is 1%

$\alpha = 1\% = \frac{1}{100} = 0.01$

$\alpha = 0.01$

$t_{0.01} = 2.95$

confidence limit of mean at 95%

$= \bar{x} \pm t_{0.05} \frac{s}{n} = 53 \pm 2.13 \times \frac{3}{16}$

$= 51.4025 , 54.5975$

confidence limit of mean at 99%

$= \bar{x} \pm t_{0.01} \frac{s}{n} = 53 \pm 2.95 \times \frac{3}{16}$

$= 50.7875 , 55.2125$

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GATEWAY CLASSES

$t_{0.05} = 2.13$

## Table 2 : SIGNIFICANT VALUES $t_\nu(\alpha)$ OF t-DISTRIBUTION (TWO TAIL AREAS) $[|t| > t_\nu(\alpha)] = \alpha$

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| | 0.50 | 0.10 | 0.05 | 0.02 | 0.01 | 0.001 |
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| 2 | 0.82 | 0.92 | 4.30 | 6.97 | 6.93 | 31.60 |
| 3 | 0.77 | 2.32 | 3.18 | 4.54 | 5.84 | 12.94 |
| 4 | 0.74 | 2.13 | 2.78 | 3.75 | 4.60 | 8.61 |
| 5 | 0.73 | 2.02 | 2.57 | 3.37 | 4.03 | 6.86 |
| 6 | 0.72 | 1.94 | 2.45 | 3.14 | 3.71 | 5.96 |
| 7 | 0.71 | 1.90 | 2.37 | 3.00 | 3.50 | 5.41 |
| 8 | 0.71 | 1.80 | 2.31 | 2.90 | 3.36 | 5.04 |
| 9 | 0.70 | 1.83 | 2.26 | 2.82 | 3.25 | 4.78 |
| 10 | 0.70 | 1.81 | 2.23 | 2.76 | 3.17 | 4.59 |
| 11 | 0.70 | 1.80 | 2.20 | 2.72 | 3.11 | 4.44 |
| 12 | 0.70 | 1.78 | 2.18 | 2.68 | 3.06 | 4.32 |
| 13 | 0.69 | 1.77 | 2.16 | 2.05 | 3.01 | 4.22 |
| 14 | 0.69 | 1.76 | 2.15 | 2.62 | 2.98 | 4.14 |
| 15 | 0.69 | 1.75 | 2.13 | 2.60 | 2.95 | 4.07 |
| 16 | 0.69 | 1.75 | 2.12 | 2.58 | 2.92 | 4.02 |
| 17 | 0.69 | 1.74 | 2.11 | 2.57 | 2.90 | 3.97 |
| 18 | 0.69 | 1.73 | 2.10 | 2.55 | 2.88 | 3.92 |
| 19 | 0.69 | 1.73 | 2.09 | 2.54 | 2.86 | 3.88 |
| 20 | 0.69 | 1.73 | 2.09 | 2.53 | 2.85 | 3.85 |
| 21 | 0.69 | 1.72 | 2.08 | 2.52 | 2.83 | 3.83 |
| 22 | 0.69 | 1.72 | 2.07 | 2.51 | 2.42 | 3.79 |
| 23 | 0.69 | 1.71 | 2.07 | 2.50 | 2.81 | 3.77 |
| 24 | 0.69 | 1.71 | 2.06 | 2.49 | 2.80 | 3.75 |
| 25 | 0.68 | 1.71 | 2.06 | 2.49 | 2.79 | 3.73 |
| 26 | 0.68 | 1.71 | 2.06 | 2.48 | 2.78 | 3.71 |
| 27 | 0.68 | 1.70 | 2.05 | 2.47 | 2.77 | 3.69 |
| 28 | 0.68 | 1.70 | 2.05 | 2.47 | 2.76 | 3.67 |
| 29 | 0.68 | 1.70 | 2.05 | 2.46 | 2.76 | 3.66 |
| 30 | 0.68 | 1.70 | 2.04 | 2.46 | 2.75 | 3.65 |
| ∞ | 0.67 | 1.65 | 1.96 | 2.33 | 2.58 | 3.29 |

Gateway Classes, 7498 961 284

## UNIT-5 : DPP- 02

**Topic :** $t-test\ for\ one\ sample$

**Q. 1** *A sample of 20 items has mean 42 units and S. D. 5 units. Test the hypothesis that it is a random sample from a normal population with mean 45 units.*

**Q. 2** *Ten individuals are chosen at random from a normal population of students and their marks are found to be 63, 63, 66, 67, 68, 69, 70, 70, 71, 71. In the light of these data, discuss the that mean mark of the population of students is 66.*

Gateway Classes : 7455 9612 84

# ENGINEERING MATHEMATICS

## UNIT-5 : *Statistical Techniques – III*

## *(Testing a Hypothesis and Statistical Quality Control)*

### *Lec-03*

**Today's Target**

- $t - test\ for\ two\ sample$
- PYQ
- DPP

Gateway Classes :7455 561284

## $t$ – test for two sample

- To test the significant difference between mean of two independent samples.
- Let two independent samples are sample – 1 and sample – 2

| Sample – 1 | Sample – 2 |
|---|---|
| (i) $x_1, x_2, x_3 \dots \dots x_n$ | (i) $y_1, y_2, y_3 \dots \dots y_n$ |
| (ii) Mean $\bar{x} = \frac{\Sigma x}{n_1}$ | (ii) Mean $\bar{y} = \frac{\Sigma y}{n_2}$ |
| (iii) $S_1 = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n_1-1}}$ | (iii) $S_2 = \sqrt{\frac{\Sigma(y-\bar{y})^2}{n_2-1}}$ |
| (iv) Size $n_1 \leq 30$ | (iv) Size $n_2 \leq 30$ |
| (v) Population mean $\mu_1$ | (v) Population mean $\mu_2$ |

Gateway Classes : 7455 961284

## Step to sovle Questions

**STEP – 1: Set Null Hypothesis ($H_0$)**

$H_0$: The means of the two Population are same

$H_0: \mu_1 = \mu_2$

**STEP – 2: Alternative Hypothesis($H_1$)**

$H_1: \mu_1 \neq \mu_2$

**STEP – 3: Calculate | t |**

$$t = \frac{\bar{x} - \bar{y}}{s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

where $\bar{x}$ = Mean of Sample-1

$\bar{y}$ = Mean of Sample-2

$n_1$ = Size of Sample-1

$n_1$ = Size of sample-2

$S_1 = S.D\ of\ sample - 1$

$S_2 = S.D\ of\ sample - 2$

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(i) If S.D $S_1$ and $S_2$ are given

$$S = \sqrt{\frac{n_1 S_1^2 + n_2 S_2^2}{n_1 + n_2 - 2}}$$

(ii) If S.D $S_1$ and $S_2$ are not given

$$S = \sqrt{\frac{\Sigma(x-\bar{x})^2 + \Sigma(y-\bar{y})^2}{n_1 + n_2 - 2}}$$

**STEP – 4**: Write degree of freedom $(n_1 + n_2 - 2)$ and level of significance $\propto$ $(\propto = 5\%\ OR \propto = 1\%)$

By Default

**STEP – 5** : From the t – test table

Find the value of $t_{\propto}$ at level of significance for degree of freedom $(n_1 + n_2 - 2)$

Gateway Classes : 7455 9612 84

**STEP – 6: Conclusion**

**(i)** If $|t| < t_{\alpha}$ then $H_0$ is accepted

There is no significant difference between ~~between~~ their means

**(ii)** If $|t| > t_{\alpha}$ then $H_0$ is rejected

There is significant difference between ~~between~~ their means

Gateway Classes : 7455 96 12 84

**Q. 1** *Samples of sizes 10 and 14 were taken from two normal populations with S.D. 3.5 and 5.2 The sample means were found to be 20.3 and 18.6. Test whether the means of the two populations are the same at 5% level.*

Given

$n_1 = 10$

$n_2 = 14$

$s_1 = 3.5$

$s_2 = 5.2$

$\bar{x} = 20.3$

$\bar{y} = 18.6$

We know that

$$s = \sqrt{\frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}}$$

$$s = \sqrt{\frac{10 \times (3.5)^2 + 14 \times (5.2)^2}{10 + 14 - 2}}$$

$$s = 4.772$$

Null Hypothesis ($H_0$)

$H_0$ :

$H_0 : \mu_1 = \mu_2$

Alternative Hypothesis ($H_1$)

$H_1 : \mu_1 \neq \mu_2$

Gateway Classes 7455 96 12 84

Calculation of |t|

$$t = \frac{\bar{x} - \bar{y}}{s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$$= \frac{20.3 - 18.6}{4.772\sqrt{\frac{1}{10} + \frac{1}{14}}}$$

$$t = 0.8604$$

$$|t| = 0.8604$$

Degree of freedom $= n_1 + n_2 - 2$

$= 10 + 14 - 2$

$= 22$

Level of significance ($\alpha$)

$$\alpha = 5\% = \frac{5}{100} = 0.05$$

$$\alpha = 0.05$$

From t-test table

$$t_{0.05} = 2.07$$

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Gateway Classes 7455 961284

## conclusion

$|t| = 0.8604$ and $t_{0.05} = 2.07$

$\because |t| < t_{0.05}$

$\therefore H_0$ Hypothesis is accepted

Gateway Classes : 7455 9612 84

$t_{0.05} = 2.07$

## Table 2 : SIGNIFICANT VALUES $t_\nu(\alpha)$ OF t-DISTRIBUTION (TWO TAIL AREAS) $[|t| > t_\nu(\alpha)] = \alpha$

| d.f. (ν) | Probability (Level of Significance) | | | | | |
|---|---|---|---|---|---|---|
| | 0.50 | 0.10 | 0.05 | 0.02 | 0.01 | 0.001 |
| 1 | 1.00 | 6.31 | 12.71 | 31.82 | 63.66 | 636.62 |
| 2 | 0.82 | 0.92 | 4.30 | 6.97 | 6.93 | 31.60 |
| 3 | 0.77 | 2.32 | 3.18 | 4.54 | 5.84 | 12.94 |
| 4 | 0.74 | 2.13 | 2.78 | 3.75 | 4.60 | 8.61 |
| 5 | 0.73 | 2.02 | 2.57 | 3.37 | 4.03 | 6.86 |
| 6 | 0.72 | 1.94 | 2.45 | 3.14 | 3.71 | 5.96 |
| 7 | 0.71 | 1.90 | 2.37 | 3.00 | 3.50 | 5.41 |
| 8 | 0.71 | 1.80 | 2.31 | 2.90 | 3.36 | 5.04 |
| 9 | 0.70 | 1.83 | 2.26 | 2.82 | 3.25 | 4.78 |
| 10 | 0.70 | 1.81 | 2.23 | 2.76 | 3.17 | 4.59 |
| 11 | 0.70 | 1.80 | 2.20 | 2.72 | 3.11 | 4.44 |
| 12 | 0.70 | 1.78 | 2.18 | 2.68 | 3.06 | 4.32 |
| 13 | 0.69 | 1.77 | 2.16 | 2.05 | 3.01 | 4.22 |
| 14 | 0.69 | 1.76 | 2.15 | 2.62 | 2.98 | 4.14 |
| 15 | 0.69 | 1.75 | 2.13 | 2.60 | 2.95 | 4.07 |
| 16 | 0.69 | 1.75 | 2.12 | 2.58 | 2.92 | 4.02 |
| 17 | 0.69 | 1.74 | 2.11 | 2.57 | 2.90 | 3.97 |
| 18 | 0.69 | 1.73 | 2.10 | 2.55 | 2.88 | 3.92 |
| 19 | 0.69 | 1.73 | 2.09 | 2.54 | 2.86 | 3.88 |
| 20 | 0.69 | 1.73 | 2.09 | 2.53 | 2.85 | 3.85 |
| 21 | 0.69 | 1.72 | 2.08 | 2.52 | 2.83 | 3.83 |
| 22 | 0.69 | 1.72 | 2.07 | 2.51 | 2.42 | 3.79 |
| 23 | 0.69 | 1.71 | 2.07 | 2.50 | 2.81 | 3.77 |
| 24 | 0.69 | 1.71 | 2.06 | 2.49 | 2.80 | 3.75 |
| 25 | 0.68 | 1.71 | 2.06 | 2.49 | 2.79 | 3.73 |
| 26 | 0.68 | 1.71 | 2.06 | 2.48 | 2.78 | 3.71 |
| 27 | 0.68 | 1.70 | 2.05 | 2.47 | 2.77 | 3.69 |
| 28 | 0.68 | 1.70 | 2.05 | 2.47 | 2.76 | 3.67 |
| 29 | 0.68 | 1.70 | 2.05 | 2.46 | 2.76 | 3.66 |
| 30 | 0.68 | 1.70 | 2.04 | 2.46 | 2.75 | 3.65 |
| ∞ | 0.67 | 1.65 | 1.96 | 2.33 | 2.58 | 3.29 |

