ENGINEERING MATHEMATICS

UNIT-4 : *Statistical Techniques – II*

Lec-08

Today's Target

- $Normal\ Probability\ Distribution (Part - I)$
- PYQs
- DPPs

Normal Distribnution

- The normal distribution is a continous distribution based on continous random variable
- A continous random variable X is said to have a normal distribution with parameters $\mu(Mean)$ and $\sigma^2(Variance)$ if the probability density function is given by

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

(i) $f(x) \geq 0$

(ii) $\int_{-\infty}^{\infty} f(x)dx = 1.$

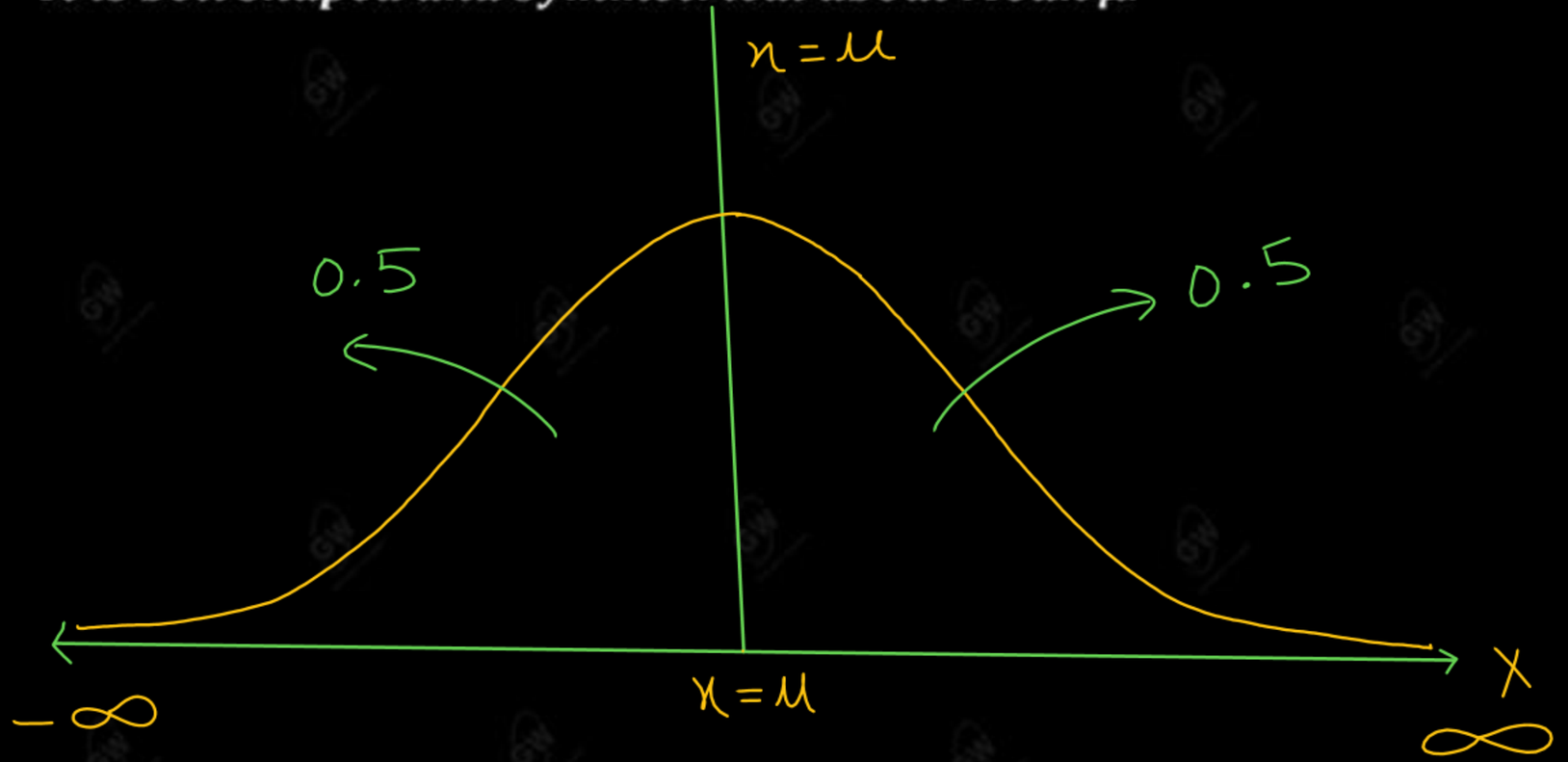
$-\infty < x < \infty$

μ = Mean

σ = Standard deviation

Normal Curve

