MATHS-4

ONE SHOT

Unit-I

Partial Diff. Equations



Live 8 March 10 AM

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GATEWAY CLASSES

UNIT-1 Partial Differential Equations

Syllabus

Module-I: Partial Differential Equations

- Origin of Partial Differential Equations, Linear and Non-Linear Partial Differential Equations of first order
- Lagrange's Equations method to solve Linear Partial Differential Equations
- Charpit's method to solve Non-Linear Partial Differential Equations,
- Solution of Linear Partial Differential Equation of Higher-order with constant coefficients
- Equations reducible to linear partial differential equations with constant coefficients.

Engineering Mathematics-IV

Unit: Partial Differential Equations (PDE)

Today's Target

- Introduction to Partial Differential Equations
- Univ. Questions
- DDPs

GATEWAY CLASSES



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Differential Equation

An equation containing the dependent variable, independent variable and the derivative of the dependent variable with respect to the independent variable is known as a differential equation.

$y = f(x)$

$\frac{dy}{dx}$

$\frac{d^2y}{dx^2}$

$\frac{d^ny}{dx^n}$

$z = f(x, y)$

$\frac{\partial z}{\partial x}$

$\frac{\partial z}{\partial y}$

$\frac{\partial^2 z}{\partial x^2}$

$\frac{\partial^2 z}{\partial y^2}$

$\frac{\partial^2 z}{\partial x \partial y}$

GATEWAY CLASSES

Differential Equation

Ordinary Differential Equation

Partial Differential Equation

Ordinary Differential Equation (ODE)

A differential equation containing ordinary derivatives :

(i) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$

(ii) $\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = x^2$

GATEWAY CLASSES

Partial Differential Equation (PDE)

A differential equation containing partial derivatives :

(i) $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = x$

(ii) $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = xy$

(iii) $\left(\frac{\partial z}{\partial y}\right)^2 = 2x\frac{\partial^3 z}{\partial x^3}$

(iv) $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = xyz$

GATEWAY CLASSES

Note :

$$\frac{\partial z}{\partial x} = p$$

$$\frac{\partial z}{\partial y} = q$$

Partial Derivative of first order

$$\frac{\partial^2 z}{\partial x^2} = r$$

$$\frac{\partial^2 z}{\partial x \partial y} = s$$

$$\frac{\partial^2 z}{\partial y^2} = t$$

Partial Derivative of second order

GATEWAY CLASSES

Order and Degree of a PDE

Order of PDE

Order of PDE is the order of highest order derivatives :

(i) $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$ $\quad O = 1$

(ii) $\frac{\partial^2 z}{\partial x^2} + x\frac{\partial z}{\partial y}$ $\quad O = 2$

(iii) $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = xy$ $\quad O = 2$

(iv) $\left(\frac{\partial z}{\partial y}\right)^2 = 2x\frac{\partial^3 z}{\partial x^3}$ $\quad O = 3$

GATEWAY CLASSES

Degree of PDE

Degree of PDE is degree of highest order derivatives in equation, provided it is free from radical and fractional power.

(i) $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = xy$ — $O = 1$, $D = 1$

(ii) $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^3 z}{\partial y^3} = 2x \frac{\partial z}{\partial x}$ — $O = 3$, $D = 1$

(iii) $\frac{\partial^2 z}{\partial x^2} = \left(1 + \frac{\partial z}{\partial y}\right)^{1/3}$ — $O = 2$, $D = 3$

(iv) $\frac{\partial^3 z}{\partial y^3} = \left(1 - \frac{\partial z}{\partial x}\right)^{1/2}$ — $O = 3$, $D = 2$

$\left(\frac{\partial^2 z}{\partial x^2}\right)^3 = \left(1 + \frac{\partial z}{\partial y}\right)^{3/3}$

$\left(\frac{\partial^2 z}{\partial x^2}\right)^3 = 1 + \frac{\partial z}{\partial y}$

$\left(\frac{\partial^3 z}{\partial y^3}\right)^2 = \left(1 - \frac{\partial z}{\partial x}\right)^{2/2}$

GATEWAY CLASSES

Linear PDE

A PDE is said to be linear, if dependent variable and its partial derivatives present only in first degree and not multiplied together.

Example

(A) $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = xyz$ — *Linear*

(B) $yp + xq = x$ — *Linear*

$p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$

(C) $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = z + xy$ — *Linear*

$\frac{\partial^2 u}{\partial x^2} + y\frac{\partial^2 u}{\partial y^2} + z\frac{\partial^2 u}{\partial z^2} = xyz$

GATEWAY CLASSES

Non-Linear PDE

A PDE which is not linear is called non-linear PDE

Example

(1) $\left(\frac{\partial z}{\partial x}\right)^2 + \frac{\partial^3 z}{\partial y^3} = 2x\left(\frac{\partial z}{\partial x}\right)$ — *Non-Linear*

(2) $z\frac{\partial z}{\partial x} + \frac{\partial^2 z}{\partial y^2} = x$ — *Non-Linear*

(3) $u\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = x$ — *Non-Linear*

$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = xy + z^2$

GATEWAY CLASSES

Type of first order PDE

(i) Linear
(ii) Semi – Linear
(iii) Quasi – Linear

$p = \frac{\partial z}{\partial x}$

$q = \frac{\partial z}{\partial y}$

(1) Linear :

A first order PDE $f(x, y, z, p, q) = 0$ is known as linear if it is linear in p, q, and z and is of the form

$$P(x,y)\,p + Q(x,y)\,q = R(x,y)\,z + s(x,y)$$

GATEWAY CLASSES

Example

$$xp + yq = z + xy$$

$$y^2x^2\,p + x^3y^2\,q = xyz + x^3y^3$$

(2) Semi – Linear:

A first order PDE $f(x, y, z, p, q) = 0$ is known as semi-linear if it is linear in p, q, and is of the form

$$P(x, y)\,p + Q(x, y)\,q = R(x, y, z)$$

Example

$$x^2 y p + x^2 y^2 q = x^2 y^2 z^2$$

GATEWAY CLASSES

(3) Quasi – Linear :

A first order PDE $f(x, y, z, p, q) = 0$ is known as Quasi-linear if it is linear in p and q and is of the form

$$P(x, y, z)\, p + Q(x, y, z)\, q = R(x, y, z)$$

Example

$$x^2 z^2 p + y^2 x q = x y z$$

$$(x^2 - yz)\, p + (y^2 - zx)\, q = z^2 - x y$$

GATEWAY CLASSES

Engineering Mathematics

Unit: Partial Differential Equations (PDE)

Lec-2

Today's Target

- Formation of PDE [Part-1]
- Univ. Questions
- DDPs

GATEWAY CLASSES



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Note : $z = f(x, y)$

$$\frac{\partial z}{\partial x} = p$$

$$\frac{\partial z}{\partial y} = q$$

Partial Derivative of first order

$$\frac{\partial^2 z}{\partial x^2} = r$$

$$\frac{\partial^2 z}{\partial x \partial y} = s$$

$$\frac{\partial^2 z}{\partial y^2} = t$$

Partial Derivative of second order

GATEWAY CLASSES

Formation of PDE

By elimination of arbitrary constants

By elimination of arbitrary functions

By elimination of arbitrary constants

Let $F(x, y, z, a, b) = 0$ ——— (1)

be the given function, where a and b are arbitrary constants

Step 1 - Find $\frac{\partial F}{\partial x} = 0$ ——— (2)

Step 2 - Find $\frac{\partial F}{\partial y} = 0$ ——— (3)

Step 3 - Eliminate a, b (arbitrary constants) from equations (1), (2) and (3)

GATEWAY CLASSES

Now, we get required PDE

CASE - I

If No. of arbitrary constants < No. of independent variables

Then we get more than one PDE of order one

GATEWAY CLASSES

Q.1 Find PDE by eleminating a from $z = ax + y$.

Given

$$z = ax + y \quad -(1)$$

Partially Diff. wrt x and y

$$\frac{\partial z}{\partial x} = a \quad -(2)$$

$$\frac{\partial z}{\partial y} = 1 \quad -(3)$$

From eqn (1) and (2)

$$z = x\frac{\partial z}{\partial x} + y$$

$$\boxed{z = px + y}$$

$$\boxed{q = 1}$$

} Required PDE

GATEWAY CLASSES

CASE - II

If No. of arbitrary constants = No. of independent variables

Then we get only one PDE of order one

GATEWAY CLASSES

Q.2 Form partial differential equation by eliminating the arbitrary constants:

$$z = ax + a^2 + b^2 + by$$

Given

$$z = ax + a^2 + b^2 + by \quad \text{—①}$$

Partially diff. w.r.t. x and y

$$\frac{\partial z}{\partial x} = a + 0 + 0 + 0$$

$$\frac{\partial z}{\partial x} = a \quad \text{—②}$$

$$\frac{\partial z}{\partial y} = 0 + 0 + 0 + b$$

$$\frac{\partial z}{\partial y} = b \quad \text{—③}$$

From eqn ①, ② and ③

$$z = x\frac{\partial z}{\partial x} + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + y\frac{\partial z}{\partial y}$$

$$\boxed{z = px + p^2 + q^2 + qy}$$

GATEWAY CLASSES

Q.3 Form partial differential equation by eliminating the arbitrary constants:

$$z = (x + a)(y + b)$$

Given

$z = (x+a)(y+b)$ — ①

Partially diff w.r.t x and y

$\frac{\partial z}{\partial x} = (y+b)(1+0)$

$\frac{\partial z}{\partial x} = (y+b)$ — ②

$\frac{\partial z}{\partial y} = (x+a)(1+0)$

$\frac{\partial z}{\partial y} = (x+a)$ — ③

From ①, ② and ③

$z = \frac{\partial z}{\partial y} \times \frac{\partial z}{\partial x}$

$\boxed{z = pq}$

GATEWAY CLASSES

Q.4 Form the partial differential equation by eliminating a and b from $z = (x^2 + a)(y^2 + b)$. [2012]

Given

$$z = (x^2+a)(y^2+b) \quad -①$$

Partially diff. w.r.t x and y

$$\frac{\partial z}{\partial x} = (y^2+b)(2x+0)$$

$$\frac{\partial z}{\partial x} = 2x(y^2+b)$$

$$(y^2+b) = \frac{1}{2x} \times \frac{\partial z}{\partial x} \quad -②$$

$$\frac{\partial z}{\partial y} = (x^2+a)(2y+0)$$

$$\frac{\partial z}{\partial y} = 2y(x^2+a)$$

$$(x^2+a) = \frac{1}{2y} \times \frac{\partial z}{\partial y} \quad -③$$

From ①, ② and ③

$$z = \frac{1}{2y} \times \frac{\partial z}{\partial y} \times \frac{1}{2x} \times \frac{\partial z}{\partial x}$$

$$4xyz = \frac{\partial z}{\partial y} \times \frac{\partial z}{\partial x}$$

$$4xy = pq$$

GATEWAY CLASSES

Q.5 ***Find the partial differential equation of all spheres whose centers lie on z-axis and given by equations*** $x^2 + y^2 + (z-a)^2 = b^2$ ***; a and b being constants.*** **[2017]**

Given

$$x^2+y^2+(z-a)^2 = b^2 \quad \text{—①}$$

Partially diff w.r.t x and y

$$2x + 0 + 2(z-a)\frac{\partial z}{\partial x} = 0$$

$$2x + 2(z-a)\frac{\partial z}{\partial x} = 0$$

$$2(z-a)\frac{\partial z}{\partial x} = -2x$$

$$(z-a)p = -x$$

$$(z-a) = -\frac{x}{p} \quad \text{—②}$$

$$0 + 2y + 2(z-a)\frac{\partial z}{\partial y} = 0$$

$$2(z-a)q = -2y$$

$$z-a = -\frac{y}{q} \quad \text{—③}$$

From equation ② and ③

$$+\frac{x}{p} = +\frac{y}{q}$$

$$\boxed{qx - py = 0}$$

GATEWAY CLASSES

CASE - III

If No. of arbitrary constants > No. of independent variables

more than one

Then we get higher order PDE

GATEWAY CLASSES

Q.6 Find PDE by eleminating a, b, c : $z = ax + by + cxy$

$z = ax + by + cxy$ — (1)

Partially diff wrt x

$\frac{\partial z}{\partial x} = a + 0 + cy$

$\frac{\partial z}{\partial x} = a + cy$ — (2)

Again partially diff wrt x

$\frac{\partial^2 z}{\partial x^2} = 0$ — (3)

partially diff (1) wrt y

$\frac{\partial z}{\partial y} = 0 + b + cx$ — (4)

Again partially diff wrt y

$\frac{\partial^2 z}{\partial y^2} = 0$ — (5)

Multiply eqn (2) by x and (4) by y

$x\frac{\partial z}{\partial x} = ax + cxy$ — (6)

$y\frac{\partial z}{\partial y} = by + cxy$ — (7)

Adding (6) and (7)

GATEWAY CLASSES

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = ax + cxy + by + cxy$$

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = z + cxy \quad -(8)$$

From (2)

$$\frac{\partial z}{\partial x} = a + cy$$

Partially diff wrt y

$$\frac{\partial^2 z}{\partial x \partial y} = c$$

Put c in (8)

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = z + \frac{\partial^2 z}{\partial x \partial y} \times xy$$

$$\boxed{x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = z + xy\frac{\partial^2 z}{\partial x \partial y}}$$

GATEWAY CLASSES

Engineering Mathematics

Unit: Partial Differential Equations (PDE)

Lec-3

Today's Target

- Formation of PDE [Part-2]
- Univ. Questions
- DDPs

GATEWAY CLASSES



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Formation of PDE

By elimination of arbitrary constants

By elimination of arbitrary functions

Note :

$$\frac{\partial z}{\partial x} = p \qquad \frac{\partial z}{\partial y} = q$$

$$\frac{\partial^2 z}{\partial x^2} = r \qquad \frac{\partial^2 z}{\partial x \partial y} = s \qquad \frac{\partial^2 z}{\partial y^2} = t$$