Gateway Classes, 7415 961 284

**Q.2** *The height of 6 randomly chosen sailors in inches are 63, 65, 68, 69, 71 and 72. Those of 9 randomly chosen soldiers are 61, 62, 65, 66, 69, 70, 71, 72 and 73. Test whether the sailors are on the average taller than soldiers.*

Given

Sample-1 → Height of Sailors

Sample-2 → Height of Soldiers

$n_1 = 6$

$n_2 = 9$

| $x$ | $x-\bar{x}$ | $(x-\bar{x})^2$ | $y$ | $y-\bar{y}$ | $(y-\bar{y})^2$ |
|---|---|---|---|---|---|
| 63 | −5 | 25 | 61 | −6.67 | 44.4889 |
| 65 | −3 | 9 | 62 | −5.67 | 32.1489 |
| 68 | 0 | 0 | 65 | −2.67 | 7.1289 |
| 69 | 1 | 1 | 66 | −1.67 | 2.7889 |
| 71 | 3 | 9 | 69 | 1.33 | 1.7689 |
| 72 | 4 | 16 | 70 | 2.33 | 5.4289 |
| | | | 71 | 3.33 | 11.0889 |
| | | | 72 | 4.33 | 18.7489 |
| | | | 73 | 5.33 | 28.4089 |
| $\Sigma x =$ 408 | | $\Sigma(x-\bar{x})^2 = 60$ | $\Sigma y =$ 609 | | $\Sigma(y-\bar{y})^2 =$ 151.9995 |

Gateway Classes 7455961284

## Mean

$$\bar{x} = \frac{\Sigma x}{n_1} = \frac{408}{6}$$

$$\boxed{\bar{x} = 68}$$

$$\bar{y} = \frac{\Sigma y}{n_2}$$

$$\bar{y} = \frac{609}{9}$$

$$\boxed{\bar{y} = 67.67}$$

We know that

$$s = \sqrt{\frac{\Sigma(x-\bar{x})^2 + \Sigma(y-\bar{y})^2}{n_1 + n_2 - 2}}$$

$$s = \sqrt{\frac{60 + 151.9995}{6+9-2}}$$

$$\boxed{s = 4.038}$$

Null Hypothesis ($H_0$)

$H_0$: _____

$H_0: \mu_1 = \mu_2$

Calculation of $|t|$

$$t = \frac{\bar{x} - \bar{y}}{s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$$t = \frac{68 - 67.67}{4.038\sqrt{\frac{1}{6} + \frac{1}{9}}}$$

$$\boxed{|t| = 0.150\overline{5}}$$

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GatewayClasses : 7455961284

Degree of freedom $= n_1 + n_2 - 2$

$= 6 + 9 - 2$

$= (13)$

Level of significance $\alpha$

$\alpha = 5\% = \frac{5}{100} = 0.05$

$\alpha = 0.05$

From the t-test table

$$t_{0.05} = 2.16$$

conclusion

$|t| = 0.1505$ and $t_{0.05} = 2.16$

$\because |t| < t_{0.05}$

$\therefore H_0$ Hypothesis is accepted

Means of the two population are same

Hence,

Average Height of Sailors is not taller than soldiers

GW GATEWAY CLASSES
Gateway Classes: 7455 961284

GW GATEWAY CLASSES

$t_{0.05} = 2.16$

## Table 2 : SIGNIFICANT VALUES $t_\nu(\alpha)$ OF t-DISTRIBUTION (TWO TAIL AREAS) $[|t| > t_\nu(\alpha)] = \alpha$

| d.f. (ν) | Probability (Level of Significance) | | | | | |
|---|---|---|---|---|---|---|
| | 0.50 | 0.10 | 0.05 | 0.02 | 0.01 | 0.001 |
| 1 | 1.00 | 6.31 | 12.71 | 31.82 | 63.66 | 636.62 |
| 2 | 0.82 | 0.92 | 4.30 | 6.97 | 6.93 | 31.60 |
| 3 | 0.77 | 2.32 | 3.18 | 4.54 | 5.84 | 12.94 |
| 4 | 0.74 | 2.13 | 2.78 | 3.75 | 4.60 | 8.61 |
| 5 | 0.73 | 2.02 | 2.57 | 3.37 | 4.03 | 6.86 |
| 6 | 0.72 | 1.94 | 2.45 | 3.14 | 3.71 | 5.96 |
| 7 | 0.71 | 1.90 | 2.37 | 3.00 | 3.50 | 5.41 |
| 8 | 0.71 | 1.80 | 2.31 | 2.90 | 3.36 | 5.04 |
| 9 | 0.70 | 1.83 | 2.26 | 2.82 | 3.25 | 4.78 |
| 10 | 0.70 | 1.81 | 2.23 | 2.76 | 3.17 | 4.59 |
| 11 | 0.70 | 1.80 | 2.20 | 2.72 | 3.11 | 4.44 |
| 12 | 0.70 | 1.78 | 2.18 | 2.68 | 3.06 | 4.32 |
| 13 | 0.69 | 1.77 | 2.16 | 2.05 | 3.01 | 4.22 |
| 14 | 0.69 | 1.76 | 2.15 | 2.62 | 2.98 | 4.14 |
| 15 | 0.69 | 1.75 | 2.13 | 2.60 | 2.95 | 4.07 |
| 16 | 0.69 | 1.75 | 2.12 | 2.58 | 2.92 | 4.02 |
| 17 | 0.69 | 1.74 | 2.11 | 2.57 | 2.90 | 3.97 |
| 18 | 0.69 | 1.73 | 2.10 | 2.55 | 2.88 | 3.92 |
| 19 | 0.69 | 1.73 | 2.09 | 2.54 | 2.86 | 3.88 |
| 20 | 0.69 | 1.73 | 2.09 | 2.53 | 2.85 | 3.85 |
| 21 | 0.69 | 1.72 | 2.08 | 2.52 | 2.83 | 3.83 |
| 22 | 0.69 | 1.72 | 2.07 | 2.51 | 2.42 | 3.79 |
| 23 | 0.69 | 1.71 | 2.07 | 2.50 | 2.81 | 3.77 |
| 24 | 0.69 | 1.71 | 2.06 | 2.49 | 2.80 | 3.75 |
| 25 | 0.68 | 1.71 | 2.06 | 2.49 | 2.79 | 3.73 |
| 26 | 0.68 | 1.71 | 2.06 | 2.48 | 2.78 | 3.71 |
| 27 | 0.68 | 1.70 | 2.05 | 2.47 | 2.77 | 3.69 |
| 28 | 0.68 | 1.70 | 2.05 | 2.47 | 2.76 | 3.67 |
| 29 | 0.68 | 1.70 | 2.05 | 2.46 | 2.76 | 3.66 |
| 30 | 0.68 | 1.70 | 2.04 | 2.46 | 2.75 | 3.65 |
| ∞ | 0.67 | 1.65 | 1.96 | 2.33 | 2.58 | 3.29 |

Gateway Classes 7415 961 84

**Q.1** ***Two samples of sodium vapour bulbs were tested for length of life and the following results were got:***

| | Size | Sample Mean | Sample S.D. |
|---|---|---|---|
| Type I | 8 | 1234 hrs | 36 hrs |
| Type II | 7 | 1036 hrs | 40 hrs |

***Is the difference in the means significant to generalise that Type I is superior to Type II regarding length of life?***

**Q.2** ***The marks obtained by a group of 9 regular course students and another group of 11 part time course students in a test are given below:***

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| *Regular*: | 56 | 62 | 63 | 54 | 60 | 51 | 67 | 69 | 58 | | |
| *Part Time*: | 62 | 70 | 71 | 62 | 60 | 56 | 75 | 64 | 72 | 68 | 66 |

***Examine whether the marks obtained by regular students and part time students differ significantly at 5% and 1% level of significance.***

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# ENGINEERING MATHEMATICS

## UNIT-5 : *Statistical Techniques – III*

*(Testing a Hypothesis and Statistical Quality Control)*

*Lec-04*

**Today's Target**

- $Z-test$
- PYQ
- DPP

Gateway Classes :7455 561284

# Hypothesis Testing

## Test of significance

(i) $t$ – test

(ii) $z$ – test

(iii) $F$ – test

(iv) chi – square test

## Z–test is applicable when

| | Population S.D ($\sigma$) is Known | Population S.D ($\sigma$) is Unknown |
|---|---|---|
| $n \leq 30$ | $z$ – test | $t$ – test v. imp |
| $n > 30$ | $z$ – test v. imp | $z$ – test |

Gateway Classes : 7455 9612 84

# Z–test

- For one sample
- For two sample

## Z–test for one sample

*To test the significant difference between the sample mean and population mean.*

### Steps to solve Questions

**STEP – 1 : Set Null Hypothesis ($H_0$)**

$H_0$ : There is no significant difference between sample mean and population mean

$H_0 : \mu = \mu_0$ (given)

**STEP – 2 : Alternative Hypothesis ($H_1$)**

$H_1 : \mu \neq \mu_0$ (Two tailed test)

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## Step – 3 : Calculate Z

$$Z_{cal} = \frac{\bar{x} - \mu}{\frac{S}{\sqrt{n}}} \quad OR \quad Z_{cal} = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

$\bar{x}$ = Sample mean

$\mu$ = Population mean

$n$ = Sample size

$\sigma$ = Population S.D

$S$ = Sample S.D

**Step – 4 : Write level of significance** $\propto$ ($\propto = 5\%$ **OR** $\propto = 1\%$) ↓ By Default

$Z_{tab} = 1.96$

$\propto = 5\%$

$\frac{100-5}{2}\% = 47.5\% = 0.475$

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**Step – 5: From the Z – test table**

**Find the tabulated value $Z_{tab}$ at the level of significance $\propto = 5\%$**

**Step – 6: Conclusion**

**(i) If $Z_{cal} < Z_{tab}$ then $H_0$ is accepted**

- **There is no significant difference between sample mean and population mean**
- **Sample have been taken from correct population.**

**(ii) If $Z_{cal} > Z_{tab}$ then $H_0$ is rejected**

- **There is significant difference between sample mean and population mean**
- **Sample could not have come from this population.**