- The graph of normal *Distribnution is called normal curve.*
- *It is bell shaped and symmetrical about Mean μ.*



- *In normal distribution Mean, median and mode coincide.*
- *Total area under the curve above x − axis is 1*

Standard Normal Distribution

- If X is a Normal random variable with mean μ and standard deviation σ, then a random Variable Z with Mean $= 0$ and standard deviation $= 1$ is called standard normal random variable.

$$Z = \frac{x-\mu}{\sigma},$$

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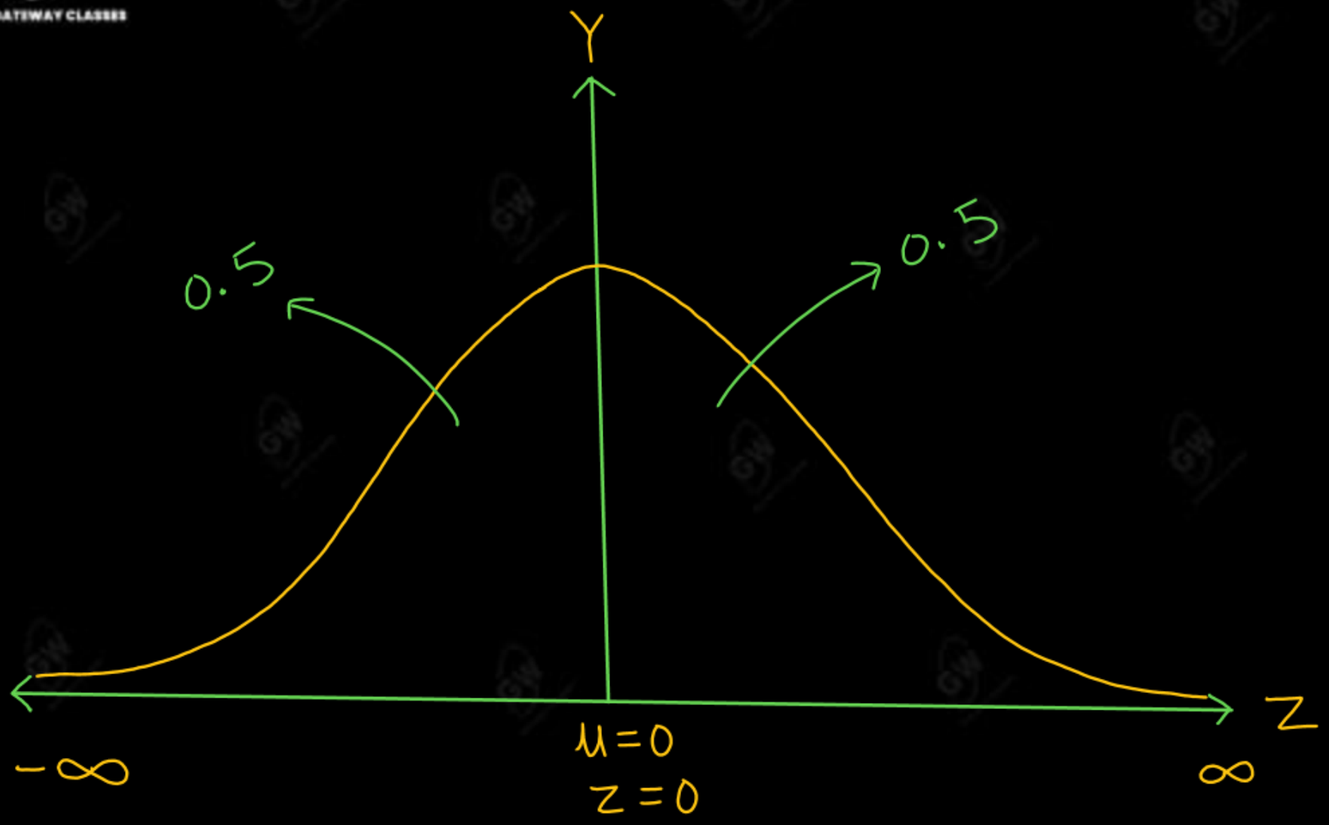
- The probability density function for the normal distribution in standard form is given by