GATEWAY CLASSES

By elimination of arbitrary function

CASE - 1

When given function is of the form of

$$z = f(x, y)$$

When single function is given

NOTE : Order of PDE = 1

GATEWAY CLASSES

Q.1 ***Form the partial differential equation by eliminating the arbitrary function from*** $z = f(x^2 - y^2)$ ***(Sem. 2015)***

$$z = f(x^2 - y^2) \quad \text{—— (1)}$$

Partially Diff wrt x

$$\frac{\partial z}{\partial x} = f'(x^2 - y^2) \times \frac{\partial}{\partial x}(x^2 - y^2)$$

$$\frac{\partial z}{\partial x} = f'(x^2 - y^2) \times (2x - 0)$$

$$p = 2x\, f'(x^2 - y^2) \quad \text{—— (2)}$$

Partially Diff wrt y

$$\frac{\partial z}{\partial y} = f'(x^2 - y^2) \times \frac{\partial}{\partial y}(x^2 - y^2)$$

$$\frac{\partial z}{\partial y} = f'(x^2 - y^2) \times (0 - 2y)$$

$$q = -2y\, f'(x^2 - y^2) \quad \text{— (3)}$$

Divide (2) ÷ (3)

$$\frac{p}{q} = \frac{2x\, f'(x^2 - y^2)}{-2y\, f'(x^2 - y^2)}$$

$$\frac{p}{q} = \frac{x}{-y}$$

$$qx = -py$$

$$\boxed{qx + py = 0}$$

GATEWAY CLASSES

CASE - 2

When given function is of the form of

$$z = f_1(x, y) + f_2(x, y)$$

Note: Order of PDE > 1

GATEWAY CLASSES

Q.2 ***Form the partial differential equation by eliminating the arbitrary function from the following : $z = f(x + it) + g(x - it)$.*** **(Sem. 2011)**

$$z = f(x+it) + g(x-it) \quad \text{—①}$$

Partially diff wrt x

$$\frac{\partial z}{\partial x} = f'(x+it)\times 1 + g'(x-it)\times 1$$

Again partially diff wrt x

$$\frac{\partial^2 z}{\partial x^2} = f''(x+it) + g''(x-it)$$

—②

Partially Diff wrt t

$$\frac{\partial z}{\partial t} = f'(x+it)\times i + g'(x-it)\times(-i)$$

Again partially diff wrt t

$$\frac{\partial^2 z}{\partial t^2} = f''(x+it)\times i^2 + g''(x-it)\times i^2$$

$$\frac{\partial^2 z}{\partial t^2} = i^2\left[f''(x+it) + g''(x-it)\right]$$

$$\frac{\partial^2 z}{\partial t^2} = -\left[f''(x+it) + g''(x-it)\right] \quad \text{—③}$$

GATEWAY CLASSES

Adding equation ② and ③

$$\frac{\partial^2 z}{\partial n^2} + \frac{\partial^2 z}{\partial t^2} = 0$$

order = 2

GATEWAY CLASSES

Q.3 ***Form the partial differential equation by eliminating the arbitrary function from the following : $Z = x + y + f(xy)$.***

$$z = x + y + f(xy) \quad \text{——①}$$

Partially diff w.r.t x

$$\frac{\partial z}{\partial x} = 1 + 0 + f'(xy) \times y$$

$$p - 1 = f'(xy)xy \quad \text{—②}$$

Partially diff w.r.t y

$$\frac{\partial z}{\partial y} = 0 + 1 + f'(xy) \times x$$

$$q - 1 = f'(xy)x \quad \text{—③}$$

Divide eqn ② by ③

$$\frac{p-1}{q-1} = \frac{\cancel{f'(xy)}xy}{\cancel{f'(xy)}x}$$

$$\frac{p-1}{q-1} = \frac{y}{x}$$

$$px - x = qy - y$$

$$\boxed{px - qy = x - y}$$

order = 1

GATEWAY CLASSES

Q.4 Form the partial differential equation by eliminating the arbitrary function from the following : $z = \varphi(x) \cdot \Psi(y)$

$$z = \phi(x) \times \psi(y) \quad -(1)$$

Partially diff wrt x

$$\frac{\partial z}{\partial x} = \phi'(x) \times \psi(y) \quad -(2)$$

Partially diff wrt y

$$\frac{\partial z}{\partial y} = \phi(x) \times \psi'(y) \quad -(3)$$

Partially diff eqn (3) wrt x

$$\frac{\partial^2 z}{\partial x \partial y} = \phi'(x) \times \psi'(y) \quad -(4)$$

Multiply eqn (2) and (3)

$$\frac{\partial z}{\partial x} \times \frac{\partial z}{\partial y} = \phi(x) \times \psi(y) \times \phi(x) \times \psi'(y)$$

$$p \times q = \phi'(x) \times \psi'(y) \times \phi(x) \times \psi(y)$$

$$p \times q = \frac{\partial^2 z}{\partial x \partial y} \times z$$

$$\boxed{p \times q = sz}$$

order = 2

GATEWAY CLASSES

CASE - 3

When given function is of the form of $\varphi(u, v) = 0$

$p = \frac{\partial z}{\partial x}$ $q = \frac{\partial z}{\partial y}$

Where u and v are the function of x, y, z

Steps - (1) Given $\varphi(u, v) = 0$ Where $u = u(x, y, z)$ $v = v(x, y, z)$

(2) Then PDE can be formed by $Pp + Qq = R$ ——— (1)

$$P = \frac{\partial(u, v)}{\partial(y, z)} = \begin{vmatrix} \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \end{vmatrix}$$

$$Q = \frac{\partial(u, v)}{\partial(z, x)} = \begin{vmatrix} \frac{\partial u}{\partial z} & \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial z} & \frac{\partial v}{\partial x} \end{vmatrix}$$

$$R = \frac{\partial(u, v)}{\partial(x, y)} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix}$$

Put P, Q, R in (1) to get required PDE

GATEWAY CLASSES

Q. 5 Formulate the PDE by eliminating the arbitrary function from $\varphi(x^2+y^2, y^2+z^2)=0$.

$$\phi(x^2+y^2, y^2+z^2)=0$$

Where $u = x^2+y^2$

$$v = y^2+z^2$$

Then required PDE is

$$\boxed{Pp+Qq=R} \quad -(1)$$

$$P=\frac{\partial(u,v)}{\partial(y,z)}=\begin{vmatrix}\frac{\partial u}{\partial y} & \frac{\partial u}{\partial z}\\ \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z}\end{vmatrix}$$

$$P=\begin{vmatrix}2y & 0\\ 2y & 2z\end{vmatrix}=4yz$$

$$Q=\frac{\partial(u,v)}{\partial(z,x)}=\begin{vmatrix}\frac{\partial u}{\partial z} & \frac{\partial u}{\partial x}\\ \frac{\partial v}{\partial z} & \frac{\partial v}{\partial x}\end{vmatrix}$$

$$Q=\begin{vmatrix}0 & 2x\\ 2z & 0\end{vmatrix}=-4xz$$

$$R=\frac{\partial(u,v)}{\partial(x,y)}=\begin{vmatrix}\frac{\partial u}{\partial x} & \frac{\partial u}{\partial y}\\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y}\end{vmatrix}$$

$$R=\begin{vmatrix}2x & 2y\\ 0 & 2y\end{vmatrix}=4xy$$

Put P, Q and R in eqn (1)

$$4yzp-4xzq=4xy$$

$$\boxed{yzp-xzq=xy}$$

order = 1

GATEWAY CLASSES

Engineering Mathematics

Unit: Partial Differential Equations (PDE)

Lec-4

Today's Target

- Lagrange's Method [Part-1]
- Univ. Questions
- Practice Questions

GATEWAY CLASSES



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Lagrange's Method

First order Quasi-Linear PDE in standard form is called Lagrange's Equation

$$Pp + Qq = R$$

Where P, Q, R are function of x, y, z and $p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$

Step to Solve questions

Step-1 Write down the PDE in standard form

Step-2 Write down the Auxiliary Equation

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R} \quad \text{— (1)}$$

(I) (II) (III)

GATEWAY CLASSES

Step-3 Solve above auxiliary Equation by

(i) Method of Grouping

(ii) Method of Multipliers

Now we get two independent solutions in the form of

$$u(x, y, z) = c_1 \quad \text{— (2)}$$

$$v(x, y, z) = c_2 \quad \text{— (3)}$$

GATEWAY CLASSES

Step-4 General Solution

$\phi(u, v) = 0$ **OR** $u = \phi(v)$ **OR** $v = \phi(u)$

Method of Grouping

Type-1

Q.1 Solve : $p + q = z$

$p + q = z$

Here $P = 1$

$Q = 1$

$R = z$

Aux. Eqⁿ

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$$

$$\frac{dx}{1} = \frac{dy}{1} = \frac{dz}{z}$$

I II III

└ ①

(I, II)

$$\frac{dx}{1} = \frac{dy}{1}$$

Integrate both side

$$\int dx = \int dy$$

$$x = y + c_1$$

$$x - y = c_1 \quad \text{— ②}$$

(I, III)

$$\frac{dx}{1} = \frac{dz}{z}$$

Integrate both side

$$\int dx = \int \frac{dz}{z}$$

$$x = \log z + c_2$$

$$x - \log z = c_2$$

└ ③

General solution

$$\phi(x - y, x - \log z) = 0$$

GATEWAY CLASSES

Q.2 Solve : $px + qy = z$

$Pn + qy = z$

Here

$P = n$, $Q = y$, $R = z$

Aux. Eqn

$$\frac{dn}{n} = \frac{dy}{y} = \frac{dz}{z}$$

I II III

(I, II)

$$\frac{dn}{n} = \frac{dy}{y}$$

$$\int \frac{dn}{n} = \int \frac{dy}{y}$$

$$\log n = \log y + \log c_1$$

$$\log n = \log(y \times c_1)$$

$$n = y \times c_1$$

$$\frac{n}{y} = c_1 \quad \text{— (1)}$$

(II, III)

$$\frac{dy}{y} = \frac{dz}{z}$$

$$\int \frac{dy}{y} = \int \frac{dz}{z}$$

$$\log y = \log z + \log c_2$$

$$\log y = \log(z \times c_2)$$

$$y = z \times c_2$$

$$\frac{y}{z} = c_2 \quad \text{— (2)}$$

General solution

$$\phi\left(\frac{n}{y}, \frac{y}{z}\right) = 0$$

GATEWAY CLASSES

Q.3 Solve the following partial differential equations : $yzp - xzq = xy$

$$yzp - xzq = xy$$

Here

$P = yz$, $Q = -xz$, $R = xy$

Aux Eqn

$$\frac{dx}{yz} = \frac{dy}{-xz} = \frac{dz}{xy}$$

I II III

(I, II)

$$\frac{dx}{yz} = \frac{dy}{-xz}$$

$$\frac{dx}{y} = \frac{dy}{-x}$$

$$y\,dy = -x\,dx$$

$$\int y\,dy = -\int x\,dx$$

$$\frac{y^2}{2} = -\frac{x^2}{2} + C$$

$$\frac{y^2}{2} + \frac{x^2}{2} = C$$

$$x^2 + y^2 = 2C$$

$$x^2 + y^2 = C_1 \quad \text{— (1)}$$

(II, III)

$$\frac{dy}{-xz} = \frac{dz}{xy}$$

$$\frac{dy}{-z} = \frac{dz}{y}$$

$$y\,dy = -z\,dz$$

$$\int y\,dy = -\int z\,dz$$

$$\frac{y^2}{2} = -\frac{z^2}{2} + C$$

$$y^2 + z^2 = C_2 \quad \text{— (2)}$$

General solution

$$\phi(x^2+y^2,\ y^2+z^2) = 0$$

(I, III)

$$\frac{dx}{yz} = \frac{dz}{xy}$$

$$x\,dx = z\,dz$$

$$\frac{x^2}{2} = \frac{z^2}{2} + C_2$$

$$x^2 - z^2 = C_2$$

GATEWAY CLASSES

Q.4 Solve the following partial differential equations : $p \tan x + q \tan y = \tan z$

$p \tan x + q \tan y = \tan z$

Here

$P = \tan x$, $Q = \tan y$, $R = \tan z$

Aux. Eqn

$$\frac{dx}{\tan x} = \frac{dy}{\tan y} = \frac{dz}{\tan z}$$

I II III

(I, II)

$$\frac{dx}{\tan x} = \frac{dy}{\tan y}$$

$$\cot x\, dx = \cot y\, dy$$

$$\int \cot x\, dx = \int \cot y\, dy$$

$$\log \sin x = \log \sin y + \log c_1$$

$$\log \sin x = \log \sin y \times c_1$$

$$\sin x = \sin y \times c_1$$

$$\frac{\sin x}{\sin y} = c_1 \quad \text{— (1)}$$

(II, III)

$$\frac{dy}{\tan y} = \frac{dz}{\tan z}$$

$$\cot y\, dy = \cot z\, dz$$

$$\int \cot y\, dy = \int \cot z\, dz$$

$$\log \sin y = \log \sin z + \log c_2$$

$$\log \sin y = \log \sin z \times c_2$$

$$\frac{\sin y}{\sin z} = c_2 \quad \text{— (2)}$$

$$\phi = \left(\frac{\sin x}{\sin y}, \frac{\sin y}{\sin z}\right) = 0$$

GATEWAY CLASSES

Q.5 *Solve the following differential equations :* $\frac{y^2z}{x}p + xzq = y^2$

$$\frac{y^2z}{x}p + xzq = y^2$$

$$\frac{y^2zp + x^2zq}{x} = y^2$$

$$y^2zp + x^2zq = xy^2$$

Here

$P = y^2z$, $Q = x^2z$, $R = xy^2$

Aux. Eqn

$$\frac{dx}{y^2z} = \frac{dy}{x^2z} = \frac{dz}{xy^2}$$

(I, II)

$$\frac{dx}{y^2\cancel{z}} = \frac{dy}{x^2\cancel{z}}$$

$$x^2dx = y^2dy$$

$$\int x^2dx = \int y^2dy$$

$$\frac{x^3}{3} = \frac{y^3}{3} + c$$

$$\frac{x^3 - y^3}{3} = c$$

$$x^3 - y^3 = c_1 \quad -(1)$$

(I, III)

$$\frac{dx}{y^{\cancel{4}}z} = \frac{dz}{xy^{\cancel{2}}}$$

$$xdx = zdz$$

$$\int xdx = \int zdz$$

$$\frac{x^2}{2} = \frac{z^2}{2} + c$$

$$\frac{x^2 - z^2}{2} = c$$

$$x^2 - z^2 = c_1 \quad -(2)$$

General solution

$$\phi(x^3 - y^3,\ x^2 - z^2) = 0$$

GATEWAY CLASSES

Q.6 Solve the following partial differential equations : $y^2 p - xyq = x(z-2y)$ (Sem. 2014)