Gateway Classes : 7455 9612 84

# Z–test for two sample

**To test the significant difference between mean of two independent samples.**

## Step to sovle Questions

**STEP – 1: Set Null Hypothesis ($H_0$)**

$H_0$: The means of the two Population are same

$H_0: \mu_1 = \mu_2$

**STEP – 2: Alternative Hypothesis($H_1$)**

$H_1: \mu_1 \neq \mu_2$

**STEP – 3: Calculate $|Z|$**

$$Z_{cal} = \frac{\bar{x}-\bar{y}}{s\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} \quad OR \quad Z_{cal} = \frac{\bar{x}-\bar{y}}{\sigma\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$$

*where*

$\bar{x}$ = Mean of Sample-1

$\bar{y}$ = Mean of Sample-2

$n_1$ = Size of Sample-1

$n_1$ = Size of sample-2

$S_1 = S.D\ of\ sample - 1$

$S_2 = S.D\ of\ sample - 2$

$\sigma_1 = S.D\ of\ Population - 1$

$\sigma_2 = S.D\ of\ Population - 2$

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(i) If S.D $S_1$ and $S_2$ are given

$$S = \sqrt{\frac{n_1 S_1^2 + n_2 S_2^2}{n_1 + n_2 - 2}}$$

(ii) If S.D $\sigma_1$ and $\sigma_2$ are given

$$S = \sqrt{\frac{n_1 \sigma_1^2 + n_2 \sigma_2^2}{n_1 + n_2 - 2}}$$

**STEP – 4**: Write level of significance $\propto$ ($\propto = 5\%$ OR $\propto = 1\%$)

$\frac{100 - 5}{2}\% = 47.5\% = 0.475$

$Z_{tab} = 1.96$

**STEP – 5** : From the Z – test table

Find the tabulated value $Z_{tab}$ at the level of significance $\propto$

Gateway Classes : 7455 9612 84

**STEP – 6: Conclusion**

(i) If $Z_{cal} < Z_{tab}$ then $H_0$ is accepted

There is no significant difference between between their means

(ii) If $Z_{cal} > Z_{tab}$ then $H_0$ is rejected

There is no significant difference between between their means

Gateway Classes : 7455 961284

**Q.1** *A random sample of 50 items gives the mean 8 and variabce 15 can it be regarded as drawn from a normal population with 7 as mean at 5% of level of significance?*

Given

$n = 50$

$\bar{x} = 8$

$\sigma^2 = 15$

$\sigma = \sqrt{15}$

$\mu = 7$

$\alpha = 5\%$

Null Hypothesis ($H_0$)

$H_0$: There is no significant diffeunce between Sample mean and population mean

$H_0 : \mu = 7$

Aternative Hypothesis ($H_1$)

$H_1 : \mu \neq 7$ (Two tailed test)

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## Calculation of z

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

$$= \frac{8-7}{\frac{\sqrt{15}}{\sqrt{50}}}$$

$$= \frac{1 \times \sqrt{50}}{\sqrt{15}}$$

$$= \sqrt{\frac{50}{15}}$$

$$\boxed{z_{cal} = 1.826}$$

Level of significance

$\alpha = 5\%$

$$\frac{100 - 5\%}{2} = 47.5\%$$

$$= 0.475$$

From z-test table

$z_{tab} = 1.96$

## Conclusion:

$z_{cal} = 1.826$

$z_{tab} = 1.96$

$\because z_{cal} < z_{tab}$.

$\therefore H_0$ Hypothesis is accepted

Gateway Classes : 7455 961284

UNIT-5 : DPP- 04

Topic : Z–*test*

$Q.1$ *A random sample of* 60 *items gives the mean* 10 *and variabce* 20 *can it be regarded as drawn from a normal population with* 8 *as mean at* 5% *of level of significance*?

Gateway Classes : 7455 96 12 84

# ENGINEERING MATHEMATICS

## UNIT-5 : *Statistical Techniques – III*

## *(Testing a Hypothesis and Statistical Quality Control)*

### *Lec-05*

**Today's Target**

- $\chi^2 - test\ of\ goodness\ of\ FIT\ (Part - 1)$
- PYQ
- DPP

Gateway Classes :7455 961284

# Hypothesis Testing

## Test of significance

(i) $t$ – test

(ii) $z$ – test

(iii) $F$ – test

(iv) chi – square test ($\chi^2$ – test) v. imp

## Hpothesis Testing

### Parametric test

(Sample drawn from normal population)

(i) $t$ – test

(ii) $z$ – test

(iii) $F$ – test

### Non – Parametric test

(No information about population)

(i) $\chi^2$ – test

Gateway Classes : 7455 9612 84

# Chi – Square test ($\chi^2 - test$)

$\chi^2 - test$

- $\chi^2 - test$ of goodness of FIT — L-5, L-6
- $\chi^2 - test$ as a test of independence — L-7

**$\chi^2 - test$ of goodness of Fit** : Given Data can be fitted in

(i) Poisson Distribution

(ii) Binomial Distribution

(iii) Normal Distribution

Fitting is good or not is test by $\chi^2 - test$.

In $\chi^2 - test$ $o_1, o_2 \ldots . o_n$ is a set of observed frequencies and $E_1, E_2, E_3 \ldots E_n$ are the corresponding set of expected frequencies such that $\sum o_i = \sum E_i$

Given in Question

Find

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## Conditions for applying $\chi^2$ – test

(i) Sum of frequencies must be greater than equal to 50.

(ii) The expected frequency $E_i$ of any item must be greater than 5.

**Note** : If any item has frequency less than 5 then it should be combined with the next or preceding item until the total frequency exceed 5.

### Steps to solve questions

**Step – 1** : Null Hypothesis $(H_o)$

$(H_o)$: There is no significant difference between observed and expected frequency.

**Step – 2** : Calculate $\chi^2$ under $H_o$

$$\chi^2 = \sum \left[\frac{(O_i - E_i)^2}{E_i}\right]$$

Where $O_i$ = observed frequency

$E_i$ = Expected frequency

Gateway Classes : 7455 961 284

**Step – 3 :** Degree of freedom = Number of variables − number of constraints

$$\vartheta = n - k = n - 1$$

Level of significance $\propto = 5\% = 0.05$

**Step – 4 :** From $\chi^2$ – table

Calculate $\chi^2_{0.05}$ at the given level of significance for the given degree of freedom

**Step – 5 :** conclusion

(i) If $\chi^2 < \chi^2_{0.05}$, then $H_o$ accepted

- There is no significants difference between observed frequency and expected frequency.
- Fit of the data is considered to be good.

(ii) $\chi^2 > \chi^2_{0.05}$, then $H_o$ is rejected.

- There is significants difference between observed frequency and expected frequency.
- Fit of the data is not good.

Gateway Classes : 7455 9612 84

**Q.1** ***A die is thrown 276 times and the results of these throws are given below:***

| *No. appeared on the die* | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| *Frequency* | 40 | 32 | 29 | 59 | 57 | 59 |

***Test whether the die is biased or not.*** ***(A.K.T.U. 2019)***

Null Hypothesis ($H_0$)

$H_0$: The die is an unbiased die

Calculation of $\chi^2$, under $H_0$

$$E_i = \frac{\sum O_i}{n} = \frac{276}{6}$$

$$E_i = 46$$

| $O_i$ | $E_i$ | $O_i - E_i$ | $(O_i - E_i)^2$ |
|---|---|---|---|
| 40 | 46 | −6 | 36 |
| 32 | 46 | −14 | 196 |
| 29 | 46 | −17 | 289 |
| 59 | 46 | 13 | 169 |
| 57 | 46 | 11 | 121 |
| 59 | 46 | 13 | 169 |
| $\Sigma O_i = 276$ | $\Sigma E_i = 276$ | | $\Sigma (O_i - E_i)^2 = 980$ |

Gateway Classes : 7455 9612 84

$$\chi^2 = \sum \left[ \frac{(O_i - E_i)^2}{E_i} \right]$$

$$\chi^2 = \frac{\sum (O_i - E_i)^2}{E_i}$$

$$= \frac{980}{46}$$

$$\boxed{\chi^2 = 21.30}$$

Degree of Freedom $= n - 1$

$= 6 - 1$

$= 5$

Level of significance

$\alpha = 5\% = 0.05$

From $\chi^2$-test table

$$\chi^2_{0.05} = 11.070$$

## conclusion

$\chi^2 = 21.30$, $\chi^2 = 11.070$

$\because \chi^2 > \chi^2_{0.05}$

$\therefore H_0$ is rejected

Hence,

Given die is a biased die

Gateway Classes: 7455 961284

## Table 4 : CHI-SQUARE

**Significant Values $\chi^2(\alpha)$ of Chi-Square Distribution Right Tail Areas for Given Probability $\alpha$,**

**$P = P_r(\chi^2 > \chi^2(\alpha)) = \alpha$**

**And $v$ is Degrees of Freedom (d.f.)**

| *Degree of freedom (v)* | *Probability (Level of Significance)* | | | | | | |
|---|---|---|---|---|---|---|---|
| | *0.99* | *0.95* | *0.50* | *0.10* | *0.05* | *0.02* | *0.01* |
| 1 | .000157 | .00393 | .455 | 2.706 | 3.841 | 5.214 | 6.635 |
| 2 | .0201 | .103 | 1.386 | 4.605 | 5.991 | 7.824 | 9.210 |
| 3 | .115 | .352 | 2.366 | 6.251 | 7.815 | 9.837 | 11.341 |
| 4 | .297 | .711 | 3.357 | 7.779 | 9.488 | 11.668 | 13.277 |
| 5 | .554 | 1.145 | 4.351 | 9.236 | 11.070 | 13.388 | 15.086 |
| 6 | .872 | 2.635 | 5.348 | 10.645 | 12.592 | 15.033 | 16.812 |
| 7 | 1.239 | 2.167 | 6.346 | 12.017 | 14.067 | 16.622 | 18.475 |
| 8 | 1.646 | 2.733 | 7.344 | 13.362 | 15.507 | 18.168 | 20.090 |
| 9 | 2.088 | 3.325 | 8.343 | 14.684 | 16.919 | 19.679 | 21.669 |
| 10 | 2.558 | 3.940 | 9.340 | 15.987 | 18.307 | 21.161 | 23.209 |
| 11 | 3.053 | 4.575 | 10.341 | 17.275 | 19.675 | 22.618 | 24.725 |
| 12 | 3.571 | 5.226 | 11.340 | 18.549 | 21.026 | 24.054 | 26.217 |
| 13 | 4.107 | 5.892 | 12.340 | 19.812 | 22.362 | 25.472 | 27.688 |
| 14 | 4.660 | 6.571 | 13.339 | 21.064 | 23.685 | 26.873 | 29.141 |
| 15 | 4.229 | 7.261 | 14.339 | 22.307 | 24.996 | 28.259 | 30.578 |
| 16 | 5.812 | 7.962 | 15.338 | 23.542 | 26.296 | 29.633 | 32.000 |
| 17 | 6.408 | 8.672 | 15.338 | 24.769 | 27.587 | 30.995 | 33.409 |
| 18 | 7.015 | 9.390 | 17.338 | 25.989 | 28.869 | 32.346 | 34.805 |
| 19 | 7.633 | 10.117 | 18.338 | 27.204 | 30.144 | 33.687 | 36.191 |
| 20 | 8.260 | 10.851 | 19.337 | 28.412 | 31.410 | 35.020 | 37.566 |
| 21 | 8.897 | 11.591 | 20.337 | 29.615 | 32.671 | 36.343 | 38.932 |
| 22 | 9.542 | 12.338 | 21.337 | 30.813 | 33.924 | 37.659 | 40.289 |
| 23 | 10.196 | 13.091 | 22.337 | 32.007 | 35.172 | 38.968 | 41.638 |
| 24 | 10.856 | 13.848 | 23.337 | 32.196 | 36.415 | 40.270 | 42.980 |
| 25 | 11.524 | 14.611 | 24.337 | 34.382 | 37.65 | 41.566 | 44.314 |
| 26 | 12.198 | 15.379 | 25.336 | 35.363 | 38.885 | 41.856 | 45.642 |
| 27 | 12.879 | 16.151 | 26.336 | 36.741 | 40.113 | 41.140 | 46.963 |
| 28 | 13.565 | 16.928 | 27.336 | 37.916 | 41.337 | 45.419 | 48.278 |
| 29 | 14.256 | 17.708 | 28.336 | 39.087 | 42.557 | 46.693 | 49.588 |
| 30 | 14.933 | 18.493 | 29.336 | 40.256 | 43.773 | 47.962 | 50.892 |