$$f(Z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(Z)^2}$$

- Probability

$$P(z_1 < z < z_2) = \int_{z_1}^{z_2} f(z)dz$$

Standard Normal Curve



Probability

$P(z_1 < z < z) =$ *Area under standard normal curve between* $z = z_1$ *and* $z = z_2$

Q.1 *If X is a normal variate with mean 80 and standard deviation 10, find probability $P(X \leq 100)$.*

Given

$\mu = 80$

$\sigma = 10$

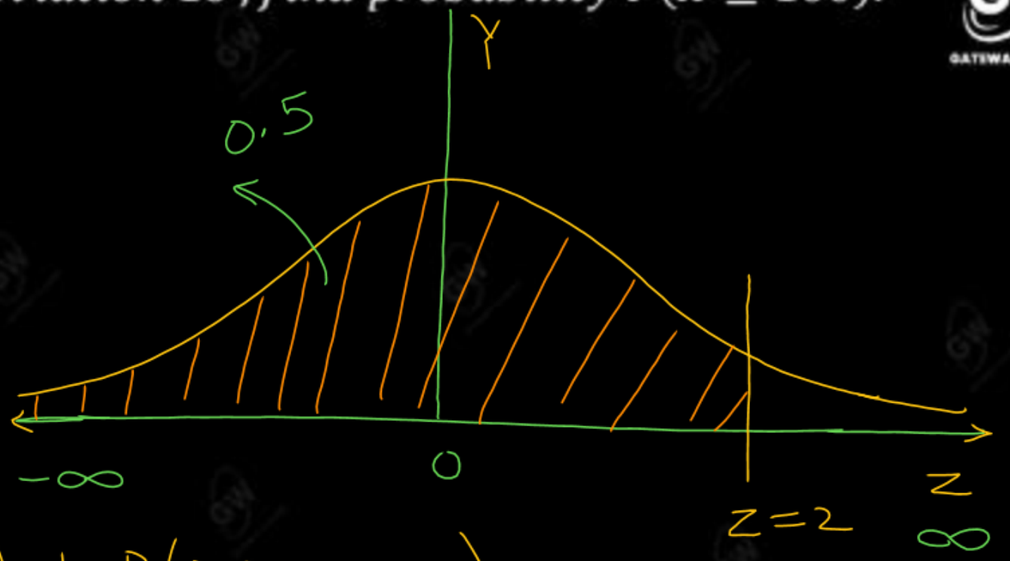
$x = 100$

$$z = \frac{x - \mu}{\sigma}$$

$$z = \frac{100 - 80}{10}$$

$$\boxed{z = 2}$$

$$P(x \leq 100) = P(z \leq 2)$$



$$= P(-\infty < z < 0) + P(0 < z \leq 2)$$

$$= 0.5 + P(0 < z \leq 2)$$

$$= 0.5 + 0.4772$$

$$\boxed{P(x \leq 100) = 0.9772}$$

Q.2 *The life of army shoes is normally distributed with mean 8 months and standard deviation 2 months. If 5000 pairs are insured, how many pairs would be expected to need replacement after 12 months?* $\left[\text{Given that } P(z \geq 2) = 0.0228 \text{ and } z = \frac{x-\mu}{\sigma}\right]$. **(A.K.T.U. 2018)**