$y^2 p - xyq = x(z-2y)$

Here

$P = y^2$, $Q = -xy$, $R = x(z-2y)$

Aux Eqn

$$\frac{dx}{y^2} = \frac{dy}{-xy} = \frac{dz}{x(z-2y)}$$

I II III

(I, II)

$$\frac{dx}{y^2} = \frac{dy}{-xy}$$

$$y\,dy = -x\,dx$$

$$\int y\,dy = -\int x\,dx$$

$$\frac{y^2}{2} = -\frac{x^2}{2} + c$$

$$\frac{x^2+y^2}{2} = c$$

$$x^2 + y^2 = c_1 \quad -(1)$$

II, III

$$\frac{dy}{-xy} = \frac{dz}{x(z-2y)}$$

$$\frac{dy}{-y} = \frac{dz}{z-2y}$$

$$(z-2y)\,dy = -y\,dz$$

$$z\,dy - 2y\,dy = -y\,dz$$

$$z\,dy + y\,dz = 2y\,dy$$

$$d(yz) = 2y\,dy$$

$$\int d(yz) = \int 2y\,dy$$

$$yz = \frac{2y^2}{2} + c_2$$

GATEWAY CLASSES

$$yz = y^2 + c_2$$

$$yz - y^2 = c_2$$

$$y^2 - yz = c_2$$

General solution

$$\phi\left(x^2+y^2,\ y^2-yz\right) = 0$$

GATEWAY CLASSES

Type-2

Q.7 Solve : $p+3q=5z+\tan(y-3x)$ **(Sem. 2017)**

$P+3q=5z+\tan(y-3x)$

Here

$P=1$

$Q=3$

$R=5z+\tan(y-3x)$

Aux. Eqn

$$\frac{dx}{1}=\frac{dy}{3}=\frac{dz}{5z+\tan(y-3x)}$$

I II III

(I, II)

$$\frac{dx}{1}=\frac{dy}{3}$$

$$\int dx=\frac{1}{3}\int dy$$

$$x=\frac{y}{3}+c$$

$$3x=y+3c$$

$$3x-y=3c$$

$$-1(y-3x)=3c$$

$$y-3x=\frac{3c}{-1}$$

$$y-3x=c_1 \quad -(1)$$

(I, III)

$$\frac{dx}{1}=\frac{dz}{5z+\tan(y-3x)}$$

Put $y-3x=c_1$

GATEWAY CLASSES

$$\frac{dx}{1} = \frac{dz}{5z + \tan c_1}$$

$$\int dx = \int \frac{dz}{5z + \tan c_1}$$

Put $5z + \tan c_1 = t$

$$5dz = dt$$

$$dz = \frac{dt}{5}$$

$$\int dx = \frac{1}{5}\int \frac{dt}{t}$$

$$x = \frac{1}{5}\log t + c$$

$$x = \frac{\log t + 5c}{5}$$

$$5x = \log(5z + \tan c) + 5c$$

$$5x - \log(5z + \tan(y - 3x)) = c_2$$

$$5x - \log[5z + \tan(y - 3x) = c_2$$

General solution

$$\phi\left[y - 3x, 5x - \log\{5z + \tan(y - 3x)\}\right] = 0$$

GATEWAY CLASSES

Q.8 Solve the following differential equations : $pz - qz = z^2 + (x+y)^2$

$pz - qz = z^2 + (x+y)^2$

Here

$P = z$, $Q = -z$

$R = z^2 + (x+y)^2$

Aux Eqn

$$\frac{dx}{z} = \frac{dy}{-z} = \frac{dz}{z^2 + (x+y)^2}$$

I II III

(I, II)

$$\frac{dx}{z} = \frac{dy}{-z}$$

$dy = -dx$

$\int dy = -\int dx$

$y = -x + c_1$

$x + y = c_1$

(I, III)

$$\frac{dx}{z} = \frac{dz}{z^2 + (x+y)^2}$$

Put $x + y = c_1$

$$\frac{dx}{z} = \frac{dz}{z^2 + c_1^2}$$

$$\int dx = \int \frac{z\,dz}{z^2 + c_1^2}$$

Put $z^2 + c_1^2 = t$

$2z\,dz = dt$

$z\,dz = \frac{dt}{2}$

$$\int dx = \frac{1}{2}\int \frac{dt}{t}$$

GATEWAY CLASSES

$$x = \frac{1}{2}\log t + c$$

$$2x = \log(z^2 + c_1^2) + 2c$$

$$2x - \log(z^2 + c_1^2) = c_2$$

$$2x - \log\left(z^2 + (x+y)^2\right) = c_2$$

$$2x - \log\left(z^2 + x^2 + y^2 + 2xy\right) = c_2$$

$$\log\left(z^2 + x^2 + y^2 + 2xy\right) - 2x = c_2$$

General solution

$$\phi\left[x+y,\ 2x - \log(z^2 + x^2 + y^2 + 2xy)\right]$$

OR

$$\phi\left[x+y,\ \log(z^2 + x^2 + y^2 + 2xy) - 2x\right] = 0$$

GATEWAY CLASSES

Q.9 *Solve the following differential equations :* $zxp + yzq = xy$

GATEWAY CLASSES

Engineering Mathematics

Unit: Partial Differential Equations (PDE)

Lec-5

Today's Target

- Lagrange's Method [Part-2]
- Univ. Questions
- Practice Questions

GATEWAY CLASSES



By Gulshan sir

Lagrange's Method

First order Quasi-Linear PDE in standard form is called Lagrange's Equation

$$Pp + Qq = R$$

Where P, Q, R are function of x, y, z and $p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$

Step to Solve questions

Step-1 Write down the PDE in standard form

Step-2 Write down the Auxiliary Equations

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R} \quad \text{——— (1)}$$

(I) (II) (III)

GATEWAY CLASSES

Step-3 Solve above auxiliary by

(i) Method of Grouping (I, II) , (II, III) , (I, III)

(ii) Method of Multipliers

Let two independent solutions are

$u(x, y, z) = c_1$ — (2)

$v(x, y, z) = c_2$ — (3)

GATEWAY CLASSES

Step-4 General Solution

$\phi(u, v) = 0$ **OR** $u = \phi(v)$ **OR** $v = \phi(u)$

Method of Multiplier

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$$

Let Multipliers (P_1, Q_1, R_1)

Each fraction = $\frac{P_1dx + Q_1dy + R_1dz}{PP_1 + QQ_1 + RR_1}$

Type - 1

Select Multipliers in such a way that

$$PP_1 + QQ_1 + RR_1 = 0$$

Hence each fraction = $\frac{P_1dx + Q_1dy + R_1dz}{0}$

$$P_1dx + Q_1dy + R_1dz = 0$$

Integrate above equation to get solution

Type - 2

Select Multiplier in such a way that

Each fraction = $\frac{P_1dx + Q_1dy + R_1dz}{PP_1 + QQ_1 + RR_1}$

Numerator = Derivative of Denominator

GATEWAY CLASSES

Type - 1

Q.1 Solve the partial differential equation $(y^2+z^2)p - xyq = -zx$

$(y^2+z^2)p - xyq = -zx$

Here

$P = y^2+z^2$

$Q = -xy$

$R = -zx$

Aux. Eqn

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$$

$$\frac{dx}{y^2+z^2} = \frac{dy}{-xy} = \frac{dz}{-zx} \quad \text{--- (1)}$$

I II III

(II, III)

$$\frac{dy}{+xy} = \frac{dz}{+zx}$$

$$\frac{dy}{y} = \frac{dz}{z}$$

$$\int \frac{dy}{y} = \int \frac{dz}{z}$$

$$\log y = \log z + \log c_1$$

$$\log y - \log z = \log c_1$$

$$\log \frac{y}{z} = \log c_1$$

$$\boxed{\frac{y}{z} = c_1} \quad \text{--- (2)}$$

GATEWAY CLASSES

Taking x, y, z as Multiplier

$$\text{each fraction} = \frac{x\,dx + y\,dy + z\,dy}{xy^2 + xz^2 - xy^2 - z^2x}$$

$$\text{each fraction} = \frac{x\,dx + y\,dy + z\,dy}{0}$$

$$x\,dx + y\,dy + z\,dz = 0$$

$$\int x\,dx + \int y\,dy + \int z\,dz = 0$$

$$\frac{x^2}{2} + \frac{y^2}{2} + \frac{z^2}{2} = C$$

$$x^2 + y^2 + z^2 = 2C$$

$$x^2 + y^2 + z^2 = c_2 \quad \text{— (3)}$$

General solution

$$\phi\left(\frac{y}{z},\ x^2 + y^2 + z^2\right) = 0$$

GATEWAY CLASSES

Q.2 Solve : $(y+zx)p-(x+yz)q=x^2-y^2$

(Sem. 2022)

$$(y+zx)p-(x+yz)q=x^2-y^2$$

Aux Eqⁿ

$$\frac{dx}{y+zx}=\frac{dy}{-x-yz}=\frac{dz}{x^2-y^2} \quad \text{—①}$$

Taking $(x, y, -z)$ as a Multipliers

$$\text{each fraction}=\frac{x\,dx+y\,dy-z\,dz}{xy+x^2z-xy-y^2z-zx^2+zy^2}$$

$$\text{each fraction}=\frac{x\,dx+y\,dy-z\,dz}{0}$$

$$x\,dx+y\,dy-z\,dz$$

$$\int x\,dx+\int y\,dy-\int z\,dz=0$$

$$\frac{x^2}{2}+\frac{y^2}{2}-\frac{z^2}{2}=c$$

$$\boxed{x^2+y^2-z^2=c_1} \quad \text{—②}$$

GATEWAY CLASSES

Taking $(y, x, 1)$ as a Multiplier

$$\text{each fraction} = \frac{y\,dx + x\,dy + dz}{y^2 + xy^2 - x^2 - xyz + x^2 - y^2}$$

$$\text{each fraction} = \frac{y\,dx + x\,dy + dz}{0}$$

$$y\,dx + x\,dy + dz = 0$$

$$d(xy) + dz = 0$$

$$\int d(xy) + \int dz = 0$$

$$xy + z = c_2$$

General solution

$$\phi(x^2 + y^2 - z^2,\ xy + z) = 0$$

GATEWAY CLASSES

Q.3 Solve the following differential equations $x^2(y-z)p + y^2(z-x)q = z^2(x-y)$. **(Sem. 2013)**

$x^2(y-z)p + y^2(z-x)q = z^2(x-y)$

Aux Eq^n

$$\frac{dx}{x^2(y-z)} = \frac{dy}{y^2(z-x)} = \frac{dz}{z^2(x-y)} \quad \text{—①}$$

Taking $\left(\frac{1}{x^2}, \frac{1}{y^2}, \frac{1}{z^2}\right)$ as a multiplier

$$\text{each fraction} = \frac{\frac{dx}{x^2} + \frac{dy}{y^2} + \frac{dz}{z^2}}{y-z+z-x+x-y}$$

$$\text{each fraction} = \frac{\frac{dx}{x^2} + \frac{dy}{y^2} + \frac{dz}{z^2}}{0}$$

$$\frac{dx}{x^2} + \frac{dy}{y^2} + \frac{dz}{z^2} = 0$$

$$\int \frac{dx}{x^2} + \int \frac{dy}{y^2} + \int \frac{dz}{z^2} = 0$$

$$-\frac{1}{x} - \frac{1}{y} - \frac{1}{z} = c$$

$$\boxed{\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = c_1} \quad \text{—②}$$

GATEWAY CLASSES

Taking $\left(\frac{1}{x}, \frac{1}{y}, \frac{1}{z}\right)$ as a Multiplier

$$\text{each fraction} = \frac{\frac{dx}{x} + \frac{dy}{y} + \frac{dz}{z}}{xy - yz + yz - yx + zx - zy}$$

$$\text{each fraction} = \frac{\frac{dx}{x} + \frac{dy}{y} + \frac{dz}{z}}{0}$$

$$\frac{dx}{x} + \frac{dy}{y} + \frac{dz}{z} = 0$$

$$\int \frac{dx}{x} + \int \frac{dy}{y} + \frac{dz}{z}$$

$$\log x + \log y + \log z = \log c_2$$

$$\log(xyz) = \log c_2$$

$$xyz = c_2 \quad \text{— (2)}$$

General solution

$$\phi\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}, xyz\right) = 0$$

GATEWAY CLASSES

Type - 2

Q.4 Solve the following partial differential equations : $x^2p + y^2q = (x+y)z$ (Sem. 2015)

$x^2p + y^2q = (x+y)z$

Aux Eq^n

$$\frac{dx}{x^2} = \frac{dy}{y^2} = \frac{dz}{(x+y)z}$$

I II III

(I, II)

$$\frac{dx}{x^2} = \frac{dy}{y^2}$$

$$\int \frac{dx}{x^2} = \int \frac{dy}{y^2}$$

$$-\frac{1}{x} = -\frac{1}{y} + C$$

$$\frac{1}{y} - \frac{1}{x} = C$$

$$\boxed{\frac{1}{x} - \frac{1}{y} = C_1} \quad -(2)$$

Taking (1, −1, 0)

each fraction $= \frac{dx - dy}{x^2 - y^2}$

each fraction $= \frac{dx - dy}{(x-y)(x+y)}$

GATEWAY CLASSES

$$\frac{dx}{x^2} = \frac{dy}{y^2} = \frac{dz}{(x+y)z} = \frac{dx-dy}{(x-y)(x+y)}$$

I II III IV

(III, IV)

$$\frac{dz}{(x+y)z} = \frac{dx-dy}{(x-y)(x+y)}$$

$$\frac{dz}{z} = \frac{dx-dy}{x-y}$$

$$\int \frac{dz}{z} = \int \frac{dx-dy}{x-y}$$

$$\log z = \log(x-y) + \log c_2$$

$$\log z = \log(x-y) \times c_2$$

$$z = (x-y)\, c_2$$

$$\boxed{\frac{z}{x-y} = c_2} \quad -(3)$$

General solution

$$\boxed{\phi\left(\frac{1}{x} - \frac{1}{y}, \frac{z}{x-y}\right) = 0}$$

GATEWAY CLASSES

Q.5 *Solve the following differential equations :* $(y^2 + z^2 - x^2)p - 2xyq + 2xz = 0$

$$(y^2+z^2-x^2)p - 2xyq + 2xz = 0$$

$$(y^2+z^2-x^2)p - 2xyq = -2xz$$

Aux. Eqn

$$\frac{dx}{y^2+z^2-x^2} = \frac{dy}{-2xy} = \frac{dz}{-2xz} \quad \text{— (1)}$$

I II III

(II, III)

$$\frac{dy}{-2xy} = \frac{dz}{-2xz}$$

$$\frac{dy}{y} = \frac{dz}{z}$$

$$\int \frac{dy}{y} = \int \frac{dz}{z}$$

$$\log y = \log z + \log c_1$$

$$\log y = \log z c_1$$

$$y = z c_1$$

$$\boxed{\frac{y}{z} = c_1} \quad \text{— (2)}$$

GATEWAY CLASSES

Taking (x, y, z) as a multiplier

$$\text{each fraction} = \frac{x\,dx + y\,dy + z\,dz}{xy^2 + xz^2 - x^3 - 2xy^2 - 2xz^2}$$

$$\text{each fraction} = \frac{x\,dx + y\,dy + z\,dz}{-xy^2 - xz^2 - x^3}$$

$$\frac{dx}{y^2+z^2-x^2} = \frac{dy}{-2xy} = \frac{dz}{-2xz} = \frac{x\,dx + y\,dy + z\,dz}{-x^3 - xy^2 - xz^2}$$