Gateway Classes 9455 6282 84

**Q.2** *The following table gives the number of accidents that took place in an industry during various days of the week. Test if accidents are uniformly distributed over the week.*

| Day | Mon | Tue | Wed | Thu | Fri | Sat |
|---|---|---|---|---|---|---|
| No. of accidents | 14 | 18 | 12 | 11 | 15 | 14 |

Null Hypothesis ($H_0$)

$H_0$: Accidents are uniformly distributed over the week

calculation of $\chi^2$, under $H_0$

$$E_i = \frac{\sum O_i}{n} = \frac{84}{6} = 14$$

| $O_i$ | $E_i$ | $O_i - E_i$ | $(O_i - E_i)^2$ |
|---|---|---|---|
| 14 | 14 | 0 | 0 |
| 18 | 14 | 4 | 16 |
| 12 | 14 | −2 | 4 |
| 11 | 14 | −3 | 9 |
| 15 | 14 | 1 | 1 |
| 14 | 14 | 0 | 0 |
| $\sum O_i =$ 84 | | | $\sum (O_i - E_i)^2 =$ 30 |

Gateway Classes: 7455 96 1384

$$\chi^2 = \sum \left[ \frac{(O_i - E_i)^2}{E_i} \right]$$

$$= \frac{\sum (O_i - E_i)^2}{E_i}$$

$$= \frac{30}{14}$$

$$\boxed{\chi^2 = 2.1428}$$

Degree of freedom $= n - 1$

$= 6 - 1$

$= 5$

Level of significance ($\alpha$)

$\alpha = 5\% = 0.05$

From $\chi^2$ test table

$$\boxed{\chi^2_{0.05} = 11.070}$$

## conclusion

$\chi^2 = 2.1428$, $\chi^2_{0.05} = 11.070$

$\therefore \chi^2 < \chi^2_{0.05}$

$\therefore H_0$ is accepted

Hence,

Accidents are uniformly distributed over the week

Gateway Classes : 7455961284

## Table 4 : CHI-SQUARE

**Significant Values $\chi^2(\alpha)$ of Chi-Square Distribution Right Tail Areas for Given Probability $\alpha$,**

**$P = P_r(\chi^2 > \chi^2(\alpha)) = \alpha$**

**And $v$ is Degrees of Freedom (d.f.)**

| *Degree of freedom (v)* | *Probability (Level of Significance)* | | | | | | |
|---|---|---|---|---|---|---|---|
| | *0.99* | *0.95* | *0.50* | *0.10* | *0.05* | *0.02* | *0.01* |
| 1 | .000157 | .00393 | .455 | 2.706 | 3.841 | 5.214 | 6.635 |
| 2 | .0201 | .103 | 1.386 | 4.605 | 5.991 | 7.824 | 9.210 |
| 3 | .115 | .352 | 2.366 | 6.251 | 7.815 | 9.837 | 11.341 |
| 4 | .297 | .711 | 3.357 | 7.779 | 9.488 | 11.668 | 13.277 |
| 5 | .554 | 1.145 | 4.351 | 9.236 | 11.070 | 13.388 | 15.086 |
| 6 | .872 | 2.635 | 5.348 | 10.645 | 12.592 | 15.033 | 16.812 |
| 7 | 1.239 | 2.167 | 6.346 | 12.017 | 14.067 | 16.622 | 18.475 |
| 8 | 1.646 | 2.733 | 7.344 | 13.362 | 15.507 | 18.168 | 20.090 |
| 9 | 2.088 | 3.325 | 8.343 | 14.684 | 16.919 | 19.679 | 21.669 |
| 10 | 2.558 | 3.940 | 9.340 | 15.987 | 18.307 | 21.161 | 23.209 |
| 11 | 3.053 | 4.575 | 10.341 | 17.275 | 19.675 | 22.618 | 24.725 |
| 12 | 3.571 | 5.226 | 11.340 | 18.549 | 21.026 | 24.054 | 26.217 |
| 13 | 4.107 | 5.892 | 12.340 | 19.812 | 22.362 | 25.472 | 27.688 |
| 14 | 4.660 | 6.571 | 13.339 | 21.064 | 23.685 | 26.873 | 29.141 |
| 15 | 4.229 | 7.261 | 14.339 | 22.307 | 24.996 | 28.259 | 30.578 |
| 16 | 5.812 | 7.962 | 15.338 | 23.542 | 26.296 | 29.633 | 32.000 |
| 17 | 6.408 | 8.672 | 15.338 | 24.769 | 27.587 | 30.995 | 33.409 |
| 18 | 7.015 | 9.390 | 17.338 | 25.989 | 28.869 | 32.346 | 34.805 |
| 19 | 7.633 | 10.117 | 18.338 | 27.204 | 30.144 | 33.687 | 36.191 |
| 20 | 8.260 | 10.851 | 19.337 | 28.412 | 31.410 | 35.020 | 37.566 |
| 21 | 8.897 | 11.591 | 20.337 | 29.615 | 32.671 | 36.343 | 38.932 |
| 22 | 9.542 | 12.338 | 21.337 | 30.813 | 33.924 | 37.659 | 40.289 |
| 23 | 10.196 | 13.091 | 22.337 | 32.007 | 35.172 | 38.968 | 41.638 |
| 24 | 10.856 | 13.848 | 23.337 | 32.196 | 36.415 | 40.270 | 42.980 |
| 25 | 11.524 | 14.611 | 24.337 | 34.382 | 37.65 | 41.566 | 44.314 |
| 26 | 12.198 | 15.379 | 25.336 | 35.363 | 38.885 | 41.856 | 45.642 |
| 27 | 12.879 | 16.151 | 26.336 | 36.741 | 40.113 | 41.140 | 46.963 |
| 28 | 13.565 | 16.928 | 27.336 | 37.916 | 41.337 | 45.419 | 48.278 |
| 29 | 14.256 | 17.708 | 28.336 | 39.087 | 42.557 | 46.693 | 49.588 |
| 30 | 14.933 | 18.493 | 29.336 | 40.256 | 43.773 | 47.962 | 50.892 |

Gateway Classes 9455 612 84

**Q.3** ***In experiments on pea breeding, the following frequencies of seeds were obtained:***

| *Round and yellow* | *Wrinkled and yellow* | *Round and green* | *Wrinkled and green* | *Total* |
|---|---|---|---|---|
| 315 | 101 | 108 | 32 | 556 |

***Theory predicts that the frequencies should be in proportions 9: 3::3: 1.***

***Examine the correspondence between theory and experiment.***

Null Hypothesis ($H_0$)

$H_0$: Experimental Results support theory

Calculation of $\chi^2$ under $H_0$

$E_1 = \frac{9}{16} \times 556 = 312.75$

| $O_i$ | $E_i$ | $O_i - E_i$ | $(O_i - E_i)^2$ | $\frac{(O_i - E_i)^2}{E_i}$ |
|---|---|---|---|---|
| 315 | 312.75 | 2.25 | 5.0625 | 0.016187 |
| 101 | 104.25 | -3.25 | 10.5625 | 0.101319 |
| 108 | 104.25 | 3.75 | 14.0625 | 0.134892 |
| 32 | 34.75 | -2.75 | 7.5625 | 0.217626 |
| $\Sigma O_i = 556$ | $\Sigma E_i = 556$ | | | $\sum\left[\frac{(O_i - E_i)^2}{E_i}\right] = ?$ |

Gateway Classes: 7455 26 2 84

$$E_2 = \frac{3}{16} \times 556 = 104.25$$

$$E_3 = \frac{3}{16} \times 556 = 104.25$$

$$E_4 = \frac{1}{16} \times 556 = 34.75$$

$$\chi^2 = \sum \left[ \frac{(O_i - E_i)^2}{E_i} \right]$$

$$\boxed{\chi^2 = 0.470024}$$

Degree of freedom $= n - 1$

$= 4 - 1$

$= 3$

Level of significance ($\alpha$)

$\alpha = 5\% = 0.05$

From $\chi^2$-test table

$$\chi^2_{0.05} = 7.185$$

Gateway Classes +7455961284

# Conclusion

$\chi_{cal} = 0.470024$, $\chi_{tab} = 7.185$

$\therefore \chi_{cal} < \chi_{tab}$

$\therefore H_0$ is Accepted

Hence,

Experimental Results support theoritical results

Gateway Classes : 7455 9612 84

## Table 4 : CHI-SQUARE

**Significant Values $\chi^2(\alpha)$ of Chi-Square Distribution Right Tail Areas for Given Probability $\alpha$,**

**$P = P_r(\chi^2 > \chi^2(\alpha)) = \alpha$**

**And $v$ is Degrees of Freedom (d.f.)**

| *Degree of freedom (v)* | *Probability (Level of Significance)* | | | | | | |
|---|---|---|---|---|---|---|---|
| | *0.99* | *0.95* | *0.50* | *0.10* | *0.05* | *0.02* | *0.01* |
| 1 | .000157 | .00393 | .455 | 2.706 | 3.841 | 5.214 | 6.635 |
| 2 | .0201 | .103 | 1.386 | 4.605 | 5.991 | 7.824 | 9.210 |
| 3 | .115 | .352 | 2.366 | 6.251 | 7.815 | 9.837 | 11.341 |
| 4 | .297 | .711 | 3.357 | 7.779 | 9.488 | 11.668 | 13.277 |
| 5 | .554 | 1.145 | 4.351 | 9.236 | 11.070 | 13.388 | 15.086 |
| 6 | .872 | 2.635 | 5.348 | 10.645 | 12.592 | 15.033 | 16.812 |
| 7 | 1.239 | 2.167 | 6.346 | 12.017 | 14.067 | 16.622 | 18.475 |
| 8 | 1.646 | 2.733 | 7.344 | 13.362 | 15.507 | 18.168 | 20.090 |
| 9 | 2.088 | 3.325 | 8.343 | 14.684 | 16.919 | 19.679 | 21.669 |
| 10 | 2.558 | 3.940 | 9.340 | 15.987 | 18.307 | 21.161 | 23.209 |
| 11 | 3.053 | 4.575 | 10.341 | 17.275 | 19.675 | 22.618 | 24.725 |
| 12 | 3.571 | 5.226 | 11.340 | 18.549 | 21.026 | 24.054 | 26.217 |
| 13 | 4.107 | 5.892 | 12.340 | 19.812 | 22.362 | 25.472 | 27.688 |
| 14 | 4.660 | 6.571 | 13.339 | 21.064 | 23.685 | 26.873 | 29.141 |
| 15 | 4.229 | 7.261 | 14.339 | 22.307 | 24.996 | 28.259 | 30.578 |
| 16 | 5.812 | 7.962 | 15.338 | 23.542 | 26.296 | 29.633 | 32.000 |
| 17 | 6.408 | 8.672 | 15.338 | 24.769 | 27.587 | 30.995 | 33.409 |
| 18 | 7.015 | 9.390 | 17.338 | 25.989 | 28.869 | 32.346 | 34.805 |
| 19 | 7.633 | 10.117 | 18.338 | 27.204 | 30.144 | 33.687 | 36.191 |
| 20 | 8.260 | 10.851 | 19.337 | 28.412 | 31.410 | 35.020 | 37.566 |
| 21 | 8.897 | 11.591 | 20.337 | 29.615 | 32.671 | 36.343 | 38.932 |
| 22 | 9.542 | 12.338 | 21.337 | 30.813 | 33.924 | 37.659 | 40.289 |
| 23 | 10.196 | 13.091 | 22.337 | 32.007 | 35.172 | 38.968 | 41.638 |
| 24 | 10.856 | 13.848 | 23.337 | 32.196 | 36.415 | 40.270 | 42.980 |
| 25 | 11.524 | 14.611 | 24.337 | 34.382 | 37.65 | 41.566 | 44.314 |
| 26 | 12.198 | 15.379 | 25.336 | 35.363 | 38.885 | 41.856 | 45.642 |
| 27 | 12.879 | 16.151 | 26.336 | 36.741 | 40.113 | 41.140 | 46.963 |
| 28 | 13.565 | 16.928 | 27.336 | 37.916 | 41.337 | 45.419 | 48.278 |
| 29 | 14.256 | 17.708 | 28.336 | 39.087 | 42.557 | 46.693 | 49.588 |
| 30 | 14.933 | 18.493 | 29.336 | 40.256 | 43.773 | 47.962 | 50.892 |