Given

$\mu = 8$

$\sigma = 2$

$N = 5000$

Let $x = 12$

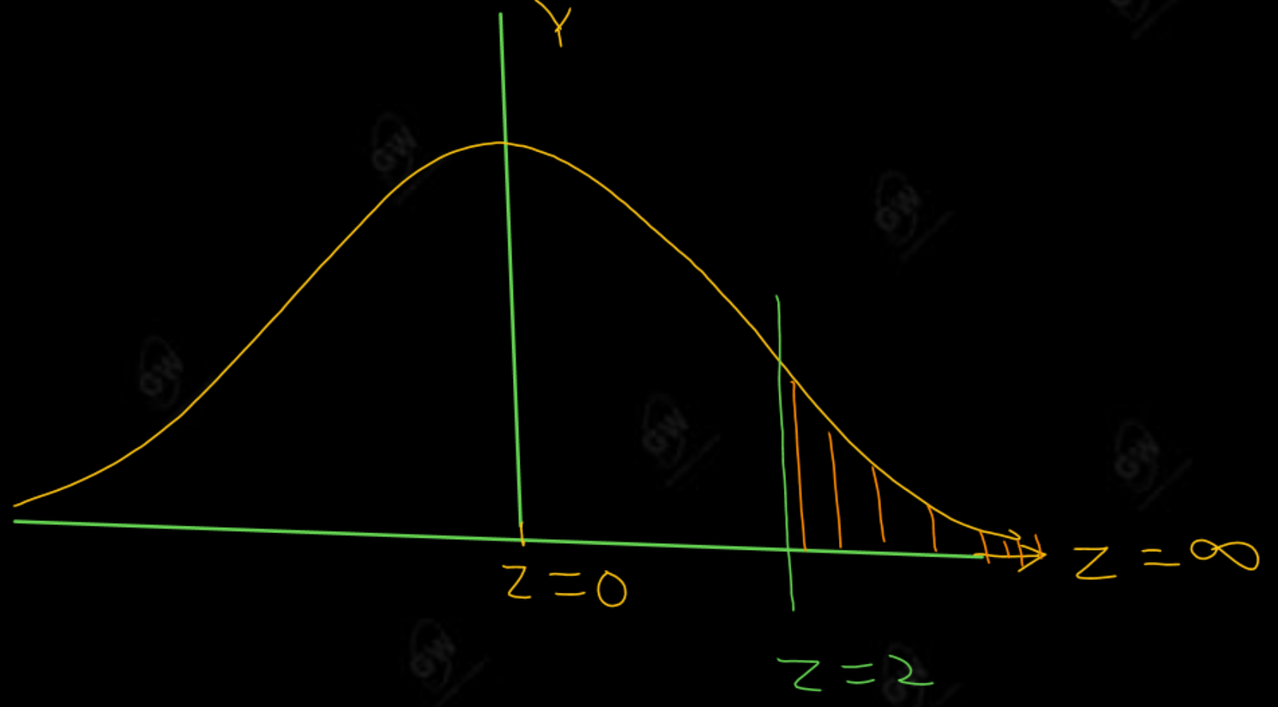
$P(x \geq 12) = ?$

$$z = \frac{x-\mu}{\sigma}$$

$$= \frac{12-8}{2}$$

$$\boxed{z = 2}$$

$$P(x \geq 12) = P(z \geq 2)$$



$$P(x \geq 12) = P(0 < z < \infty) - P(0 < z < 2)$$

$P(x \geq 12) = 0.5 - 0.4772$

$= 0.0228$

Number of pairs having life more then 12 months

$= 5000 \times 0.0228$

$= 114$

No. of pairs replaced after 12 months $= 5000 - 114$

$= 4886$

Q. 3 *Assume mean height of soldiers to be 68.22 inches with a variance of 10.8 inches square. How many soldiers in a regiment of 1,000 would you expect to be over 6 feet tall, given that the area under the standard normal curve between z = 0 and z = 0.35 is 0.1368 and between z = 0 and z = 1.15 is 0.3746.* [AKTU (C.O.) 2021