I II III IV

GATEWAY CLASSES

III, IV

$$\frac{dz}{+2xz} = \frac{x\,dx + y\,dy + z\,dz}{+x(x^2+y^2+z^2)}$$

$$\frac{dz}{z} = \frac{2x\,dx + 2y\,dy + 2z\,dz}{x^2+y^2+z^2}$$

$$\int \frac{dz}{z} = \int \frac{2x\,dx + 2y\,dy + 2z\,dz}{x^2+y^2+z^2}$$

$$\log z = \log(x^2+y^2+z^2) + \log c$$

$$\log z = \log(x^2+y^2+z^2) \times C$$

$$z = (x^2+y^2+z^2) \times C$$

$$C = \frac{z}{x^2+y^2+z^2}$$

$$\left(\frac{1}{C}\right) = \frac{x^2+y^2+z^2}{z}$$

$$C_2 = \frac{x^2+y^2+z^2}{z}$$

General solution

$$\phi\left(\frac{y}{z}, \frac{x^2+y^2+z^2}{z}\right) = 0$$

GATEWAY CLASSES

Q. 6 Solve : $(y+z)p+(z+x)q=x+y$

$$(y+z)p+(z+x)q=x+y$$

Aux Eqⁿ

$$\frac{dx}{y+z}=\frac{dy}{z+x}=\frac{dz}{x+y} \quad -①$$

Taking $(1,-1,0)$ as a multiplier

each fraction $=\dfrac{dx-dy}{y+z-z-x}$

$$=\frac{dx-dy}{-(x-y)}$$

Taking $(0,1,-1)$

each fraction $=\dfrac{dy-dz}{z+x-x-y}$

$$=\frac{dy-dz}{-(y-z)}$$

Taking $(1,1,1)$ as a multiplier

each fraction $=\dfrac{dx+dy+dz}{y+z+z+x+x+y}$

$$=\frac{dx+dy+dz}{2(x+y+z)}$$

GATEWAY CLASSES

$$\frac{dx}{y+z} = \frac{dy}{z+x} = \frac{dz}{x+y} = \frac{dx-dy}{-(x-y)} = \frac{dy-dz}{-(y-z)} = \frac{dx+dy+dz}{2(x+y+z)}$$

I II III IV V VI

(IV, V)

$$\frac{dx-dy}{+(x-y)} = \frac{dy-dz}{+(y-z)}$$

$$\int \frac{dx-dy}{x-y} = \int \frac{dy-dz}{y-z}$$

$$\log(x-y) = \log(y-z) + \log 4$$

$$\log(x-y) = \log(y-z) \times 4$$

$$x-y = (y-z)\,4$$

$$\boxed{\frac{x-y}{y-z} = 4} \quad -(2)$$

(IV, VI)

$$\frac{dx-dy}{-(x-y)} = \frac{dx+dy+dz}{2(x+y+z)}$$

GATEWAY CLASSES

$$\frac{dx-dy}{(x-y)} + \frac{dx+dy+dz}{2(x+y+z)} = 0$$

$$\int \frac{dx-dy}{x-y} + \frac{1}{2}\int \frac{dx+dy+dz}{x+y+z} = 0$$

$$\log(x-y) + \frac{1}{2}\log(x+y+z) = \log c$$

$$2\log(x-y) + \log(x+y+z) = 2\log c$$

$$\log(x-y)^2 + \log(x+y+z) = \log c_2$$

$$\log(x-y)^2 \times (x+y+z) = \log c_2$$

$$\boxed{(x-y)^2(x+y+z) = c_2} \quad -(3)$$

General solution

$$\boxed{\phi\left[\frac{x-y}{y-z}, (x-y)^2(x+y+z)\right] = 0}$$

GATEWAY CLASSES

Practice Q.1 Solve the following differential equation : $9mz - ny)p + (n\,x - lz)q = ly - m\,x$

Practice Q.2 Solve the following differential equations : $(x^2 - yz)p + (y^2 - z\,x)q = z^2 - xy$

GATEWAY CLASSES

Engineering Mathematics

UNIT : Partial Differential Equation

Today's Target

- Non Linear Partial Differential Equation (Some Standard Forms)
- PYQs
- DPP

GATEWAY CLASSES

Non – Linear Partial Differential Equations

Some Standard Forms

Type – I

Equation containing p and q only

$$f(p,\ q) = 0$$

Steps to solve

Step-I $f(p,\ q) = 0$ ——— (1)

Step-II *Replace p by a and q by b*

$$f(a,\ b) = 0 \quad \text{——— (2)}$$

Step-III *Find b from (2) in terms of a*

Step-IV *Complete solution is given by*

$$\boxed{z = ax + by + c} \quad \text{——— (3)}$$

Step-V Put b in (3)

GATEWAY CLASSES

Q.1 Solve : $p + q = 1$

$$p+q=1 \quad \text{---} \quad (1)$$

Replace p by a and q by b

$$a+b=1 \quad \text{---} \quad (2)$$

$$b=1-a$$

complete solution is

$$z=ax+by+c \quad \text{---} \quad (3)$$

$$\boxed{z=ax+(1-a)y+c}$$

Q.2 Solve : $\sqrt{p}+\sqrt{q}=1$

$$\sqrt{p}+\sqrt{q}=1 \quad \text{---} \quad (1)$$

Replace p by a and q by b

$$\sqrt{a}+\sqrt{b}=1 \quad \text{---} \quad (2)$$

$$\sqrt{b}=1-\sqrt{a}$$

$$b=(1-\sqrt{a})^2$$

complete solution is

$$z=ax+by+c$$

$$\boxed{z=ax+(1-\sqrt{a})^2 y+c}$$

GATEWAY CLASSES

Q.3 Solve the partial differential equation : $p^2 + p = q^2$

$$p^2 + p = q^2 \quad \text{—— (1)}$$

Replace p by a and q by b

$$a^2 + a = b^2 \quad \text{—— (2)}$$

$$b^2 = a^2 + a$$

$$b = \sqrt{a^2 + a}$$

Complete solution is

$$z = ax + by + c \quad \text{— (3)}$$

Put b in (3)

$$\boxed{z = ax + \left(\sqrt{a^2 + a}\right) y + c}$$

GATEWAY CLASSES

Type – II

Clairaut Form $z = px + qy + f(p, q)$

Steps to solve

Step-I $z = px + qy + f(p, q)$ ——— (1)

Step-II Replace p by a and q by b in (1)

Step-III Complete solution is given by

$$z = ax + by + f(a, b)$$

GATEWAY CLASSES

Q.4 Solve : $z = px + qy + \sqrt{1 + p^2 + q^2}$.

$$z = px + qy + \sqrt{1 + p^2 + q^2} \quad \text{—— (1)}$$

Replace p by a and q by b

$$\boxed{z = ax + by + \sqrt{1 + a^2 + b^2}}$$

Q.5 Solve : $z = px + qy + \frac{p}{p+q}$.

$$z = px + qy + \frac{p}{p+q} \quad \text{—— (1)}$$

Replace p by a and q by b

$$\boxed{z = ax + by + \frac{a}{a+b}}$$

GATEWAY CLASSES

Type – III

Equations not containing x and y

$$f(z,\ p,\ q) = 0$$

Steps to solve

Step-I Let $u = x + ay$ so that $p = \frac{dz}{du}$ $q = a\frac{dz}{du}$

Step-II Put the values of p and q in the given equation

Step-III Solve the ordinary differential equation in z and u

Step-IV Replace u by $x + ay$ in the solution of ODE

$$u = x + ay$$

$$\frac{\partial u}{\partial x} = 1 \qquad \frac{\partial u}{\partial y} = a$$

We know that

$$p = \frac{\partial z}{\partial x} = \frac{\partial z}{\partial u} \times \frac{\partial u}{\partial x} = \frac{\partial z}{\partial u} = \frac{dz}{du}$$

$$q = \frac{\partial z}{\partial y} = \frac{\partial z}{\partial u} \times \frac{\partial u}{\partial y} = a\frac{\partial z}{\partial u} = a\frac{dz}{du}$$

GATEWAY CLASSES

Q.6 Solve the partial differential equation : $p + q = z$

$$p+q=z \quad \text{—①}$$

Let $u = x+ay$ so that

$$p=\frac{dz}{du} \quad \text{and} \quad q=a\frac{dz}{du}$$

Put p and q in ①

$$\frac{dz}{du}+a\frac{dz}{du}=z$$

$$(1+a)\frac{dz}{du}=z$$

$$(1+a)\frac{dz}{z}=du$$

Integrate both side

$$(1+a)\int\frac{dz}{z}=\int du$$

$$(1+a)\log z = u+b$$

Put $u = x+ay$

$$\boxed{(1+a)\log z = x+ay+b}$$

GATEWAY CLASSES

Q.7 Solve : $p(1 + q^2) = q\,(z-a)$

$$p(1+q^2) = q(z-a) \quad \text{—— ①}$$

Let $u = x + by$ so that

$$p = \frac{dz}{du} \quad \text{and} \quad q = b\frac{dz}{du}$$

Put p and q in ①

$$\frac{dz}{du}\left[1+\left(b\frac{dz}{du}\right)^2\right] = b\frac{dz}{du}(z-a)$$

$$1 + b^2\left(\frac{dz}{du}\right)^2 = bz - ab$$

$$b^2\left(\frac{dz}{du}\right)^2 = bz - ab - 1$$

$$b\frac{dz}{du} = \sqrt{bz-ab-1}$$

$$\frac{dz}{du} = \frac{1}{b}\sqrt{bz-ab-1}$$

$$\frac{b\,dz}{\sqrt{bz-ab-1}} = du$$

Integrate both side

$$\int \frac{b\,dz}{\sqrt{bz-ab-1}} = \int du$$

$$b\int (bz-ab-1)^{-\frac{1}{2}} = u + c$$

$$\frac{b(bz-ab-1)^{\frac{1}{2}}}{\frac{1}{2}\times b} = u + c$$

$$2(bz-ab-1)^{\frac{1}{2}} = u + c$$

$$4(bz-ab-1) = (u+c)^2$$

$$\boxed{4(bz-ab-1) = (x+by+c)^2}$$

GATEWAY CLASSES

Type – IV

Equations of the form

$$f_1(x,\ p) = f_2(y,\ q)$$

In these equations, z is absent and the terms containing x and p can be written on one side and terms containing y and q can be written on other side.

Steps to solve

Step-I $f_1(x,\ p) = f_2(y,\ q) = a$ ——— (1)

Step-II $f_1(x,\ p) = a$ Solve it for p ~~let $p = f_1(x)$~~

Step-III $f_2(y,\ q) = a$ Solve it for q ~~let $q = f_2(y)$~~

Step-IV We know that $\boxed{dz = \frac{\partial z}{\partial x}\,dx + \frac{\partial z}{\partial y}\,dy = p\,dx + q\,dy}$

Step-V Put p and q in (2) and integrate it $z = \int \cancel{F_1(x)}dx + \int \cancel{F_2(y)}dy + C$

p q

GATEWAY CLASSES

Q.8 Solve : $p^2 - q^2 = x - y$

$$p^2 - q^2 = x - y$$

$$p^2 - x = q^2 - y$$

Let $p^2 - x = q^2 - y = a$ —— ①

$$p^2 - x = a \qquad q^2 - y = a$$

$$p^2 = x + a \qquad q^2 = y + a$$

$$p = \sqrt{x+a} \qquad q = \sqrt{y+a}$$

We know that

$$dz = p\,dx + q\,dy$$

Put p and q in above equation

$$dz = \sqrt{x+a}\,dx + \sqrt{y+a}\,dy$$

Integrate both side

$$z = \int (x+a)^{1/2}\,dx + \int (y+a)^{1/2}\,dy$$

$$z = \frac{2}{3}(x+a)^{3/2} + \frac{2}{3}(y+a)^{3/2} + c$$

GATEWAY CLASSES

PDE : DPP-6

Topic : Non Linear Partial Differential Equation(Some Standard Forms)

Q .1 *Solve : $p + q = pq$*

Q .2 *Solve the partial differential equation : $p^2 + q^2 = 2$*

Q .3 *Solve the partial differential equation $z = px + qy + \sin(p+q)$*

Q .4 *Solve : $9(p^2z + q^2) = 4$*

Q .5 *Solve : $p - 3x^2 = q^2 - y$*

GATEWAY CLASSES

Engineering Mathematics

UNIT : Partial Differential Equation

Today's Target

- Solution of Non Linear Partial Differential Equation by **CHARPIT METHOD**
- PYQs
- DPP

GATEWAY CLASSES

CHARPIT METHOD

It is a general method to solve non-linear first order partial differential equation

STEPS TO SOLVE

Step-I Write the given PDE in the form given below

$$f(x, y, z, p, q) = 0 \quad \text{---------} \ (1)$$

Step-II Partially differentiate equation (1) w.r.t. x, y, z, p and q to find,

$$\frac{\partial f}{\partial x}, \quad \frac{\partial f}{\partial y}, \quad \frac{\partial f}{\partial z}, \quad \frac{\partial f}{\partial p}, \quad \frac{\partial f}{\partial q}$$

Step-III Write Charpit Auxiliary equation

$$\frac{dx}{-\frac{\partial f}{\partial p}} = \frac{dy}{-\frac{\partial f}{\partial q}} = \frac{dz}{-p\frac{\partial f}{\partial p} - q\frac{\partial f}{\partial q}} = \frac{dp}{\frac{\partial f}{\partial x} + p\frac{\partial f}{\partial z}} = \frac{dq}{\frac{\partial f}{\partial y} + q\frac{\partial f}{\partial z}}$$

GATEWAY CLASSES

Step-IV Select two proper fraction so that resulting integral may be simplest relation involving at least one of p and q

Step-V With the help of relation in step 4 and equation (1) find p and q in terms of x, y, z

Step-VI Put the value of p and q in $\boxed{dz = pdx + qdy}$

Step-VII Integrate above equation to get required Answer.