Gateway Classes 7455 612 284

Topic : $\chi^2 - test\ of\ goodness\ of\ FIT\ (Part-1)$

**Q.1** *What is* $\chi^2 - test$?

*A die is thrown 90 times with the following results:* [AKTU 2010]

| Face | : | 1 | 2 | 3 | 4 | 5 | 6 | Total |
|---|---|---|---|---|---|---|---|---|
| Frequency | : | 10 | 12 | 16 | 14 | 18 | 20 | 90 |

*Use* $\chi^2 - test$ *to test whether these data are consistent with the hypothesis that die is unbiased.*

**Q.2** *The demand for a particular spare part in a factory was found to vary from day – to – day. In a sample study, the following information was obtained:*

| Days : | Mon | Tue | Wed | Thurs | Fri | Sat |
|---|---|---|---|---|---|---|
| No. of parts demanded : | 1124 | 1125 | 1110 | 1120 | 1126 | 1115 |

*Test the hypothesis that the number of parts demanded does not depend on the day of the week.*

**Q.3** *The theory predicts the proportion of beans in the four groups,* $G_1, G_2, G_3, G_4$ *should be in the ratio* 9:3:3:1. *In an experiment with* 1600 *beans the numbers in the four groups were* 882, 313, 287 *and* 118. *Does the experimental result support the theory* ?

Gateway Classes: 7455 9612 84

# ENGINEERING MATHEMATICS

## UNIT-5 : *Statistical Techniques – III*

## *(Testing a Hypothesis and Statistical Quality Control)*

### *Lec-06*

**Today's Target**

- $\chi^2 - test\ of\ goodness\ of\ FIT\ (Part - 2)$
- PYQ
- DPP

Gateway Classes :7455 961284

# Hpothesis Testing

## Parametric test

(Sample drawn from normal population)

(i) $t-$ test

(ii) $z-$ test

(iii) $F-$ test

## Non – Parametric test

(No information about population)

(i) $\chi^2-$ test

## $\chi^2-$ test

### $\chi^2-$ test of goodness of FIT

L-5, L-6

### $\chi^2-$ test as a test of independence

L-7

Gateway Classes : 7455 961 2 84

## $\chi^2$ – test of goodness of Fit :

- Given Data can be fitted in

(i) Poisson Distribution

(ii) Binomial Distribution

~~(iii) Normal Distribution~~

- Fitting is good or not is test by $\chi^2$ – test.
- In $\chi^2$ – test $o_1, o_2 \ldots . o_n$ is a set of observed frequencies and $E_1, E_2, E_3 \ldots E_n$ are the corresponding set of expected frequencies such that $\sum o_i = \sum E_i$

### Conditions for applying $\chi^2$ – test

(i) Sum of frequencies must be greater than equal to 50.

(ii) The expected frequency $E_i$ of any item must be greater than 5.

**Note** : If any item has frequency less than 5 then it should be combined with the next or preceding item until the total frequency exceed 5.

Gateway Classes : 7455 9612 84

## Steps to solve questions

**Step – 1 : Null Hypothesis $(H_o)$**

$(H_o)$: There is no significant difference between observed and expected frequency.

**Step – 2 : Calculate $\chi^2$ under $H_o$**

$$\chi^2 = \sum \left[\frac{(O_i - E_i)^2}{E_i}\right]$$

Where $O_i$ = observed frequency

$E_i$ = Expected frequency

**Step – 3 :** Degree of freedom = Number of variables − number of constraints

$$\vartheta = n - k$$

Level of significance $\propto = 5\% = 0.05$

Gateway Classes : 7455 961234

**Step – 4 :** From $\chi^2$ – table

Calculate $\chi^2_{0.05}$ at the given level of significance for the given degree of freedom

**Step – 5 :**

(i) If $\chi^2 < \chi^2_{0.05}$, then $H_o$ accepted

- There is no significants difference between observed frequency and expected frequency.
- Fit of the data is considered to be good.

(ii) $\chi^2 > \chi^2_{0.05}$, then $H_o$ is rejected.

- There is significants difference between observed frequency and expected frequency.
- Fit of the data is not good.

Gateway Classes : 7455 9612 84

**Q.1** *A survey of 320 families with 5 children shows the following distribution:*

| No of boys & girls : | 5 boys & 0 girl | 4 boys & 1 girl | 3 boys & 2 girl | 2 boys & 3 girl | 1 boys & 4 girl | 0 boys & 5 girl | Total |
|---|---|---|---|---|---|---|---|
| No. of families : | 18 | 56 | 110 | 88 | 40 | 8 | 320 |

***Test the hypothesis that data are binomially distributed and male and female births are equal probability.*** **(G.B.T.U. 2010)**

Null Hypothesis ($H_0$)

$H_0$: Data are binomially distributed and probability of Male and Female birth are equal

Let $p$ = Prob. of Girl (success)

$q$ = Prob. of Boy (Failure)

$p = q = \frac{1}{2}$

$N = 320$

$n = 5$

$r = 0, 1, 2, 3, 4, 5$

Binomial Distribution

$P(X=r) = {}^{n}C_{r}\, p^{r} q^{n-r}$

Gateway Classes: 7455 9612 84

# Expected Frequencies

$$E_i = N\,{}^{n}C_r\, p^r q^{n-r}$$

$$E_i = 320 \times {}^{5}C_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{5-r} \quad \text{---} (1)$$

$$E_1 = 320 \times {}^{5}C_0 \times \left(\frac{1}{2}\right)^0 \times \left(\frac{1}{2}\right)^5$$

$$= \frac{\overset{10}{\cancel{320}}}{32} \times 1 \times 1 \times \frac{1}{\cancel{32}}$$

$$E_1 = 10$$

$$E_2 = 320 \times {}^{5}C_1 \times \left(\frac{1}{2}\right)^1 \times \left(\frac{1}{4}\right)^4 = \frac{\overset{10}{\cancel{320}}}{32} \times 5 \times \frac{1}{\cancel{32}} = 50$$

$$E_2 = 50$$

$$E_3 = 320 \times {}^{5}C_2 \times \left(\frac{1}{2}\right)^2 \times \left(\frac{1}{2}\right)^3 = \frac{\overset{10}{\cancel{320}}}{32} \times \frac{5 \times \overset{2}{\cancel{4}}}{\cancel{2}} \times \frac{1}{\cancel{32}} = 100$$

$$E_3 = 100$$

$$E_4 = 320 \times {}^{5}C_3 \times \left(\frac{1}{2}\right)^3 \times \left(\frac{1}{2}\right)^2 = \frac{\overset{10}{\cancel{320}}}{32} \times \frac{5 \times \overset{2}{\cancel{4}}}{\cancel{2}} \times \frac{1}{\cancel{32}}$$

$$E_4 = 100$$

$$E_5 = 320 \times {}^{5}C_4 \times \left(\frac{1}{2}\right)^4 \times \left(\frac{1}{2}\right)^1 = \frac{\overset{10}{\cancel{320}}}{32} \times 5 \times \frac{1}{\cancel{32}} = 50$$

Gateway Classes : 7455 9612 84

$E_5 = 50$

$$E_6 = 320 \times {}^{5}C_{5} \times \left(\frac{1}{2}\right)^{5} \times \left(\frac{1}{2}\right)^{0}$$

$$= \cancel{320}^{10} \times 1 \times \frac{1}{\cancel{32}}$$

$$E_6 = 10$$

calculation of $\chi^2$, under $H_0$

| $O_i$ | $E_i$ | $O_i - E_i$ | $(O_i - E_i)^2$ | $\frac{(O_i - E_i)^2}{E_i}$ |
|---|---|---|---|---|
| 18 | 10 | 8 | 64 | 6.4 |
| 56 | 50 | 6 | 36 | 0.72 |
| 110 | 100 | 10 | 100 | 1 |
| 88 | 100 | −12 | 144 | 1.44 |
| 40 | 50 | −10 | 100 | 2 |
| 8 | 10 | −2 | 4 | 0.4 |
| | | | | $\sum\left[\frac{(O_i - E_i)^2}{E_i}\right] = 11.96$ |

Gateway Classes: 7455 96 12 84

$\chi^2_{cal} = 11.96$

$\chi_{tab} = 11.070$

Degree of freedom $= n - k$

$= 6 - 1$

$= 5$

Level of significance ($\alpha$)

$\alpha = 5\% = 0.05$

From $\chi^2$-table

$\chi_{tab} =$

**conclusion**

$\chi^2_{cal} = 11.96$ and $\chi_{tab} = 11.070$

$\because \chi^2_{cal} > \chi^2_{tab}$

$\therefore H_0$ is rejected

Hence,

Data are not Binomially Distributed and Probability of birth of boy and Girl are not equal