Given

$\mu = 68.22$

$\sigma^2 = 10.8$

$\sigma = \sqrt{10.8}$

$N = 1000$

$x = 6$ feet

$x = 6 \times 12$

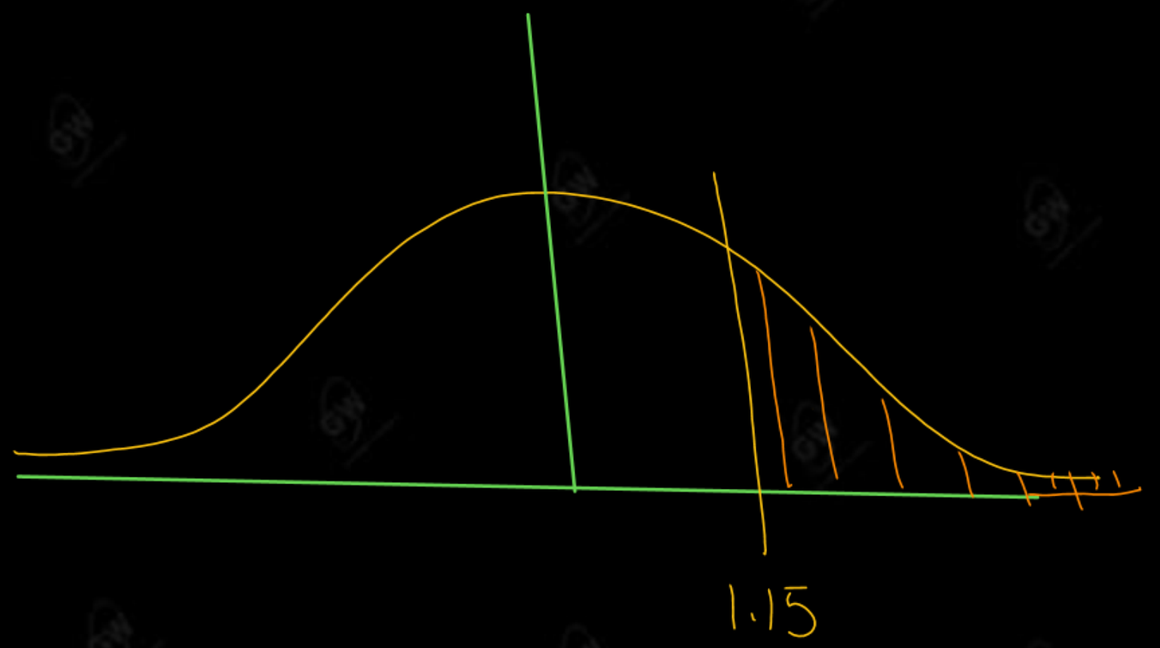
$x = 72$ inches

$$z = \frac{x - \mu}{\sigma}$$

$$z = \frac{72 - 68.22}{\sqrt{10.8}}$$

$z = 1.15$

$P(x > 72) = P(z > 1.15)$



$$P(x > 72) = P(z > 1.15)$$

$$= P(0 < z < \infty) - P(0 < z < 1.15)$$

$$= 0.5 - 0.3746$$

$$= 0.1254$$

$$\text{No. of soldiers} = 1000 \times 0.1254$$

$$= 125 \text{ (approx)}$$

Q.4 *The daily wages of 1000 workers are distributed around a mean of Rs. 140 and with a standard deviation of Rs. 10. Estimate the number of workers whose daily wages*