GATEWAY CLASSES

$$dz = \left(\frac{\partial z}{\partial x}\right)dx + \left(\frac{\partial z}{\partial y}\right)dy$$

p q

Q.1 By Charpit's method, find the complete solution of PDE : $px + qy = pq$

$$px + qy = pq$$

$$px + qy - pq = 0 \quad \text{——} (1)$$

Let $f(x, y, z, p, q) = px + qy - pq = 0$

$$\frac{\partial f}{\partial x} = p, \quad \frac{\partial f}{\partial y} = q, \quad \frac{\partial f}{\partial z} = 0, \quad \frac{\partial f}{\partial p} = x - q, \quad \frac{\partial f}{\partial q} = y - p$$

Charpit Aux. Eqn

$$\frac{dx}{-\frac{\partial f}{\partial p}} = \frac{dy}{-\frac{\partial f}{\partial q}} = \frac{dz}{-p\frac{\partial f}{\partial p} - q\frac{\partial f}{\partial q}} = \frac{dp}{\frac{\partial f}{\partial x} + p\frac{\partial f}{\partial z}} = \frac{dq}{\frac{\partial f}{\partial y} + q\frac{\partial f}{\partial z}}$$

GATEWAY CLASSES

$$\frac{dx}{-(x-a)} = \frac{dy}{-(y-b)} = \frac{dz}{-b(x-a)-a(y-b)} = \underline{\frac{dp}{p+0} = \frac{dq}{q+0}} \quad \text{— (2)}$$

$$\frac{dp}{p} = \frac{dq}{q}$$

Integrate both side

$$\int \frac{dp}{p} = \int \frac{dq}{q}$$

$$\log p = \log q + \log a$$

$$\log p = \log aq$$

$$p = aq \quad \text{— (3)}$$

Put p in (1)

$$px + qy - pq = 0$$

$$aqx + qy - aq^2 = 0$$

$$(ax+y)q = aq^2$$

$$aq = ax+y$$

$$\boxed{q = \frac{ax+y}{a}}$$

Put q in (3)

$$p = \cancel{a} \times \frac{(ax+y)}{\cancel{a}}$$

$$\boxed{p = ax+y}$$

GW GATEWAY CLASSES
GATEWAY CLASSES

We know that

$$dz = p\,dx + q\,dy$$

$$dz = (ax+y)\,dx + \left(\frac{ax+y}{a}\right)dy$$

$$\int dz = \int (ax+y)\,dx + \left(\frac{ax+y}{a}\right)dy$$

$$z = \int (ax+y)\left(dx + \frac{dy}{a}\right)$$

$$z = \int (ax+y)\left(\frac{adx+dy}{a}\right)$$

$$z = \frac{1}{a}\int (ax+y)(a\,dx+dy)$$

Put $ax+y = t$

$a\,dx + dy = dt$

$$z = \frac{1}{a}\int t\,dt = \frac{1}{a}\frac{t^2}{2} + b$$

$$\boxed{az = \frac{(ax+y)^2}{2} + b}$$

GATEWAY CLASSES

Q.2 Solve the following equation by Charpit's method : $pxy + pq + qy = yz$

$$pxy + pq + qy = yz$$

$$pxy + pq + qy - yz = 0 \quad \text{---} (1)$$

Let $f(x, y, z, p, q) = pxy + pq + qy - yz = 0$

$$\frac{\partial f}{\partial x} = py, \quad \frac{\partial f}{\partial y} = px + q - z, \quad \frac{\partial f}{\partial z} = -y, \quad \frac{\partial f}{\partial p} = xy + q, \quad \frac{\partial f}{\partial q} = p + y$$

Charpit Aux. Eqn

$$\frac{dx}{-\frac{\partial f}{\partial p}} = \frac{dy}{-\frac{\partial f}{\partial q}} = \frac{dz}{-p\frac{\partial f}{\partial p} - q\frac{\partial f}{\partial q}} = \frac{dp}{\frac{\partial f}{\partial x} + p\frac{\partial f}{\partial z}} = \frac{dq}{\frac{\partial f}{\partial y} + q\frac{\partial f}{\partial z}}$$

GATEWAY CLASSES

$$\frac{dx}{-(ny+q)} = \frac{dy}{-(p+y)} = \frac{dz}{-p(ny+q)-q(p+y)} = \frac{dp}{py-py} = \frac{dq}{pn+q-z-qy}$$

$$\frac{dp}{0} = \frac{dq}{pn+q-z-qy}$$

$$dp = 0$$

$$\int dp = 0$$

$$\boxed{p = a} \quad -(3)$$

Put p in (1)

$$pny + pq + qy - yz = 0$$

$$any + aq + qy - yz = 0$$

$$(a+y)q = yz - any$$

$$\boxed{q = \frac{y(z-an)}{a+y}}$$

GATEWAY CLASSES

We know that

$$dz = p\,dx + q\,dy$$

$$dz = a\,dx + \frac{y(z-ax)}{a+y}\,dy$$

$$dz - a\,dx = \frac{y(z-ax)}{a+y}\,dy$$

$$\int \frac{dz - a\,dx}{z-ax} = \int \frac{y}{a+y}\,dy$$

$$\int \frac{dz - a\,dx}{z-ax} = \int \frac{a+y-a}{a+y}\,dy$$

$$\int \frac{dz - a\,dx}{z-ax} = \int \left(1 - \frac{a}{a+y}\right) dy$$

$$\boxed{\log(z-ax) = y - a\log(a+y) + b}$$

GATEWAY CLASSES

Q.3 Find the complete integral of PDE : $(p^2 + q^2)y = qz$

$(p^2+q^2)y = qz$

$(p^2+q^2)y - qz = 0 \quad \text{—— (1)}$

Let $f(x, y, z, p, q) = (p^2+q^2)y - qz = 0$

$\frac{\partial f}{\partial x} = 0, \frac{\partial f}{\partial y} = p^2+q^2, \frac{\partial f}{\partial z} = -q, \frac{\partial f}{\partial p} = 2py, \frac{\partial f}{\partial q} = 2qy - z$

Charpit Aux. Eqn

$$\frac{dx}{-\frac{\partial f}{\partial p}} = \frac{dy}{-\frac{\partial f}{\partial q}} = \frac{dz}{-p\frac{\partial f}{\partial p} - q\frac{\partial f}{\partial q}} = \frac{dp}{\frac{\partial f}{\partial x} + p\frac{\partial f}{\partial z}} = \frac{dq}{\frac{\partial f}{\partial y} + q\frac{\partial f}{\partial z}}$$

GATEWAY CLASSES

$$\frac{dx}{-2py} = \frac{dy}{-(2qy-z)} = \frac{dz}{-2p^2y-q(2qy-z)} = \frac{dp}{-pq} = \frac{dq}{p^2+q^2-q^2}$$

$$\frac{dp}{-pq} = \frac{dq}{p^2}$$

$$\frac{dp}{-q} = \frac{dq}{p}$$

$$p\,dp = -q\,dq$$

$$p\,dp + q\,dq = 0$$

$$\int p\,dp + \int q\,dq = 0$$

$$\frac{p^2}{2} + \frac{q^2}{2} = \frac{a^2}{2}$$

$$p^2 + q^2 = a^2 \quad \text{---} \ (3)$$

Put eqn (3) in (1)

$$(p^2+q^2)y - qz = 0$$

$$a^2y - qz = 0$$

$$a^2y - qz = 0$$

$$\boxed{q = \frac{a^2y}{z}}$$

Put q in (3)

$$p^2 + \left(\frac{a^2y}{z}\right)^2 = a^2$$

$$p^2 = a^2 - \frac{a^4y^2}{z^2}$$

GATEWAY CLASSES

$$p^2 = \frac{a^2z^2 - a^4y^2}{z^2}$$

$$p^2 = \frac{a^2}{z^2}(z^2 - a^2y^2)$$

$$\boxed{p = \frac{a}{z}\sqrt{z^2 - a^2y^2}}$$

We know that

$$dz = p\,dx + q\,dy$$

$$dz = \frac{a}{z}\sqrt{z^2 - a^2y^2}\,dx + \frac{a^2y}{z}\,dy$$

$$z\,dz = a\left(\sqrt{z^2 - a^2y^2}\right)dx + a^2y\,dy$$

$$z\,dz - a^2y\,dy = a\left(\sqrt{z^2 - a^2y^2}\right)dx$$

$$\int \frac{z\,dz - a^2y\,dy}{\sqrt{z^2 - a^2y^2}} = \int a\,dx$$

$$z^2 - a^2y^2 = t$$

$$2z\,dz - 2a^2y\,dy = dt$$

$$z\,dz - a^2y\,dy = \frac{dt}{2}$$

$$\frac{1}{2}\int \frac{dt}{\sqrt{t}} = ax$$

$$\frac{1}{2}\int t^{-1/2}\,dt = ax$$

$$\frac{t^{1/2}}{\frac{1}{2}} = ax + b$$

$$t^{1/2} = ax + b$$

$$(z^2 - a^2y^2)^{1/2} = ax + b$$

$$\boxed{(z^2 - a^2y^2) = (ax + b)^2}$$

GATEWAY CLASSES

Q .1 ***Solve :*** $2zx - px^2 - 2pxy + pq = 0$

Ans $z = ay + b(x^2 - a)$

Q .2 ***Solve the following equation by charpit's method :*** $z^2 = pqxy$

Ans $z = ax^b y^{1/b}$

Q .3 ***Solve the following equation by charpit's method :*** $z = p^2x + q^2x$

Ans $\sqrt{(1+a)z} = \sqrt{ax} + \sqrt{y} + b$

GATEWAY CLASSES

Engineering Mathematics

UNIT : Partial Differential Equation

Today's Target

- Homogeneous Linear Partial Differential Equation with constant coefficient (Complementary function-CF)

- PYQs
- DPP

GATEWAY CLASSES

Unit-1

1. PDE
2. Order and degree of PDE
3. Types of PDE
4. Formation of PDE

Solution of PDE

1. Lagrange Method
2. Some standard form of non linear PDE
3. Charpit Method
4. Linear PDE with constant coefficient

(a) Homogeneous Linear PDE with constant coefficient

(b) Non-Homogeneous Linear PDE with constant coefficient

8. Equation reducible to Linear PDE with constant coefficient

GATEWAY CLASSES

Linear Partial Differential Equation with constant coefficient

Linear PDE with constant coefficient

Homogeneous Linear PDE with constant coefficient

(All partial derivatives are of same order)

Complete solution = CF + PI

Non-Homogeneous Linear PDE with constant coefficient

(All partial derivatives are not of same order)

Complete solution = CF + PI

GATEWAY CLASSES

Homogeneous Linear Partial Differential Equation with constant coefficient

General equation

$$a_0\frac{\partial^n z}{\partial x^n} + a_1\frac{\partial^n z}{\partial x^{n-1}\partial y} + \cdots\cdots\cdots\cdots + a_n\frac{\partial^n z}{\partial y^n} = F(x,y)$$

Put $\frac{\partial}{\partial x} = D$ and $\frac{\partial}{\partial y} = D'$

$$(a_0D^n + a_1D^{n-1}D' + \cdots\cdots\cdots\cdots a_nD'^n)z = F(x,y)$$

$$f(D,D')z = F(x,y)$$

CF

Complementary Function

Number of arbitrary function = order of PDF

PI

Particular integral

Does not contain any arbitrary function or constant

GATEWAY CLASSES

Steps to find CF

STEP-1: Convert the given PDE in the form

$$f(D,\ D')z = F(x,\ y) \quad \text{——} \quad (1)$$

Using $\frac{\partial}{\partial x} = D$, $\frac{\partial^2}{\partial x^2} = D^2$, $\frac{\partial}{\partial y} = D'$, $\frac{\partial^2}{\partial x \partial y} = DD'$ $\frac{\partial^2}{\partial y^2} = D'^2$ $etc.$

STEP-2: Put $D = m$ and $D' = 1$ in (1)

$$f(m,\ 1)z = F(x,\ y) \quad \text{——} \quad (2)$$

STEP-3: Form Aux. Equation

$$f(m,\ 1) = 0$$

STEP-4: Solve to find roots of Aux. Equation

GATEWAY CLASSES

CASE-1: When roots are different

S. No.	Roots of A E	C. F.
1.	m_1, m_2	$f_1(y + m_1x) + f_2(y + m_2x)$
2.	m_1, m_2, m_3	$f_1(y + m_1x) + f_2(y + m_2x) + f_3(y + m_3x)$
3.	$m_1, m_2, m_3\ \ m_4$	$f_1(y + m_1x) + f_2(y + m_2x) + f_3(y + m_3x) + f_4\,(y + m_4x)$

GATEWAY CLASSES

Note: Where f_1, f_2, f_3 ..... are arbitrary function

Q.1 Solve: $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = 0$

$$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = 0$$

$$(D^2 - D'^2)z = 0$$

Put $D = m$, $D' = 1$

$$(m^2 - 1)z = 0$$

Aux Eqn

$$m^2 - 1 = 0$$

$$m = 1, -1$$

$$CF = f_1(y + m_1 x) + f_2(y + m_2 x)$$

$$CF = f_1(y + x) + f_2(y - x)$$

$$PI = 0$$

complete solution

$$z = CF + PI$$

$$\boxed{z = f_1(y + x) + f_2(y - x)}$$

GATEWAY CLASSES

Q.2 :- Solve the following partial differential equation : $\frac{\partial^2 z}{\partial x^2} + 4\frac{\partial^2 z}{\partial x \partial y} - 5\frac{\partial^2 z}{\partial y^2} = 0$

$$\frac{\partial^2 z}{\partial x^2} + 4\frac{\partial^2 z}{\partial x \partial y} - 5\frac{\partial^2 z}{\partial y^2} = 0$$

$$(D^2 + 4DD' - 5D'^2)z = 0$$

Put $D = m$, $D' = 1$

$$(m^2 + 4m - 5)z = 0$$

Aux Eqn

$$m^2 + 4m - 5 = 0$$

$m = 1$

$m = -5$

$$CF = f_1(y + m_1 x) + f_2(y + m_2 x)$$

$$CF = f_1(y + x) + f_2(y - 5x)$$

$PI = 0$

Complete solution

$$z = CF + PI$$

$$\boxed{z = f_1(y + x) + f_2(y - 5x)}$$

GATEWAY CLASSES

Q.3 :- Solve the following partial differential equation $(D^3 - 6D^2D' + 11DD'^2 - 6D'^3)z = 0$