$\Rightarrow p \neq q$

Gateway Classes : 7455 9612 84

## Table 4 : CHI-SQUARE

**Significant Values $\chi^2(\alpha)$ of Chi-Square Distribution Right Tail Areas for Given Probability $\alpha$,**

**$P = P_r(\chi^2 > \chi^2(\alpha)) = \alpha$**

**And $v$ is Degrees of Freedom (d.f.)**

| *Degree of freedom (v)* | *Probability (Level of Significance)* | | | | | | |
|---|---|---|---|---|---|---|---|
| | *0.99* | *0.95* | *0.50* | *0.10* | *0.05* | *0.02* | *0.01* |
| 1 | .000157 | .00393 | .455 | 2.706 | 3.841 | 5.214 | 6.635 |
| 2 | .0201 | .103 | 1.386 | 4.605 | 5.991 | 7.824 | 9.210 |
| 3 | .115 | .352 | 2.366 | 6.251 | 7.815 | 9.837 | 11.341 |
| 4 | .297 | .711 | 3.357 | 7.779 | 9.488 | 11.668 | 13.277 |
| 5 | .554 | 1.145 | 4.351 | 9.236 | 11.070 | 13.388 | 15.086 |
| 6 | .872 | 2.635 | 5.348 | 10.645 | 12.592 | 15.033 | 16.812 |
| 7 | 1.239 | 2.167 | 6.346 | 12.017 | 14.067 | 16.622 | 18.475 |
| 8 | 1.646 | 2.733 | 7.344 | 13.362 | 15.507 | 18.168 | 20.090 |
| 9 | 2.088 | 3.325 | 8.343 | 14.684 | 16.919 | 19.679 | 21.669 |
| 10 | 2.558 | 3.940 | 9.340 | 15.987 | 18.307 | 21.161 | 23.209 |
| 11 | 3.053 | 4.575 | 10.341 | 17.275 | 19.675 | 22.618 | 24.725 |
| 12 | 3.571 | 5.226 | 11.340 | 18.549 | 21.026 | 24.054 | 26.217 |
| 13 | 4.107 | 5.892 | 12.340 | 19.812 | 22.362 | 25.472 | 27.688 |
| 14 | 4.660 | 6.571 | 13.339 | 21.064 | 23.685 | 26.873 | 29.141 |
| 15 | 4.229 | 7.261 | 14.339 | 22.307 | 24.996 | 28.259 | 30.578 |
| 16 | 5.812 | 7.962 | 15.338 | 23.542 | 26.296 | 29.633 | 32.000 |
| 17 | 6.408 | 8.672 | 15.338 | 24.769 | 27.587 | 30.995 | 33.409 |
| 18 | 7.015 | 9.390 | 17.338 | 25.989 | 28.869 | 32.346 | 34.805 |
| 19 | 7.633 | 10.117 | 18.338 | 27.204 | 30.144 | 33.687 | 36.191 |
| 20 | 8.260 | 10.851 | 19.337 | 28.412 | 31.410 | 35.020 | 37.566 |
| 21 | 8.897 | 11.591 | 20.337 | 29.615 | 32.671 | 36.343 | 38.932 |
| 22 | 9.542 | 12.338 | 21.337 | 30.813 | 33.924 | 37.659 | 40.289 |
| 23 | 10.196 | 13.091 | 22.337 | 32.007 | 35.172 | 38.968 | 41.638 |
| 24 | 10.856 | 13.848 | 23.337 | 32.196 | 36.415 | 40.270 | 42.980 |
| 25 | 11.524 | 14.611 | 24.337 | 34.382 | 37.65 | 41.566 | 44.314 |
| 26 | 12.198 | 15.379 | 25.336 | 35.363 | 38.885 | 41.856 | 45.642 |
| 27 | 12.879 | 16.151 | 26.336 | 36.741 | 40.113 | 41.140 | 46.963 |
| 28 | 13.565 | 16.928 | 27.336 | 37.916 | 41.337 | 45.419 | 48.278 |
| 29 | 14.256 | 17.708 | 28.336 | 39.087 | 42.557 | 46.693 | 49.588 |
| 30 | 14.933 | 18.493 | 29.336 | 40.256 | 43.773 | 47.962 | 50.892 |

Gateway Classes 9456 612 84

**Q.1** ***Fit a Poisson distribution to the following data and test the goodness of fit:***

| $x$ | : | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|
| $f$ | : | 109 | 65 | 22 | 3 | 1 |

Null Hypothesis ($H_0$)

$H_0$: Fitting of data in poisson Distribution is good

| $x$ | $f$ | $fx$ |
|---|---|---|
| 0 | 109 | 0 |
| 1 | 65 | 65 |
| 2 | 22 | 44 |
| 3 | 3 | 9 |
| 4 | 1 | 4 |
| | $\Sigma f = 200$ | $\Sigma fx = 122$ |

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\lambda = \frac{122}{200} = 0.61$$

By Poisson Distribution

$$P(x) = \frac{e^{-\lambda} \lambda^x}{x!}$$

$$P(x) = \frac{e^{-0.61} \times (0.61)^x}{x!}$$

Gateway Classes : 7455 9612 84

Expected Frequencies

$$E_i = \frac{N e^{-0.61} \times (0.61)^x}{x!}$$

$$E_i = 200 \times \frac{e^{-0.61} \times (0.61)^x}{x!} \quad \text{— (1)}$$

$$E_1 = \frac{200 \times e^{-0.61} \times (0.61)^0}{0!}$$

$$E_1 = \frac{200 \times e^{-0.61}}{1}$$

$$E_1 = 108.67 \approx 109$$

$$E_2 = 66.29 \approx 66$$

$$E_3 = 20.22 \approx 20$$

$$E_4 = 4.11 \approx 4$$

$$E_5 = 0.63 \approx 1$$

calculation of $\chi^2$, under $H_0$

Gateway Classes : 7455 961284

| $O_i$ | $E_i$ | $O_i - E_i$ | $(O_i - E_i)^2$ | $\frac{(O_i - E_i)^2}{E_i}$ |
|---|---|---|---|---|
| 109 | 109 | 0 | 0 | 0 |
| 65 | 66 | −1 | 1 | 0.01515 |
| 22 | 20 | 2 | 4 | 0.2 |
| 3, 1 (4) | 4, 1 (5) | −1 | 1 | 0.2 |
| | | | | $\chi^2 = 0.41515$ |

$$\chi_{cal} = 0.41515$$

Degree of freedom $= n - K$

$$d.f = 5 - 1 - 1 - 1$$

$$d.f = 2$$

Level of significance ($\alpha$)

$$\alpha = 5\% = \frac{5}{100} = 0.05$$

From $\chi^2$-table

$$\chi^2_{tab} = 5.991$$

GW GATEWAY CLASSES
Gateway Classes : 7455 961 284

## Conclusion

$\chi^2_{cal} = 0.41515$ and $\chi^2_{tab} = 5.991$

$\because \chi^2_{cal} < \chi^2_{tab}$

$\therefore$ $H_0$ is accepted

Hence, Fitting of data in poisson Distribution is good

Gateway Classes : 7455 9612 84

## Table 4 : CHI-SQUARE

**Significant Values $\chi^2(\alpha)$ of Chi-Square Distribution Right Tail Areas for Given Probability $\alpha$,**

**$P = P_r(\chi^2 > \chi^2(\alpha)) = \alpha$**

**And $v$ is Degrees of Freedom (d.f.)**

| *Degree of freedom (v)* | *Probability (Level of Significance)* | | | | | | |
|---|---|---|---|---|---|---|---|
| | *0.99* | *0.95* | *0.50* | *0.10* | *0.05* | *0.02* | *0.01* |
| 1 | .000157 | .00393 | .455 | 2.706 | 3.841 | 5.214 | 6.635 |
| 2 | .0201 | .103 | 1.386 | 4.605 | 5.991 | 7.824 | 9.210 |
| 3 | .115 | .352 | 2.366 | 6.251 | 7.815 | 9.837 | 11.341 |
| 4 | .297 | .711 | 3.357 | 7.779 | 9.488 | 11.668 | 13.277 |
| 5 | .554 | 1.145 | 4.351 | 9.236 | 11.070 | 13.388 | 15.086 |
| 6 | .872 | 2.635 | 5.348 | 10.645 | 12.592 | 15.033 | 16.812 |
| 7 | 1.239 | 2.167 | 6.346 | 12.017 | 14.067 | 16.622 | 18.475 |
| 8 | 1.646 | 2.733 | 7.344 | 13.362 | 15.507 | 18.168 | 20.090 |
| 9 | 2.088 | 3.325 | 8.343 | 14.684 | 16.919 | 19.679 | 21.669 |
| 10 | 2.558 | 3.940 | 9.340 | 15.987 | 18.307 | 21.161 | 23.209 |
| 11 | 3.053 | 4.575 | 10.341 | 17.275 | 19.675 | 22.618 | 24.725 |
| 12 | 3.571 | 5.226 | 11.340 | 18.549 | 21.026 | 24.054 | 26.217 |
| 13 | 4.107 | 5.892 | 12.340 | 19.812 | 22.362 | 25.472 | 27.688 |
| 14 | 4.660 | 6.571 | 13.339 | 21.064 | 23.685 | 26.873 | 29.141 |
| 15 | 4.229 | 7.261 | 14.339 | 22.307 | 24.996 | 28.259 | 30.578 |
| 16 | 5.812 | 7.962 | 15.338 | 23.542 | 26.296 | 29.633 | 32.000 |
| 17 | 6.408 | 8.672 | 15.338 | 24.769 | 27.587 | 30.995 | 33.409 |
| 18 | 7.015 | 9.390 | 17.338 | 25.989 | 28.869 | 32.346 | 34.805 |
| 19 | 7.633 | 10.117 | 18.338 | 27.204 | 30.144 | 33.687 | 36.191 |
| 20 | 8.260 | 10.851 | 19.337 | 28.412 | 31.410 | 35.020 | 37.566 |
| 21 | 8.897 | 11.591 | 20.337 | 29.615 | 32.671 | 36.343 | 38.932 |
| 22 | 9.542 | 12.338 | 21.337 | 30.813 | 33.924 | 37.659 | 40.289 |
| 23 | 10.196 | 13.091 | 22.337 | 32.007 | 35.172 | 38.968 | 41.638 |
| 24 | 10.856 | 13.848 | 23.337 | 32.196 | 36.415 | 40.270 | 42.980 |
| 25 | 11.524 | 14.611 | 24.337 | 34.382 | 37.65 | 41.566 | 44.314 |
| 26 | 12.198 | 15.379 | 25.336 | 35.363 | 38.885 | 41.856 | 45.642 |
| 27 | 12.879 | 16.151 | 26.336 | 36.741 | 40.113 | 41.140 | 46.963 |
| 28 | 13.565 | 16.928 | 27.336 | 37.916 | 41.337 | 45.419 | 48.278 |
| 29 | 14.256 | 17.708 | 28.336 | 39.087 | 42.557 | 46.693 | 49.588 |
| 30 | 14.933 | 18.493 | 29.336 | 40.256 | 43.773 | 47.962 | 50.892 |

Gateway Classes 9455 612 84

**Q.2** ***When the first proof of 392 pages of a book of 1200 pages were read, the distribution of printing mistakes were found to be as follows:***

| *No. of mistakes in a page ($x$)* | : | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|---|
| *No. of pages ($f$)* | : | 275 | 72 | 30 | 7 | 5 | 2 | 1 |

***Fit a poisson distribution to the above data and test the goodness of fit.***

Null Hypothesis ($H_0$)

$H_0$ : Poisson Distribution is a good fit to the data

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f} = \frac{189}{392}$$

$$\lambda = 0.4821$$

| $x$ | $f$ | $fx$ |
|---|---|---|
| 0 | 275 | 0 |
| 1 | 72 | 72 |
| 2 | 30 | 60 |
| 3 | 7 | 21 |
| 4 | 5 | 20 |
| 5 | 2 | 10 |
| 6 | 1 | 6 |
| | $\Sigma f = 392$ | $\Sigma fx = 189$ |