(i) between 140 and 144 *(ii) less than 126* *(iii) more than 160.* **(AKTU. 2021)**

Given

$N = 1000$

$\mu = 140$

$\sigma = 10$

$$z = \frac{x - \mu}{\sigma}.$$

$$z = \frac{x - 140}{10} \quad \text{---} \quad (1)$$

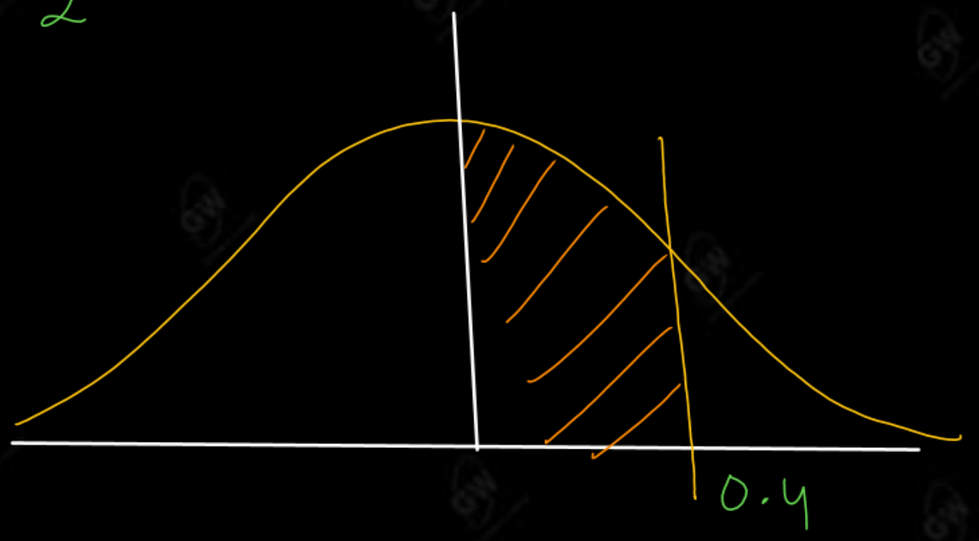
(i) $x_1 = 140$, $x_2 = 144$

$$z_1 = \frac{140 - 140}{10} = 0$$

$$z_1 = 0$$

$$z_2 = \frac{144 - 140}{10} = \frac{4}{10}$$

$z_2 = 0.4$



$$P(140 < x < 144) = P(0 < z < 0.4)$$

$$= 0.1554$$

Expected number of workers

$= 0.1554 \times 1000$

$= 155.4$

$= 155$

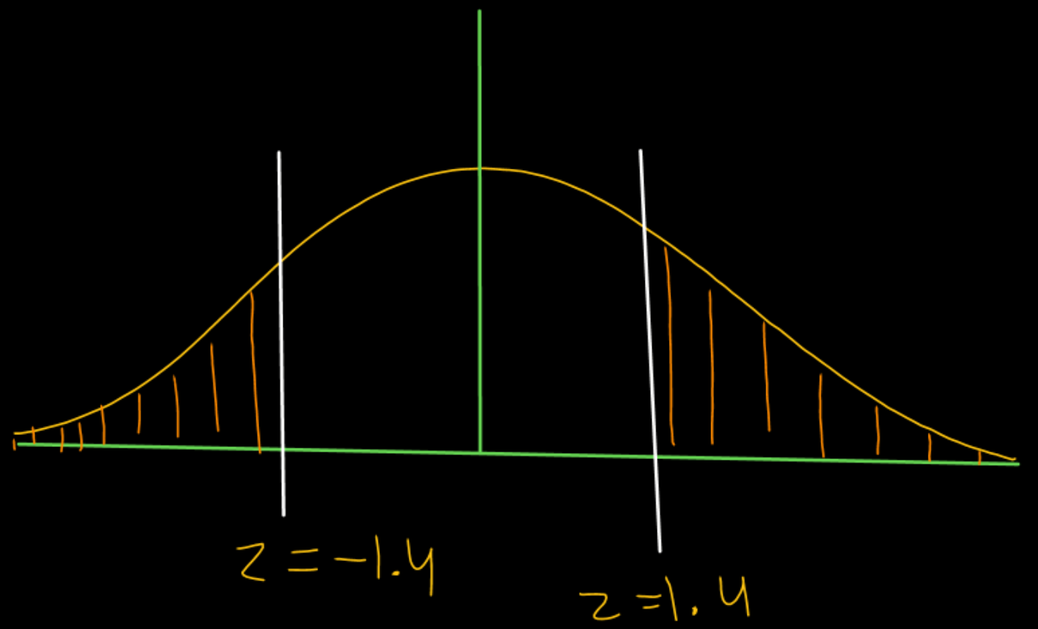
(ii) $x = 126$

$$Z = \frac{x - 140}{10}$$

$$= \frac{126 - 140}{10}$$

$z = -1.4$

$P(x < 126) = P(z < -1.4)$



$P(x < 126) = P(z > 1.4)$

$= P(0 < z < \infty) - P(0 < z < 1.4)$

$P(x<126) = 0.5 - 0.4192$

$= 0.0808$

Expected no. of workers

$= 1000 \times 0.0808$

$= 80.8$