$(D^3 - 6D^2D' + 11DD'^2 - 6D'^3)z = 0$

Put $D = m$ and $D' = 1$

$(m^3 - 6m^2 + 11m - 6)z = 0$

Aux Eq^n

$m^3 - 6m^2 + 11m - 6 = 0$

$m = 3, 1, 2$

$CF = f_1(y + m_1x) + f_2(y + m_2x) + f_3(y + m_3x)$

$CF = f_1(y + 3x) + f_2(y + x) + f_3(y + 2x)$

$PI = 0$

Complete solution

$z = CF + PI$

$$z = f_1(y + 3x) + f_2(y + x) + f_3(y + 2x)$$

GATEWAY CLASSES

Q.4 :- Solve : $\frac{\partial^4 z}{\partial x^4} - \frac{\partial^4 z}{\partial y^4} = 0$

$$\frac{\partial^4 z}{\partial x^4} - \frac{\partial^4 z}{\partial y^4} = 0$$

$$(D^4 - D'^4)z = 0$$

Put $D = m$, $D' = 1$

$$(m^4 - 1)z = 0$$

Aux. Eqn

$$m^4 - 1 = 0$$

$$(m^2 - 1)(m^2 + 1) = 0$$

$$m = \pm 1$$

$$m = \pm i$$

$$CF = f_1(y + m_1 x) + f_2(y + m_2 x) + f_3(y + m_3 x) + f_4(y + m_4 x)$$

$$CF = f_1(y + x) + f_2(y - x) + f_3(y + ix) + f_4(y - ix)$$

$$PI = 0$$

complete solution

$$z = CF + PI$$

GATEWAY CLASSES

Q.5 :- Solve the linear partial differential equation $\frac{\partial^4 z}{\partial x^4} + \frac{\partial^4 z}{\partial y^4} = 0$

$$\frac{\partial^4 z}{\partial x^4} + \frac{\partial^4 z}{\partial y^4} = 0$$

$$(D^4 + D'^4)z = 0$$

Put $D = m$, $D' = 1$

$$(m^4 + 1)z = 0$$

Aux Eqn

$$m^4 + 1 = 0$$

$$m^4 + 1 = 0$$

$$m^4 + 2m^2 + 1 = 2m^2$$

$$(m^2)^2 + 2m^2 + 1 = 2m^2$$

$$(m^2 + 1)^2 = 2m^2$$

$$(m^2 + 1)^2 - 2m^2 = 0$$

$$(m^2 + 1)^2 - (\sqrt{2}\, m)^2 = 0$$

$$(m^2 + \sqrt{2}m + 1)(m^2 - \sqrt{2}m + 1) = 0$$

$$m^2 + \sqrt{2}m + 1 = 0$$

$$m = \frac{-\sqrt{2} \pm \sqrt{2-4}}{2}$$

$$m = \frac{-\sqrt{2}}{2} \pm i\frac{\sqrt{2}}{2}$$

$$m = -\frac{1}{\sqrt{2}} \pm i\frac{1}{\sqrt{2}}$$

$$m = z_1, \overline{z_1}$$

$$m^2 - \sqrt{2}m + 1 = 0$$

$$m = \frac{\sqrt{2} \pm \sqrt{2-4}}{2}$$

$$m = \frac{\sqrt{2}}{2} \pm i\frac{\sqrt{2}}{2}$$

$$m = \frac{1}{\sqrt{2}} \pm i\frac{1}{\sqrt{2}}$$

$$m = z_2, \overline{z_2}$$

GATEWAY CLASSES

$$CF = f_1(y+m_1x) + f_2(y+m_2x) + f_3(y+m_3x) + f_4(y+m_4x)$$

$$CF = f_1(y+z_1x) + f_2(y+\overline{z}_1x) + f_3(y+z_2x) + f_4(y+\overline{z}_2x)$$

$$PI = 0$$

complete solution

$$\boxed{Z = CF + PI}$$

GATEWAY CLASSES

CASE-2: When roots are repeated

S. No.	Roots of A E	C. F.
1.	m, m	$f_1(y+mx)+xf_2(y+mx)$
2.	m, m, m	$f_1(y+mx)+xf_2(y+mx)+x^2f_3(y+mx)$

Note: Where f_1, f_2, f_3 ..... are arbitrary function

GATEWAY CLASSES

Q.6 :- Solve $25r - 40s + 16t = 0$

$25r - 40s + 16t = 0$

$$25\frac{\partial^2 z}{\partial x^2} - 40\frac{\partial^2 z}{\partial x \partial y} + 16\frac{\partial^2 z}{\partial y^2} = 0$$

$$(25D^2 - 40DD' + 16D'^2)z = 0$$

Put $D = m$, $D' = 1$

$$(25m^2 - 40m + 16)z = 0$$

Aux Eqn

$25m^2 - 40m + 16 = 0$

$(5m - 4)^2 = 0$

$(5m - 4)(5m - 4) = 0$

$m = \frac{4}{5}, \frac{4}{5}$

$CF = f_1(y + mx) + x f_2(y + mx)$

$CF = f_1(y + \frac{4}{5}x) + x f_2(y + \frac{4}{5}x)$

complete solution

$$\boxed{Z = CF + PI}$$

GATEWAY CLASSES

Q.7 :- Solve: $(D^3 - 6D^2D' + 12DD'^2 - 8D'^3)z = 0$

$(D^3 - 6D^2D' + 12DD'^2 - 8D'^3)z = 0$

Put $D = m$, $D' = 1$

$(m^3 - 6m^2 + 12m - 8)z = 0$

Aux. Eqn

$m^3 - 6m^2 + 12m - 8 = 0$

$m = 2, 2, 2$

$CF = f_1(y + mx) + x f_2(y + mx) + x^2 f_3(y + mx)$

$CF = f_1(y + 2x) + x f_2(y + 2x) + x^2 f_3(y + 2x)$

$PI = 0$

complete solution

$$z = CF + PI$$

GATEWAY CLASSES

Q.8 :- Solve: $(D + 2D')(D - 3D')^2 z = 0$

$$(D + 2D')(D - 3D')^2 z = 0$$

Put $D = m$, $D' = 1$

$$(m+2)(m-3)^2 z = 0$$

Aux Eqn

$$(m+2)(m-3)^2 = 0$$

$$m = -2$$

$$m = 3, 3$$

$$CF = f_1(y + m_1 x) + f_2(y + mx) + x f_3(y + mx)$$

$$CF = f_1(y - 2x) + f_2(y + 3x) + x f_3(y + 3x)$$

$$PI = 0$$

Complete solution

$$Z = CF + PI$$

GATEWAY CLASSES

CASE-3: When D , D' , both D and D' are common factor

(i) C F corresponding to $D'\ z = f_1(x)$

(ii) C F corresponding to $D'^{\,2}\ z = f_1(x) + yf_2(x)$

(iii) C F corresponding to $D'^{\,3}\ z = f_1(x) + yf_2(x) + y^2 f_3(x)$

(ii) C F corresponding to $Dz = f_1(y)$

(iii) C F corresponding to $D^2\ z = f_1(y) + xf_2(y)$

(iv) C F corresponding to $D^3\ z = f_1(y) + xf_2(y) + x^2 f_3(y)$

GATEWAY CLASSES

Q.9 :- Solve: $\frac{\partial^3 z}{\partial x^3} - \frac{\partial^3 z}{\partial x^2 \partial y} = 0$

$$\frac{\partial^3 z}{\partial y^3} - \frac{\partial^3 z}{\partial x^2 \partial y} = 0$$

$$(D'^3 - D^2 D')z = 0$$

$$D'(D'^2 - D^2)z = 0$$

Aux Eqn

$$D'(D'^2 - D^2) = 0$$

$$D' = 0$$

$$CF = f_1(x)$$

$$D'^2 - D^2 = 0$$

Put $D = m$

$$D' = 1$$

$$1 - m^2 = 0$$

$$m = \pm 1$$

$$CF = f_2(y+mx) + f_3(y+m_2x)$$

$$CF = f_2(y+x) + f_3(y-x)$$

Final

$$CF = f_1(x) + f_2(y+x) + f_3(y-x)$$

$PI = 0$

Complete solution

$$z = CF + PI$$

$$\boxed{z = f_1(x) + f_2(y+x) + f_3(y-x)}$$

GATEWAY CLASSES

Q.10 :- Solve: $(D^3D'^2 + D^2D'^3)z = 0$

$$(D^3D'^2 + D^2D'^3)z = 0$$

$$D^2D'^2(D + D')z = 0$$

Aux Eqn

$$D^2 = 0$$

$$CF = f_1(y) + nf_2(y)$$

$$D'^2 = 0$$

$$CF = f_3(x) + yf_4(x)$$

$$D + D' = 0$$

Put $D = m$

$D' = 1$

$$m + 1 = 0$$

$$m = -1$$

$$CF = f_5(y + mx)$$

$$CF = f_5(y - x)$$

$$CF = f_1(y) + nf_2(y) + f_3(x) + yf_4(x) + f_5(y - x)$$

Complete solution

$$\boxed{z = CF + PI}$$

GATEWAY CLASSES

PDE : DPP-8

Topic : Homogeneous Linear Partial Differential Equation with constant coefficient (Complementary function)

Q.1 Solve : $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} - 6\frac{\partial^2 z}{\partial y^2} = 0$ Ans $z = f_1(y+3x) + f_2(y-2x)$

Q.2 Solve $\frac{\partial^3 z}{\partial x^3} - 3\frac{\partial^3 z}{\partial x^2 \partial y} + 2\frac{\partial^3 z}{\partial x \partial y^2} = 0$ Ans $z = f_1(y) + f_2(y+x) + f_3(y+2x)$

Q.3 Solve: $4r - 12s + 9t = 0$ Ans $z = f_1(y+\frac{3}{2}x) + x f_2(y+\frac{3}{2}x)$

Q.4 Solve $(D^3D' - 4D^2D'^2 + 4DD'^3)z = 0$ Ans

$z = f_1(y) + f_2(x) + f_3(y+2x) + x f_4(y+2x)$

GATEWAY CLASSES

Engineering Mathematics

UNIT : Partial Differential Equation

Today's Target

- Homogeneous Linear Partial Differential Equation with constant coefficient

  Particular Integral(PI) : Case-I (TYPE-I)

- PYQs
- DPP

GATEWAY CLASSES

CASE-1: When roots are different

S. No.	Roots of A E	C. F.
1.	m_1, m_2	$f_1(y + m_1x) + f_2(y + m_2x)$
2.	m_1, m_2, m_3	$f_1(y + m_1x) + f_2(y + m_2x) + f_3(y + m_3x)$
3.	$m_1, m_2, m_3\ \ m_4$	$f_1(y + m_1x) + f_2(y + m_2x) + f_3(y + m_3x) + f_4\,(y + m_4x)$

Note: Where f_1, f_2, f_3 ..... are arbitrary function

GATEWAY CLASSES

CASE-2: When roots are repeated

S. No.	Roots of A E	C. F.
1.	m, m	$f_1(y+mx)+xf_2(y+mx)$
2.	m, m, m	$f_1(y+mx)+xf_2(y+mx)+x^2f_3(y+mx)$

Note: Where f_1, f_2, f_3 ..... are arbitrary function

GATEWAY CLASSES

CASE-3: When D , D′ , both D and D′ are common factor

(i) C F corresponding to $D'\,z = f_1(x)$

(ii) C F corresponding to $D'^{\,2}\,z = f_1(x) + yf_2(x)$

(iii) C F corresponding to $D'^{\,3}\,z = f_1(x) + yf_2(x) + y^2 f_3(x)$

(ii) C F corresponding to $Dz = f_1(y)$

(iii) C F corresponding to $D^2\,z = f_1(y) + xf_2(y)$

(iv) C F corresponding to $D^3\,z = f_1(y) + xf_2(y) + x^2 f_3(y)$

GATEWAY CLASSES

Homogeneous Linear PDE with constant coefficient

(All partial Derivatives are of same order)

Particular Integral

Case – 1

$$F(D, D') = \phi(ax + by)$$

$$\text{PI} = \frac{1}{F(D, D')}\phi(ax + by)$$

Where $F(D, D')$ *is a Homogeneous function of degree n*

Replace

$$D = a\,,\ D' = b$$

TYPE-I

if $F(a, b) \neq 0$

put $ax + by = u,$

$$\text{PI} = \frac{1}{F(a,b)} \iiint \phi(u)du\,du \ \ldots\ldots n \text{ times}$$

GATEWAY CLASSES

TYPE-2

If $F(a,b)=0$ (Next Lecture)

Type - 1	Type - 2
(i) When power of D $\geq$ power of D'	When power of D' $>$ power of D'
(ii) $\text{PI}=x\left[\frac{1\times\phi(ax+by)}{\frac{\partial}{\partial D}[F(D,D')]}\right]$	$\text{PI}=y\left[\frac{1\times\phi(ax+by)}{\frac{\partial}{\partial D'}[F(D,D')]}\right]$
Again Repeat if $\frac{\partial}{\partial D}[F(D,D')]=0$	Again Repeat if $\frac{\partial}{\partial D'}[F(D,D')]=0$

GATEWAY CLASSES

Important Formulae

① $2\sin A\cos B = \sin(A+B) + \sin(A-B)$

② $2\cos A\sin B = \sin(A+B) - \sin(A-B)$

③ $2\cos A\cos B = \cos(A+B) + \cos(A-B)$

④ $2\sin A\sin B = \cos(A-B) - \cos(A+B)$

GATEWAY CLASSES

Q.1. Solve the linear partial differential equation $\frac{\partial^3 u}{\partial x^3} - 3\frac{\partial^3 u}{\partial x^2 \partial y} + 4\frac{\partial^3 u}{\partial y^3} = e^{x+2y}$.

$$\frac{\partial^3 u}{\partial x^3} - 3\frac{\partial^3 u}{\partial x^2 \partial y} + 4\frac{\partial^3 u}{\partial y^3} = e^{x+2y}$$

$$(D^3 - 3D^2D' + 4D'^3)u = e^{x+2y}$$

Put $D = m$, $D' = 1$

$$(m^3 - 3m^2 + 4)u = e^{x+2y}$$

Aux Eqn

$$m^3 - 3m^2 + 4 = 0$$

$m = -1, 2, 2$

$$CF = f_1(y-x) + f_2(y+2x) + x f_3(y+2x)$$

$$PI = \frac{1}{D^3 - 3D^2D' + 4D'^3} e^{x+2y}$$

Put $D = 1$, $D' = 2$, $x + 2y = u$

$$PI = \frac{1}{1 - 3\times1\times2 + 4\times8} \iiint e^u \, du\,du\,du$$

GATEWAY CLASSES

$$PI = \frac{1}{27} e^{u}$$

$$\boxed{PI = \frac{1}{27} e^{x+2y}}$$

complete solution

$$u = CF + PI$$

$$\boxed{u = f_1(y-x) + f_2(y+2x) + x f_3(y+2x) + \frac{1}{27} e^{x+2y}}$$

GATEWAY CLASSES

Q.2 Solve the linear partial differential equation $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = \sin(2x+3y)$.

$$\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = \sin(2x+3y)$$

$$(D^2 - 2DD' + D'^2)z = \sin(2x+3y)$$

Put $D = m$, $D' = 1$

$$(m^2 - 2m + 1)z = \sin(2x+3y)$$

Aux Eqn

$$m^2 - 2m + 1 = 0$$

$$(m-1)^2 = 0$$

$$m = 1, 1$$

$$CF = f_1(y+x) + x f_2(y+x)$$

$$PI = \frac{1}{(D^2 - 2DD' + D'^2)} \sin(2x+3y)$$

Put $D = 2$, $D' = 3$, $2x+3y = u$

$$PI = \frac{1}{4 - 12 + 9} \iint \sin u \, du \, du$$

GATEWAY CLASSES

$$PI = \int\int \sin u \, du \, du$$

$$PI = -\int \cos u \, du$$

$$PI = -\sin u$$

$$PI = -\sin(2x+3y)$$

complete solution

$$z = CF + PI$$

$$\boxed{z = f_1(y+x) + x f_2(y+x) - \sin(2x+3y)}$$

GATEWAY CLASSES

Q.3 Solve: $r + s - 2t = \sqrt{2x + y}$. **(UPTU-2015)**

$$r + s - 2t = \sqrt{2x + y}$$

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 2\frac{\partial^2 z}{\partial y^2} = (2x + y)^{1/2}$$

$$(D^2 + DD' - 2D'^2)z = (2x + y)^{1/2}$$

Put $D = m$, $D' = 1$

$$(m^2 + m - 2)z = (2x + y)^{1/2}$$

Aux Eqn

$$m^2 + m - 2 = 0$$

$$m = 1, -2$$

$$\boxed{CF = f_1(y + x) + f_2(y - 2x)}$$

$$PI = \frac{1}{(D^2 + DD' - 2D'^2)}(2x + y)^{1/2}$$

Put $D = 2$, $D' = 1$, $2x + y = u$

$$PI = \frac{1}{(4 + 2 - 2)}\iint u^{1/2}\, du\, du$$

GATEWAY CLASSES

$$PI = \frac{1}{4}\int\int u^{1/2}\,du\,du$$

$$PI = \frac{1}{4}\int \frac{u^{3/2}}{\frac{3}{2}}\,du$$

$$PI = \frac{1}{4} \times \frac{2}{3} \times \frac{u^{5/2}}{\frac{5}{2}}$$

$$PI = \frac{1}{4} \times \frac{2}{3} \times \frac{2}{5} \times u^{5/2}$$

$$\boxed{PI = \frac{1}{15}u^{5/2}}$$

complete solution

$$z = CF + PI$$

$$\boxed{z = f_1(y+n) + f_2(y-2n) + \frac{1}{15}(2n+y)^{5/2}}$$

GATEWAY CLASSES

Q.4 Solve the linear partial differential equation $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x \partial y} = \sin x \cos 2y$