Gateway Classes : 7455 9612 84

By Poisson Distribution

$$P(x) = \frac{e^{-\lambda} \lambda^{x}}{x!}$$

Expected Frequency

$$E_i = \frac{N \times e^{-\lambda} \lambda^{x}}{x!}$$

$$E_i = 392 \times \frac{e^{-0.4821} \times (0.4821)^{x}}{x!}$$

$$E_1 = 392 \times \frac{e^{-0.4821} \times (0.4821)^{0}}{0!}$$

$E_1 = 242.05 \simeq 242.1$

$E_2 = 116.69 \simeq 116.7$

$E_3 = 28.13 \simeq 28.1$

$E_4 = 4.52 \simeq 4.5$

$E_5 = 0.5 \simeq 0.5$

$E_6 = 0.052 \simeq 0.1$

$E_7 = 0.0042 \simeq 0$

Gateway Classes : 7455 9612 84

| $O_i$ | $E_i$ | $O_i - E_i$ | $(O_i - E_i)^2$ | $\frac{(O_i - E_i)^2}{E_i}$ |
|---|---|---|---|---|
| 275 | 242.1 | 32.9 | 1082.41 | 4.471 |
| 72 | 116.7 | -44.7 | 1998.09 | 17.121 |
| 30 | 28.1 | 1.9 | 3.61 | 0.128 |
| 7 | 4.5 | | | |
| 5 = 15 | 0.5 = 5.1 | 9.9 | 98.01 | 19.217 |
| 2 | 0.1 | | | |
| 1 | 0 | | | |
| | | | | $\chi^2 = 40.937$ |

$$\chi^2_{cal} = \sum \left[ \frac{(O_i - E_i)^2}{E_i} \right]$$

$$\chi^2_{cal} = 40.937$$

Degree of Freedom $= n - K$

$$d.f = 7 - 1 - 1 - 3$$

$$d.f = 2$$

Level of significance ($\alpha$)

$$\alpha = 5\% = 0.05$$

Gateway Classes: 7455 961284

From $\chi^2$- table

$$\chi^2_{tab} = 5.991$$

conclusion

$$\chi^2_{cal} = 40.937$$

$$\chi^2_{tab} = 5.991$$

$\because \chi^2_{cal} > \chi^2_{tab}$

$\therefore H_0$ is rejected

Hence

Poisson Distribution is not a good fit ~~to~~ the given data

Gateway Classes: 7455 96 12 84

## Table 4 : CHI-SQUARE

**Significant Values $\chi^2(\alpha)$ of Chi-Square Distribution Right Tail Areas for Given Probability $\alpha$,**

**$P = P_r(\chi^2 > \chi^2(\alpha)) = \alpha$**

**And $v$ is Degrees of Freedom (d.f.)**

| *Degree of freedom (v)* | *Probability (Level of Significance)* | | | | | | |
|---|---|---|---|---|---|---|---|
| | *0.99* | *0.95* | *0.50* | *0.10* | *0.05* | *0.02* | *0.01* |
| 1 | .000157 | .00393 | .455 | 2.706 | 3.841 | 5.214 | 6.635 |
| 2 | .0201 | .103 | 1.386 | 4.605 | 5.991 | 7.824 | 9.210 |
| 3 | .115 | .352 | 2.366 | 6.251 | 7.815 | 9.837 | 11.341 |
| 4 | .297 | .711 | 3.357 | 7.779 | 9.488 | 11.668 | 13.277 |
| 5 | .554 | 1.145 | 4.351 | 9.236 | 11.070 | 13.388 | 15.086 |
| 6 | .872 | 2.635 | 5.348 | 10.645 | 12.592 | 15.033 | 16.812 |
| 7 | 1.239 | 2.167 | 6.346 | 12.017 | 14.067 | 16.622 | 18.475 |
| 8 | 1.646 | 2.733 | 7.344 | 13.362 | 15.507 | 18.168 | 20.090 |
| 9 | 2.088 | 3.325 | 8.343 | 14.684 | 16.919 | 19.679 | 21.669 |
| 10 | 2.558 | 3.940 | 9.340 | 15.987 | 18.307 | 21.161 | 23.209 |
| 11 | 3.053 | 4.575 | 10.341 | 17.275 | 19.675 | 22.618 | 24.725 |
| 12 | 3.571 | 5.226 | 11.340 | 18.549 | 21.026 | 24.054 | 26.217 |
| 13 | 4.107 | 5.892 | 12.340 | 19.812 | 22.362 | 25.472 | 27.688 |
| 14 | 4.660 | 6.571 | 13.339 | 21.064 | 23.685 | 26.873 | 29.141 |
| 15 | 4.229 | 7.261 | 14.339 | 22.307 | 24.996 | 28.259 | 30.578 |
| 16 | 5.812 | 7.962 | 15.338 | 23.542 | 26.296 | 29.633 | 32.000 |
| 17 | 6.408 | 8.672 | 15.338 | 24.769 | 27.587 | 30.995 | 33.409 |
| 18 | 7.015 | 9.390 | 17.338 | 25.989 | 28.869 | 32.346 | 34.805 |
| 19 | 7.633 | 10.117 | 18.338 | 27.204 | 30.144 | 33.687 | 36.191 |
| 20 | 8.260 | 10.851 | 19.337 | 28.412 | 31.410 | 35.020 | 37.566 |
| 21 | 8.897 | 11.591 | 20.337 | 29.615 | 32.671 | 36.343 | 38.932 |
| 22 | 9.542 | 12.338 | 21.337 | 30.813 | 33.924 | 37.659 | 40.289 |
| 23 | 10.196 | 13.091 | 22.337 | 32.007 | 35.172 | 38.968 | 41.638 |
| 24 | 10.856 | 13.848 | 23.337 | 32.196 | 36.415 | 40.270 | 42.980 |
| 25 | 11.524 | 14.611 | 24.337 | 34.382 | 37.65 | 41.566 | 44.314 |
| 26 | 12.198 | 15.379 | 25.336 | 35.363 | 38.885 | 41.856 | 45.642 |
| 27 | 12.879 | 16.151 | 26.336 | 36.741 | 40.113 | 41.140 | 46.963 |
| 28 | 13.565 | 16.928 | 27.336 | 37.916 | 41.337 | 45.419 | 48.278 |
| 29 | 14.256 | 17.708 | 28.336 | 39.087 | 42.557 | 46.693 | 49.588 |
| 30 | 14.933 | 18.493 | 29.336 | 40.256 | 43.773 | 47.962 | 50.892 |

Gateway Classes 9455 612 84

## Topic : $\chi^2 - test\ of\ goodness\ of\ FIT\ (Part - 2)$

**Q.1** *Test for goodness of fit of a poisson distribution at 5% level of significance to the following frequency distribution:*

| $x$ | : | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|---|---|
| $f$ | : | 52 | 151 | 130 | 102 | 45 | 12 | 5 | 1 | 2 |

**Q.2** *Test for goodness of fit of a poisson distribution at 5% level of significance to the following frequency distribution:*

| $x$ | : | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|---|
| $f$ | : | 275 | 138 | 75 | 7 | 4 | 1 |

**Q.3** *Records taken of the number of male and female births in 800 families having four children are as follows:*

| No. of male births | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| No. of female births | 4 | 3 | 2 | 1 | 0 |
| No. of families | 32 | 178 | 290 | 236 | 64 |

*Test whether the data are consistent with the hypothesis that the Binomial law holds and the chance of male birth is equal to that of female birth, namely* $p = q = 1/2$. [U.P.T.U. 2009]

Gateway Classes : 7455 961284

# ENGINEERING MATHEMATICS

## UNIT-5 : *Statistical Techniques – III*

*(Testing a Hypothesis and Statistical Quality Control)*

*Lec-07*

**Today's Target**

- $\chi^2 - test\ of\ as\ a\ test\ of\ independence$
- PYQ
- DPP

Gateway Classes :7455 561284

$\chi^2 - test$

$\chi^2 - test\ of\ goodness\ of\ FIT$

L-5, L-6

$\chi^2 - test\ as\ a\ test\ of\ independence$

L-7

## $\chi^2 - test\ as\ a\ test\ of\ independence$

*Two attributes A and B are given with size r and S respectively.*

### *Steps to solve questions*

***Step – 1 :*** *Null Hypothesis* $(H_o)$

$(H_o)$ : *Two attributes are independent*

Gateway Classes : 7455 961284

**Step – 2: Contingency table : Simple data is given is tabular form, called contingency table**

| | |
|---|---|
| $a$ $O_1$ | $b$ $O_2$ |
| $c$ $O_3$ | $d$ $O_4$ |

**The expected frequency for any cell in a contingency table can be calculated by using formula**

$$\text{Expected frequency} = \frac{\text{Row total X column total}}{\text{grand total}}$$

**Observed frequency**

| | | |
|---|---|---|
| $O_1$ $a$ | $O_2$ $b$ | $a+b$ |
| $O_3$ $c$ | $O_4$ $d$ | $c+d$ |
| $a+c$ | $b+d$ | $N = a+b+c+d$ |

grand total

**Expected frequency**

| | | |
|---|---|---|
| $E_1 = \frac{(a+b)(a+c)}{N}$ | $E_2 = \frac{(a+b)(b+d)}{N}$ | $a+b$ |
| $E_3 = \frac{(c+d)(a+c)}{N}$ | $E_4 = \frac{(c+d)(b+d)}{N}$ | $c+d$ |
| $a+c$ | $b+d$ | $N = a+b+c+d$ |

Gateway Classes : 7455 96 12 84

**Step – 3 : Calculate $\chi^2$ under $H_0$**

$$\chi^2 = \sum\left[\frac{(O_i - E_i)^2}{E_i}\right]$$

**Step – 4 : Degree of freedom $= (r-1)(S-1)$**

**Level of significance$(\propto)$**

$\propto = 5\% = 0.05$

From $\chi^2$-table $\chi^2_{cal} = 7$

**Step – 5 : Conclusion**

**(ii) If $\chi^2 < \chi^2_{0.05}$, then $H_0$ accepted**

**Two attributes A and B are independent**

**(ii) If $\chi^2 > \chi^2_{0.05}$, then $H_0$ is rejected.**

**Two attributes A and B are not independent**

Gateway Classes : 7455 9612 84

**Q. 1** **From the following table regarding the colour of eyes of father and son, test if the colour of son's eye is associated with that of the father.** **(A. K. T. U. 2021)**

| | Eye colour of son | |
|---|---|---|
| **Eye colour of father** | Light | Not light |
| Light | 471 | 51 |
| Not light | 148 | 230 |

Null Hypothesis($H_0$)

$H_0$: The colour of son's eye is not associated with that of father

contingency table

Observed Frequency

| | | |
|---|---|---|
| 471 $O_1$ | 51 $O_2$ | 522 |
| 148 $O_3$ | 230 $O_4$ | 378 |
| 619 | 281 | 900 |

↑ Grand total

Gateway Classes : 7455 961284

Expected Frequency

| | | |
|---|---|---|
| $E_1 = \frac{522 \times 619}{900}$ $= 359.02$ | $E_2 = \frac{522 \times 281}{900}$ $= 162.98$ | 522 |
| $E_3 = \frac{378 \times 619}{900}$ $= 259.98$ | $E_4 = \frac{378 \times 281}{900}$ $= 118.02$ | 378 |
| 619 | 281 | 900 |

$$\chi^2 = \sum \left[ \frac{(O_i - E_i)^2}{E_i} \right]$$

$$\chi^2 = \frac{(O_1 - E_1)^2}{E_1} + \frac{(O_2 - E_2)^2}{E_2} + \frac{(O_3 - E_3)^2}{E_3} + \frac{(O_4 - E_4)^2}{E_4}$$

$$= \frac{(471 - 359.02)^2}{359.02} + \frac{51 - 162.98}{162.98} + \frac{(148 - 259.98)^2}{259.98}$$