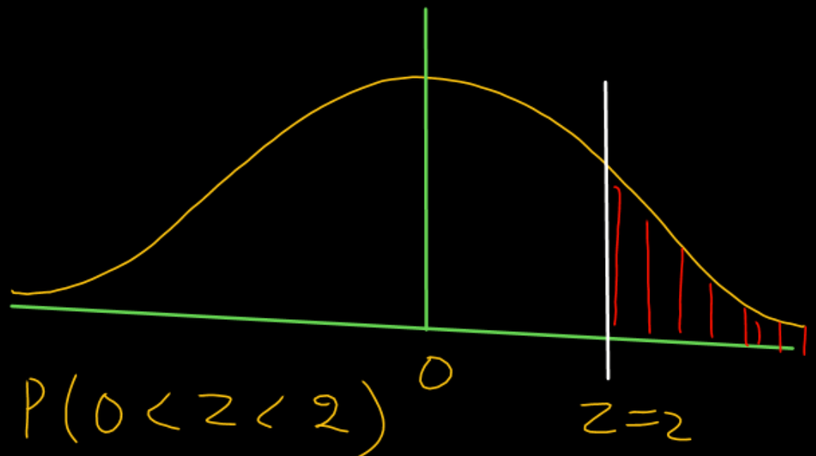
$= 81$ (approx)

(iii) $x = 160$

$z = \frac{160 - 140}{10} = \frac{20}{10} = 2$

$z = 2$

$P(x > 160) = P(z > 2)$



$= P(0 < z < \infty) - P(0 < z < 2)$

$= 0.5 - 0.4772 = 0.0228$

Expected no. of workers

$= 0.0228 \times 1000 = 22.8$

$= 23$

Topic : Normal Probability Distribution (Part – I)

Q. 1 Define:
(i) Binomial distribution
(ii) Poisson distribution
(iii) Normal distribution. (AKTU 2018)

Q. 2 If X is a normal variate with mean 30 and S. D. 5, find the probabilities that
(i) $26 \leq X \leq 40$
(ii) $X \geq 45$ and
(iii) $[X - 30 | > 5$

Ans. 2(i) 0.7653 (ii) 0.00135 (iii) 0.3174

Q. 3 If Z is a standard normal variable, find the following probabilities:

(i) $P(Z < 1.2)$
(ii) $P(Z > -1.2)$
(iii) $P(-1.2 < Z < 1.3)$.

Ans. 3(i) 0.8849 (ii) 0.8849 (iii) 0.7881

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