(AKTU-2018)

$$\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x \partial y} = \sin x \cos 2y$$

$$(D^2 - 2DD')z = \sin x \cos 2y$$

$$D(D - 2D')z = \sin x \cos 2y$$

$Dz = 0$

$CF = f_1(y)$

$(D - 2D')z = 0$

Put $D = m$, $D' = 1$

$(m - 2)z = 0$

$m - 2 = 0$

$m = 2$

$CF = f_2(y + 2x)$

$$CF = f_1(y) + f_2(y + 2x)$$

$$PI = \frac{1}{(D^2 - 2DD')} \sin x \cos 2y$$

$$PI = \frac{1}{2(D^2 - 2DD')} 2 \sin x \cos 2y$$

GATEWAY CLASSES

$$PI = \frac{1}{2(D^2-2DD')}\left[\sin(x+2y)+\sin(x-2y)\right]$$

$$PI = \frac{1}{2(D^2-2DD')}\sin(x+2y)+\frac{1}{2(D^2-2DD')}\sin(x-2y)$$

$$PI = P_1 + P_2$$

$$P_1 = \frac{1}{2(D^2-2DD')}\sin(x+2y)$$

Put $D=1$, $D'=2$, $x+2y=u$

$$P_1 = \frac{1}{2(1-4)}\int\int \sin u\, du\, dy$$

$$P_1 = -\frac{1}{6}(-\sin u)$$

$$P_1 = \frac{1}{6}\sin(x+2y)$$

$$P_2 = \frac{1}{2(D^2-2DD')}\sin(x-2y)$$

Put $D=1$, $D'=-2$

$x-2y=u$

$$P_2 = \frac{1}{2(1+4)}\int\int \sin u\, du\, dy$$

$$P_2 = \frac{1}{10}(-\sin u)$$

GATEWAY CLASSES

$$P_2 = -\frac{1}{10}\sin(x-2y)$$

$$PI = \frac{1}{6}\sin(x+2y) - \frac{1}{10}\sin(x-2y)$$

Complete solution

$$z = CF + PI$$

$$z = f_1(y) + f_2(y+2x) + \frac{1}{6}\sin(x+2y) - \frac{1}{10}\sin(x-2y)$$

GATEWAY CLASSES

Q.5 Solve the linear partial differential equation $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \cos mx \cos ny + 30(2x + y)$.

(GBTU-2011, 2012)

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \cos mx \cos ny + 30(2x+y)$$

$$(D^2 + D'^2)z = \cos mx \cos ny + 30(2x+y)$$

Put $D = m$, $D' = 1$

$$(m^2+1)z = \cos mx \cos ny + 30(2x+y)$$

Aux Eq^n

$$m^2 + 1 = 0$$

$$m = \pm i$$

$$CF = f_1(y+ix) + f_2(y-ix)$$

$$PI = \frac{1}{D^2+D'^2}\left[\cos mx \cos ny + 30(2x+y)\right]$$

$$PI = \frac{1}{(D^2+D'^2)}\left[\frac{2}{2}\cos mx \cos ny + 30(2x+y)\right]$$

GATEWAY CLASSES

$$PI = \frac{1}{(D^2+D'^2)}\left[\frac{1}{2}\{\cos(mx+ny) + \cos(mx-ny)\} + 30(2x+y)\right]$$

$$PI = \frac{1}{2(D^2+D'^2)}\cos(mx+ny) + \frac{1}{2(D^2+D'^2)}\cos(mx-ny) + \frac{1}{(D^2+D'^2)} \times 30(2x+y)$$

$$PI = P_1 + P_2 + P_3$$

$$P_1 = \frac{1}{2(D^2+D'^2)}\cos(mx+ny)$$

Put $D = m$, $D' = n$, $mx+ny = u$

$$P_1 = \frac{1}{2(m^2+n^2)}\int\int \cos u \, du \, du$$

$$P_1 = \frac{-1}{2(m^2+n^2)}\cos u$$

$$\boxed{P_1 = \frac{-1}{2(m^2+n^2)}\cos(mx+ny)}$$

GATEWAY CLASSES

$$P_2 = \frac{1}{2(D^2+D'^2)} \cos(mx-ny)$$

Put $D = m$, $D' = n$, $mx-ny = u$

$$P_2 = \frac{1}{2(m^2+n^2)} \iint \cos u \, du \, du$$

$$P_2 = \frac{-1}{2(m^2+n^2)} \cos u$$

$$\boxed{P_2 = \frac{-1}{2(m^2+n^2)} \cos(mx-ny)}$$

$$P_3 = \frac{1}{D^2+D'^2} 30(2x+y)$$

Put $D = 2$, $D' = 1$, $2x+y = u$

$$P_3 = \frac{1}{u+1} \times 30 \iint u \, du \, du$$

$$P_3 = \frac{1}{5} \times 30^{6} \times \int \frac{u^2}{2} \, du$$

$$P_3 = 6 \times \frac{u^3}{6}$$

$$\boxed{P_3 = (2x+y)^3}$$

GATEWAY CLASSES

$$PI = P_1 + P_2 + P_3$$

$$PI = \frac{-1}{2(m^2+n^2)}\cos(mx+ny) - \frac{1}{2(m^2+n^2)}\cos(mx-ny) + (2x+y)^3$$

$$PI = \frac{-1}{2(m^2+n^2)}\left[\cos(mx+ny) + \cos(mx-ny)\right] + (2x+y)^3$$

$$PI = -\frac{1}{2(m^2+n^2)}\left[2\cos mx \cos ny\right] + (2x+y)^3$$

$$PI = \frac{-1}{m^2+n^2}(\cos mx \cos ny) + (2x+y)^3$$

GATEWAY CLASSES

complete solution

$$Z = CF + PI$$

Q.6 *Solve the linear partial differential equation* $(D^2 + 7DD' + 12D'^2)z = \sinh x$.

(AKTU-2017)

$$(D^2+7DD'+12D'^2)z = \sinh x$$

Put $D = m,\ D' = 1$

$$(m^2+7m+12)z = \sinh(x+0y)$$

Aux. Eqn

$$m^2+7m+12 = 0$$

$$m = -3, -4$$

$$CF = f_1(y-3x) + f_2(y-4x)$$

$$PI = \frac{1}{(D^2+7DD'+12D'^2)} \sinh(x+0y)$$

Put $D = 1,\ D' = 0,\ x+0y = u$

$$PI = \frac{1}{(1+0+0)} \iint \sinh u\, du\, du$$

$$PI = \int \cosh u\, du$$

GATEWAY CLASSES

$PI = \sinh u$

$PI = \sinh n$

Complete solution

$Z = CF + PI$

$$Z = f_1(y-3x) + f_2(y-4x) + \sinh x$$

GATEWAY CLASSES

PDE : DPP-9

Topic : Homogeneous Linear Partial Differential Equation with constant coefficient

Particular Integral(PI) : Case-I

Q .1 *Solve the linear partial differential equation* $\frac{\partial^2 z}{\partial x^2} + 3\frac{\partial^2 z}{\partial x \partial y} + 2\frac{\partial^2 z}{\partial y^2} = x + y.$

$$z = f_1(y-x) + f_2(y-2x) + \frac{(x+y)^3}{36}$$

Q .2 *Solve :* $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = \sin x.$ *(MTU-2013)*

$$z = f_1(y+x) + x f_2(y+x) - \sin x$$

GATEWAY CLASSES

Engineering Mathematics

UNIT : Partial Differential Equation

Today's Target

- Homogeneous Linear Partial Differential Equation with constant coefficient

  Particular Integral(PI) : Case-I(Type-2)

- PYQs
- DPP

GATEWAY CLASSES

Homogeneous Linear PDE with constant coefficient
(All partial Derivatives are of same order)

Particular Integral

Case – 1

$$F(D, D') = \phi(ax + by)$$

$$\text{PI} = \frac{1}{F(D, D')}\phi(ax + by)$$

Where $F(D, D')$ *is a Homogeneous function of degree n*

Replace

$$D = a\,,\ D' = b$$

TYPE-1

if $F(a, b) \neq 0$

put $ax + by = u,$

$$\text{PI} = \frac{1}{F(a,b)} \iiint \phi(u)du\,du\ \ldots\ldots n\ times$$

GATEWAY CLASSES

Type - 2

If $F(a,b)=0$ case of failure

(i) When power of D $\geq$ power of D'	When power of D' $>$ power of D
(ii) $\mathrm{PI}=x\left[\frac{1\times\phi(ax+by)}{\frac{\partial}{\partial D}[F(D,D')]}\right]$	$\mathrm{PI}=y\left[\frac{1\times\phi(ax+by)}{\frac{\partial}{\partial D'}[F(D,D')]}\right]$
Again Repeat if $\frac{\partial}{\partial D}[F(D,D')]=0$	Again Repeat if $\frac{\partial}{\partial D'}[F(D,D')]=0$

GATEWAY CLASSES

Q.1 Solve the linear partial differential equation $(D - D')^2 z = \tan(y + x)$

$$(D-D')^2 z = \tan(y+x)$$

Put $D = m, D' = 1$

$$(m-1)^2 z = \tan(y+x)$$

Aux. Eqn

$$(m-1)^2 = 0$$

$$m = 1, 1$$

$$CF = f_1(y+x) + x f_2(y+x)$$

$$PI = \frac{1}{(D-D')^2} \tan(y+x) \quad \{\text{case of failure}\}$$

$$PI = x \times \frac{1}{2(D-D')} \tan(y+x) \quad \{\text{case of failure}\}$$

$$PI = x^2 \times \frac{1}{2 \times 1} \tan(y+x) = \frac{x^2}{2} \tan(y+x)$$

Complete solution is

$$z = CF + PI = f_1(y+x) + x f_2(y+x) + \frac{x^2}{2} \tan(y+x)$$

GATEWAY CLASSES

Q.2 Solve $4r - 4s + t = 16\log(x + 2y)$

$$4r - 4s + t = 16\log(x+2y)$$

$$4\frac{\partial^2 z}{\partial x^2} - 4\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y} = 16\log(x+2y)$$

$$(4D^2 - 4DD' + D'^2)z = 16\log(x+2y)$$

$$(2D - D')^2 z = 16\log(x+2y)$$

Put $D = m$, $D' = 1$

$$(2m-1)^2 z = 16\log(x+2y)$$

Aux. Eqn

$$(2m-1)^2 = 0$$

$$m = \frac{1}{2}, \frac{1}{2}$$

$$CF = f_1\left(y + \frac{x}{2}\right) + x f_2\left(y + \frac{x}{2}\right)$$

$$PI = \frac{1}{(2D - D')^2} 16\log(x+2y)$$

{ case of failure }

GATEWAY CLASSES

$$PI = x \times \frac{1}{2(2D-D')\times 2} \times 16 \log(x+2y)$$

{Again case of failure}

$$PI = x^2 \times \frac{1}{4\times 2} \times \cancel{16}^{2} \log(x+2y)$$

$$\boxed{PI = 2x^2 \log(x+2y)}$$

Complete solution is

$$z = CF + PI$$

$$\boxed{z = f_1(y+\tfrac{x}{2}) + x f_2(y+\tfrac{x}{2}) + 2x^2 \log(x+2y)}$$

GATEWAY CLASSES

Q.3 Solve the differential equation : $\frac{\partial^3 z}{\partial x^2 \partial y} - 2\frac{\partial^3 z}{\partial x \partial y^2} + \frac{\partial^3 z}{\partial y^3} = \frac{1}{x^2}$.

$$\frac{\partial^3 z}{\partial x^2 \partial y} - 2\frac{\partial^3 z}{\partial x \partial y^2} + \frac{\partial^3 z}{\partial y^3} = \frac{1}{x^2}$$

$$(D^2D' - 2DD'^2 + D'^3)z = \frac{1}{x^2}$$

$$D'(D^2 - 2DD' + D'^2)z = \frac{1}{x^2}$$

Aux Eqn

$D' = 0$

$CF = f_1(x)$

$D^2 - 2DD' + D'^2 = 0$

$m^2 - 2m + 1 = 0$

$(m-1)^2 = 0$

$m = 1, 1$

$CF = f_2(y+x) + x f_3(y+x)$

$$CF = f_1(x) + f_2(y+x) + x f_3(y+x)$$

$$PI = \frac{1}{(D^2D' - 2DD'^2 + D'^3)} \times \frac{1}{(x+oy)^2}$$

{ case of failure }

GATEWAY CLASSES

$$PI = y \times \frac{1}{(D^2 - 4DD' + 3D'^2)} \times \frac{1}{(x+0y)^2}$$

Put $D = 1, D' = 0, (x+0y) = u$

$$PI = y \times \frac{1}{1} \times \iint \frac{1}{u^2} du\, du$$

$$PI = y \times \int -\frac{1}{u}\, du$$

$$PI = -y \log u$$

$$\boxed{PI = -y \log x}$$

Complete solution is

$$z = CF + PI$$

$$\boxed{z = f_1(x) + f_2(y+x) + x f_3(y+x) - y \log x}$$

GATEWAY CLASSES

Q.4 Solve $(D^3 - 4D^2D' + 5DD'^2 - 2D'^3)z = e^{y+2x} + (y+x)^{1/2}$ **(AKTU-2017)**

$$(D^3 - 4D^2D' + 5DD'^2 - 2D'^3)z = e^{y+2x} + (y+x)^{1/2}$$

Put $D = m, D' = 1$

$$(m^3 - 4m^2 + 5m - 2)z = e^{y+2x} + (y+x)^{1/2}$$

Aux. Eqn

$$m^3 - 4m^2 + 5m - 2 = 0$$

$$m = 1, 1, 2$$

$$CF = f_1(y+x) + x f_2(y+x) + f_3(y+2x)$$

$$PI = \frac{1}{D^3 - 4D^2D' + 5DD'^2 - 2D'^3}\left[e^{y+2x} + (y+x)^{\frac{1}{2}}\right]$$