$$+ \frac{(230 - 118.02)^2}{118.02}$$

$$= 34.9290 + 76.9390 + 48.2326 + 106.3497$$

$$\chi^2_{cal} = 266.3497$$

Gateway Classes : 7455 961284

Degree of freedom

$d.f = (r-1)(s-1)$

$= (2-1)(2-1)$

$= 1$

Level of significance

$\alpha = 5\% = 0.05$

From $\chi^2$-table

$$\chi^2_{tab} = 3.841$$

conclusion

$\chi^2_{cal} = 266.3497$, $\chi^2_{tab} = 3.841$

$\therefore \chi^2_{cal} > \chi^2_{tab}$

$\therefore H_0$ is rejected

Hence

The colour of son's eye is associated with that of father's eye

Gateway Classes 7455 9612 84

## Table 4 : CHI-SQUARE

**Significant Values $\chi^2(\alpha)$ of Chi-Square Distribution Right Tail Areas for Given Probability $\alpha$,**

**$P = P_r(\chi^2 > \chi^2(\alpha)) = \alpha$**

**And $v$ is Degrees of Freedom (d.f.)**

| *Degree of freedom (v)* | *Probability (Level of Significance)* | | | | | | |
|---|---|---|---|---|---|---|---|
| | *0.99* | *0.95* | *0.50* | *0.10* | *0.05* | *0.02* | *0.01* |
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| 3 | .115 | .352 | 2.366 | 6.251 | 7.815 | 9.837 | 11.341 |
| 4 | .297 | .711 | 3.357 | 7.779 | 9.488 | 11.668 | 13.277 |
| 5 | .554 | 1.145 | 4.351 | 9.236 | 11.070 | 13.388 | 15.086 |
| 6 | .872 | 2.635 | 5.348 | 10.645 | 12.592 | 15.033 | 16.812 |
| 7 | 1.239 | 2.167 | 6.346 | 12.017 | 14.067 | 16.622 | 18.475 |
| 8 | 1.646 | 2.733 | 7.344 | 13.362 | 15.507 | 18.168 | 20.090 |
| 9 | 2.088 | 3.325 | 8.343 | 14.684 | 16.919 | 19.679 | 21.669 |
| 10 | 2.558 | 3.940 | 9.340 | 15.987 | 18.307 | 21.161 | 23.209 |
| 11 | 3.053 | 4.575 | 10.341 | 17.275 | 19.675 | 22.618 | 24.725 |
| 12 | 3.571 | 5.226 | 11.340 | 18.549 | 21.026 | 24.054 | 26.217 |
| 13 | 4.107 | 5.892 | 12.340 | 19.812 | 22.362 | 25.472 | 27.688 |
| 14 | 4.660 | 6.571 | 13.339 | 21.064 | 23.685 | 26.873 | 29.141 |
| 15 | 4.229 | 7.261 | 14.339 | 22.307 | 24.996 | 28.259 | 30.578 |
| 16 | 5.812 | 7.962 | 15.338 | 23.542 | 26.296 | 29.633 | 32.000 |
| 17 | 6.408 | 8.672 | 15.338 | 24.769 | 27.587 | 30.995 | 33.409 |
| 18 | 7.015 | 9.390 | 17.338 | 25.989 | 28.869 | 32.346 | 34.805 |
| 19 | 7.633 | 10.117 | 18.338 | 27.204 | 30.144 | 33.687 | 36.191 |
| 20 | 8.260 | 10.851 | 19.337 | 28.412 | 31.410 | 35.020 | 37.566 |
| 21 | 8.897 | 11.591 | 20.337 | 29.615 | 32.671 | 36.343 | 38.932 |
| 22 | 9.542 | 12.338 | 21.337 | 30.813 | 33.924 | 37.659 | 40.289 |
| 23 | 10.196 | 13.091 | 22.337 | 32.007 | 35.172 | 38.968 | 41.638 |
| 24 | 10.856 | 13.848 | 23.337 | 32.196 | 36.415 | 40.270 | 42.980 |
| 25 | 11.524 | 14.611 | 24.337 | 34.382 | 37.65 | 41.566 | 44.314 |
| 26 | 12.198 | 15.379 | 25.336 | 35.363 | 38.885 | 41.856 | 45.642 |
| 27 | 12.879 | 16.151 | 26.336 | 36.741 | 40.113 | 41.140 | 46.963 |
| 28 | 13.565 | 16.928 | 27.336 | 37.916 | 41.337 | 45.419 | 48.278 |
| 29 | 14.256 | 17.708 | 28.336 | 39.087 | 42.557 | 46.693 | 49.588 |
| 30 | 14.933 | 18.493 | 29.336 | 40.256 | 43.773 | 47.962 | 50.892 |

Gateway Classes 9455 66284

**Q. 2** *To test the effectiveness of inoculation against cholera, the following table was obtained*

| | Attacked | Not attacked | Total |
|---|---|---|---|
| Inoculated | 30 $O_1$ | 160 $O_2$ | 190 |
| Not inoculated | 140 $O_3$ | 460 $O_4$ | 600 |
| Total | 170 | 620 | 790 |

*(The figures represent the number of persons.)* **(A.K.T.U. 2009, 2018)**

*Use $\chi^2$ – test to defend or refute the statement that the inoculation prevents attack from cholera.*

Null Hypothesis ($H_0$)

$H_0$: The inoculation does not prevent from cholera attack

contingency table

observed Frequency

| | | |
|---|---|---|
| 30 $O_1$ | 160 $O_2$ | 190 |
| 140 $O_3$ | 460 $O_4$ | 600 |
| 170 | 620 | 790 |

Gateway Classes : 7455 961284

| | | |
|---|---|---|
| $E_1 = \frac{190 \times 170}{790}$ <br> $= 40.89$ | $E_2 = \frac{190 \times 620}{790}$ <br> $= 149.11$ | 190 |
| $E_3 = \frac{600 \times 170}{790}$ <br> $= 129.11$ | $E_4 = \frac{600 \times 620}{790}$ <br> $= 470.89$ | 600 |
| 170 | 620 | 790 |

$$\chi^2 = \sum \left[ \frac{(O_i - E_i)^2}{E_i} \right]$$

$$\chi^2 = \frac{(O_1 - E_1)^2}{E_1} + \frac{(O_2 - E_2)^2}{E_2} + \frac{(O_3 - E_3)^3}{E_3} + \frac{(O_4 - E_4)^2}{E_4}$$

$$\chi^2 = \frac{(30 - 40.89)^2}{40.89} + \frac{(160 - 149.11)^2}{149.11} + \frac{(140 - 129.11)^2}{129.11} + \frac{(460 - 470.89)^2}{470.89}$$

$$\chi^2 = 2.90 + 0.795 + 0.918 + 0.252$$

Gateway Classes: 7455 9612 84

$$\chi^2_{cal} = 4.865$$

Degree of freedom

$$d.f = (r-1)(s-1)$$
$$= (2-1)(2-1)$$
$$= 1$$

Level of significance

$$\alpha = 5\% = 0.05$$

From $\chi^2$-table

$$\chi^2_{tab} = 3.841$$

conclusion

$$\chi_{cal} = 4.865$$

$$\chi^2_{tab} = 3.841$$

$\therefore \chi^2_{cal} > \chi^2_{tab}$

$\therefore H_0$ is rejected

Hence

The inoculation prevent from cholera Attack

Gateway Classes : 7455 961234

## Table 4 : CHI-SQUARE

**Significant Values $\chi^2(\alpha)$ of Chi-Square Distribution Right Tail Areas for Given Probability $\alpha$,**

**$P = P_r(\chi^2 > \chi^2(\alpha)) = \alpha$**

**And $v$ is Degrees of Freedom (d.f.)**

| *Degree of freedom (v)* | *Probability (Level of Significance)* | | | | | | |
|---|---|---|---|---|---|---|---|
| | *0.99* | *0.95* | *0.50* | *0.10* | *0.05* | *0.02* | *0.01* |
| 1 | .000157 | .00393 | .455 | 2.706 | 3.841 | 5.214 | 6.635 |
| 2 | .0201 | .103 | 1.386 | 4.605 | 5.991 | 7.824 | 9.210 |
| 3 | .115 | .352 | 2.366 | 6.251 | 7.815 | 9.837 | 11.341 |
| 4 | .297 | .711 | 3.357 | 7.779 | 9.488 | 11.668 | 13.277 |
| 5 | .554 | 1.145 | 4.351 | 9.236 | 11.070 | 13.388 | 15.086 |
| 6 | .872 | 2.635 | 5.348 | 10.645 | 12.592 | 15.033 | 16.812 |
| 7 | 1.239 | 2.167 | 6.346 | 12.017 | 14.067 | 16.622 | 18.475 |
| 8 | 1.646 | 2.733 | 7.344 | 13.362 | 15.507 | 18.168 | 20.090 |
| 9 | 2.088 | 3.325 | 8.343 | 14.684 | 16.919 | 19.679 | 21.669 |
| 10 | 2.558 | 3.940 | 9.340 | 15.987 | 18.307 | 21.161 | 23.209 |
| 11 | 3.053 | 4.575 | 10.341 | 17.275 | 19.675 | 22.618 | 24.725 |
| 12 | 3.571 | 5.226 | 11.340 | 18.549 | 21.026 | 24.054 | 26.217 |
| 13 | 4.107 | 5.892 | 12.340 | 19.812 | 22.362 | 25.472 | 27.688 |
| 14 | 4.660 | 6.571 | 13.339 | 21.064 | 23.685 | 26.873 | 29.141 |
| 15 | 4.229 | 7.261 | 14.339 | 22.307 | 24.996 | 28.259 | 30.578 |
| 16 | 5.812 | 7.962 | 15.338 | 23.542 | 26.296 | 29.633 | 32.000 |
| 17 | 6.408 | 8.672 | 15.338 | 24.769 | 27.587 | 30.995 | 33.409 |
| 18 | 7.015 | 9.390 | 17.338 | 25.989 | 28.869 | 32.346 | 34.805 |
| 19 | 7.633 | 10.117 | 18.338 | 27.204 | 30.144 | 33.687 | 36.191 |
| 20 | 8.260 | 10.851 | 19.337 | 28.412 | 31.410 | 35.020 | 37.566 |
| 21 | 8.897 | 11.591 | 20.337 | 29.615 | 32.671 | 36.343 | 38.932 |
| 22 | 9.542 | 12.338 | 21.337 | 30.813 | 33.924 | 37.659 | 40.289 |
| 23 | 10.196 | 13.091 | 22.337 | 32.007 | 35.172 | 38.968 | 41.638 |
| 24 | 10.856 | 13.848 | 23.337 | 32.196 | 36.415 | 40.270 | 42.980 |
| 25 | 11.524 | 14.611 | 24.337 | 34.382 | 37.65 | 41.566 | 44.314 |
| 26 | 12.198 | 15.379 | 25.336 | 35.363 | 38.885 | 41.856 | 45.642 |
| 27 | 12.879 | 16.151 | 26.336 | 36.741 | 40.113 | 41.140 | 46.963 |
| 28 | 13.565 | 16.928 | 27.336 | 37.916 | 41.337 | 45.419 | 48.278 |
| 29 | 14.256 | 17.708 | 28.336 | 39.087 | 42.557 | 46.693 | 49.588 |
| 30 | 14.933 | 18.493 | 29.336 | 40.256 | 43.773 | 47.962 | 50.892 |

Gateway Classes 9455 62284

**Q. 3** *In an experiment on the immunisation of cattle from a disease, the following results are obtained. Derive your inferences on the efficiency of the vaccine.* **(A. K. T. U. 2021)**

| | *Affected* | *Unaffected* |
|---|---|---|
| *Inoculated with vaccine* | 12 $O_1$ | 28 $O_2$ |
| *Not incoulated* | 13 $O_3$ | 7 $O_4$ |

$$\chi^2_{cal} = 6.73$$

Gateway Classes : 7455 9612 93

## Topic : $\chi^2$ – test of as a test of independence

**Q. 1** In an experiment on the immunisation of goats from anthrox, the following results were obtained. Derive your inferences on the efficiency of the vaccine.

| | Died anthrox | Survived |
|---|---|---|
| Inoculated with vaccine | 2 | 10 |
| Not incoulated | 6 | 6 |

**Q. 2** By using $\chi^2$ – test, find out whether there is any association between income level and type of schooling:

| Income | Public School | Govt. school |
|---|---|---|
| Low | 200 | 400 |
| High | 1000 | 400 |

**Q. 3** The following data is collected on two characters:

| | Smokers | Non smokers |
|---|---|---|
| Literate | 83 | 57 |
| Illiterate | 45 | 68 |

Based on this information can you say that there is no relation between habit of smoking and literacy.

Gateway Classes: 7455 961 284

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# Thank You