$$PI = \underbrace{\frac{1}{(D^3 - 4D^2D' + 5DD'^2 - 2D'^3)} e^{y+2x}}_{P_1} + \underbrace{\frac{1}{(D^3 - 4D^2D' + 5DD'^2 - 2D'^3)} (y+x)^{\frac{1}{2}}}_{P_2}$$

$$PI = P_1 + P_2$$

GATEWAY CLASSES

$$P_1 = \frac{1}{(D^3 - 4D^2D' + 5DD'^2 - 2D'^3)} e^{y+2x}$$

Put $D = 2,\ D' = 1$

$$P_1 = \frac{1}{(8 - 16 + 10 - 2)} e^{y+2x}$$

{case of failure}

$$P_1 = x \times \frac{1}{(3D^2 - 8DD' + 5D'^2)} e^{y+2x}$$

Put $D = 2,\ D' = 1,\ y + 2x = u$

$$P_1 = \frac{x}{12 - 16 + 5} \int\int e^u \, du \, du$$

$$P_1 = x\, e^u$$

$$\boxed{P_1 = x\, e^{y+2x}}$$

$$P_2 = \frac{1}{D^3 - 4D^2D' + 5DD'^2 - 2D'^3)} (y+x)^{1/2}$$

{case of failure}

$$P_2 = x \times \frac{1}{(3D^2 - 8DD' + 5D'^2)} (y+x)^{1/2}$$

GATEWAY CLASSES

{ case of failur }

$$P_2 = x^2 \times \frac{1}{(6D - 8D')} (y+x)^{1/2}$$

Put $D = 1,\ D' = 1,\ (y+x) = u$

$$P_2 = x^2 \times \frac{1}{(6-8)} \int u^{1/2}\, du$$

$$P_2 = \frac{x^2}{-2} \times \frac{u^{3/2}}{\frac{3}{2}}$$

$$P_2 = -\frac{x^3}{3} u^{3/2}$$

$$\boxed{P_2 = -\frac{x^3}{3} (y+x)^{3/2}}$$

$$PI = x e^{y+2x} - \frac{x^3}{3} (y+x)^{3/2}$$

complete solution is

$$Z = CF + PI$$

$$\boxed{Z = f_1(y+x) + x f_2(y+x) + f_3(y+2x) + x e^{y+2x} - \frac{x^3}{3} (y+x)^{3/2}}$$

GATEWAY CLASSES

Q.5 Solve the linear partial differential equation : $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} = \sin x \cos y$

$$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} = \sin x \cos y$$

$$(D^2 - DD')z = \sin x \cos y$$

$$D(D - D')z = \sin x \cos y$$

Aux. Eqn

$D = 0$	$D - D' = 0$
$CF = f_1(y)$	$m - 1 = 0$
	$m = 1$
	$CF = f_2(y + x)$

$$PI = \frac{1}{(D^2 - DD')} \sin x \cos y$$

$$PI = \frac{1}{2(D^2 - DD')} (2 \sin x \cos y)$$

$$PI = \frac{1}{2(D^2 - DD')} \{\sin(x+y) + \sin(x-y)\}$$

$$PI = \frac{1}{2(D^2 - DD')} \sin(x+y) + \frac{1}{2(D^2 - DD')} \sin(x-y)$$

$$PI = P_1 + P_2$$

GATEWAY CLASSES

$$P_1 = \frac{1}{2(D^2 - DD')} \sin(x+y)$$

{case of failur}

$$P_1 = x \times \frac{1}{2(2D - D)} \sin(x+y)$$

Put $D = 1,\ D' = 1,\ x+y = u$

$$P_1 = x \times \frac{1}{2(2-1)} \int \sin u \, du$$

$$P_1 = \frac{x}{2}(-\cos u)$$

$$\boxed{P_1 = \frac{-x}{2}\cos(x+y)}$$

$$P_2 = \frac{1}{2(D^2 - DD')} \sin(x-y)$$

Put $D = 1,\ D' = -1,\ x-y = u$

$$P_2 = \frac{1}{2(1+1)} \int\int \sin u \, du \, du$$

$$P_2 = \frac{1}{4}(-\sin u)$$

GATEWAY CLASSES

$$P_2 = -\frac{1}{4}\sin(x-y)$$

$$PI = -\frac{x}{2}\cos(x+y) - \frac{1}{4}\sin(x-y)$$

Complete solution

$$z = CF + PI$$

$$\boxed{z = f_1(y) + f_2(y+x) - \frac{x}{2}\cos(x+y) - \frac{1}{4}\sin(x-y)}$$

GATEWAY CLASSES

PDE : DPP-10

Topic : Homogeneous Linear Partial Differential Equation with constant coefficient

Particular Integral(PI) : Case-I (Type-2)

Q .1 *Solve the linear partial differential equation* $\frac{\partial^3 z}{\partial x^3} - 4\frac{\partial^3 z}{\partial x^2 \partial y} + 4\frac{\partial^3 z}{\partial x \partial y^2} = 4\sin(2x+y)$.

Ans $f_1(y) + f_2(y+2x) + x f_3(y+2x) - x^2 \cos(2x+y)$

Q .2 *Solve :* $2r - s - 3t = 5\frac{e^x}{e^y}$

Ans $z = f_1(y-x) + f_2\left(y+\frac{3}{2}x\right) + xe^{x-y}$

Q .3 *Solve :* $\frac{\partial^3 z}{\partial x^3} - 7\frac{\partial^3 z}{\partial x \partial y^2} - 6\frac{\partial^3 z}{\partial y^3} = \sin(x+2y) + e^{3x+y}$. *(UPTU-2014)*

Ans $z = f_1(y-x) + f_2(y-2x) + f_3(y+3x) - \frac{1}{75}\cos(x+2y) + \frac{x}{20}e^{3x+y}$

GATEWAY CLASSES

Engineering Mathematics

UNIT : Partial Differential Equation

Today's Target

- Homogeneous Linear Partial Differential Equation with constant coefficient

  Particular Integral(PI) : Case-II and Case-III

- PYQs
- DPP

GATEWAY CLASSES

Homogeneous Linear PDE with constant coefficient

(All partial Derivatives are of same order)

Particular Integral

Case-II

$$F(x, y) = x^m y^n$$

$$PI = \frac{1}{f(D, D')} x^m y^n$$

$$PI = [1 + \phi(D, D')]^{-1} x^m y^n$$

Note :-

(i) If $n = m$ (Taking either D or D' as common factor)

(ii) If $n > m$ (Taking D' as a common factor)

(iii) If $m > n$ (Taking D as a common factor)

GATEWAY CLASSES

After taking common, Apply the formulae given below

(1) $(1+x)^{-1} = 1 - x + x^2 - x^3 + x^4 + \ldots\ldots$

(2) $(1-x)^{-1} = 1 + x + x^2 + x^3 + x^4 + \ldots\ldots$

(3) $(1+x)^{-2} = 1 - 2x + 3x^2 - 4x^3 + \cdots$

(4) $(1-x)^{-2} = 1 + 2x + 3x^2 + 4x^3 + \cdots$

GATEWAY CLASSES

Q.1 Solve : $\frac{\partial^2 z}{\partial x^2} + 3\frac{\partial^2 z}{\partial x \partial y} + 2\frac{\partial^2 z}{\partial y^2} = 12xy.$ **(UPTU-2015)**

$$(D^2 + 3DD' + 2D'^2)z = 12xy$$

Put $D = m, D' = 1$

$$(m^2 + 3m + 2)z = 12xy$$

Aux. Eqⁿ

$$m^2 + 3m + 2 = 0$$

$$(m+1)(m+2) = 0$$

$$m = -1, -2$$

$$CF = f_1(y - x) + f_2(y - 2x)$$

$$PI = \frac{1}{(D^2 + 3DD' + 2D'^2)} 12xy$$

$$PI = \frac{1}{D^2\left[1 + 3\frac{D'}{D} + 2\frac{D'^2}{D^2}\right]} 12xy$$

$$PI = \frac{1}{D^2}\left[1 + \left(3\frac{D'}{D} + 2\frac{D'}{D^2}\right)\right]^{-1} \times 12xy$$

GATEWAY CLASSES

$$PI = \frac{1}{D^2}\left[1 - \left(3\frac{D'}{D} + 2\frac{D'^2}{D^2}\right) + \left(3\frac{D'}{D} + 2\frac{D'^2}{D^2}\right)^2 - \cdots\right] 12xy$$

(✓ ✓ × ×)

$$PI = \frac{12}{D^2}\left[1 - 3\frac{D'}{D}\right] xy$$

$$PI = \frac{12}{D^2}\left[xy - \frac{3D'(xy)}{D}\right]$$

$$PI = \frac{12}{D^2}\left[xy - 3\times\frac{1}{D}x\right]$$

$$PI = \frac{12}{D^2}\left[xy - \frac{3x^2}{2}\right]$$

$$PI = \frac{12}{D}\left[\int\left(xy - \frac{3x^2}{2}\right)dx\right]$$

$$PI = \frac{12}{D}\left[\frac{x^2}{2}y - \frac{3x^3}{3\times 2}\right]$$

$$PI = 12\int\left(\frac{x^2}{2}y - \frac{x^3}{2}\right)dx$$

$$PI = 12\left(\frac{x^3}{6}y - \frac{x^4}{8}\right)$$

$$PI = 2x^3y - \frac{3x^4}{2}$$

complete solution

$$Z = CF + PI$$

GATEWAY CLASSES

Q.2 Solve : $\frac{\partial^3 u}{\partial x^3} - \frac{\partial^3 u}{\partial y^3} = x^3 y^3$.

$$\frac{\partial^3 u}{\partial x^3} - \frac{\partial^3 u}{\partial y^3} = x^3 y^3$$

$$(D^3 - D'^3)u = x^3 y^3$$

Put $D = m$, $D' = 1$

$$(m^3 - 1)u = x^3 y^3$$

Aux Equation

$$m^3 - 1 = 0$$

$$(m-1)(m^2 + m + 1) = 0$$

$m = 1, \omega, \omega^2$

$$CF = f_1(y+x) + f_2(y+\omega x) + f_3(y+\omega^2 x)$$

$$PI = \frac{1}{(D^3 - D'^3)} x^3 y^3$$

$$PI = \frac{1}{D^3\left[1 - \frac{D'^3}{D^3}\right]} x^3 y^3$$

$$PI = \frac{1}{D^3}\left[1 - \frac{D'^3}{D^3}\right]^{-1} x^3 y^3$$

$$PI = \frac{1}{D^3}\left[1 + \frac{D'^3}{D^3}\right] x^3 y^3$$

$$PI = \frac{1}{D^3}\left[x^3 y^3 + \frac{D'^3}{D^3} x^3 y^3\right]$$

$$PI = \frac{1}{D^3}\left[x^3 y^3 + 6x\frac{1}{D^3}\right]$$

GATEWAY CLASSES

$$PI = \frac{1}{D^3}\left[x^3y^3 + \frac{6 \times x^6}{4 \times 5 \times 6}\right]$$

$$PI = \frac{1}{D^3}\left[x^3y^3 + \frac{x^6}{20}\right]$$

$$PI = \frac{x^6y^3}{4 \times 5 \times 6} + \frac{x^9}{20 \times 7 \times 8 \times 9}$$

$$\boxed{PI = \frac{x^6y^3}{120} + \frac{x^9}{10080}}$$

Complete solution is

$$u = CF + PI$$

$$\boxed{u = f_1(y+x) + f_2(y+\omega x) + f_3(y+\omega^2 x) + \frac{x^6y^3}{120} + \frac{x^9}{10080}}$$

GATEWAY CLASSES

CASE- III :- General case

Any Function of $(x,\ y)$

$$PI = \frac{1}{F(D, D')}\ \phi(x,\ y)$$

$$PI = \frac{1}{(D - m_1 D')(D - m_2 D') \ldots (D - m_n D)}\ \phi(x,\ y)$$

$$\frac{1}{(D - m_1 D')}\ \phi(x,\ y) = \int \phi(x,\ c - m_1 x)\, dx$$

Put $y = c - m_1 x$

$y = (c + m_1 x)$

GATEWAY CLASSES

Q.3 Solve : $(D^2 - DD' - 2D'^2)z = (y-1)e^x$. **(UPTU-2014)**

$$(D^2 - DD' - 2D'^2)z = (y-1)e^x$$

Put $D = m, D' = 1$

$$(m^2 - m - 2)z = (y-1)e^x$$

Aux Eqn

$$m^2 - m - 2 = 0$$

$$(m+1)(m-2) = 0$$

$$m = -1, 2$$

$$CF = f_1(y-x) + f_2(y+2x)$$

$$PI = \frac{1}{D^2 - DD' - 2D'^2}(y-1)e^x$$

$$PI = \frac{1}{(D+D')(D-2D')}(y-1)e^x$$

$$PI = \frac{1}{(D-2D')}\left[\frac{1}{D+D'}(y-1)e^x\right]$$

GATEWAY CLASSES

$$PI = \frac{1}{(D-2D')} \int (c+x-1)\, e^x \, dx$$

(1) (2)

$$PI = \frac{1}{(D-2D')} \left[(c+x-1)\, e^x - 1 \times e^x \right]$$

$$PI = \frac{1}{D-2D'} \left[(c+x-1-1)\, e^x \right]$$

$$PI = \frac{1}{D-2D'} \left[(c+x-2)\, e^x \right]$$

$$PI = \frac{1}{D-2D'} (y-2)\, e^x$$

$$PI = \int (c-2x-2)\, e^x \, dx$$

$$PI = (c-2x-2)\, e^x - (-2)\, e^x$$

$$PI = (c-2x-2+2)\, e^x$$

$$PI = (c-2x)\, e^x$$

$$PI = y e^x$$

complete solution

$$z = CF + PI$$

GW GATEWAY CLASSES
GATEWAY CLASSES

Q.4 Solve the linear partial deferential equation $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 6\frac{\partial^2 z}{\partial y^2} = y\cos x$.

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 6\frac{\partial^2 z}{\partial y^2} = y\cos x$$

$$(D^2 + DD' - 6D'^2)z = y\cos x$$

Put $D = m$, $D' = 1$

$$(m^2 + m - 6)z = y\cos x$$

Aux. Eqn

$$m^2 + m - 6 = 0$$

$$(m-2)(m+3) = 0$$

$$m = 2, -3$$

$$CF = f_1(y+2x) + f_2(y-3x)$$

$$PI = \frac{1}{(D^2 + DD' - 6D'^2)} y\cos x$$

$$PI = \frac{1}{(D-2D')(D+3D')} y\cos x$$

$$PI = \frac{1}{(D-2D')}\left[\frac{1}{D+3D'} y\cos x\right]$$

GATEWAY CLASSES

$$PI = \frac{1}{(D-2D')} \int (c+3x)\cos x\, dx$$

$$PI = \frac{1}{(D-2D')} \int (c\cos x + 3x\cos x)\, dx$$

$$PI = \frac{1}{D-2D'} \left[c\int \cos x\, dx + 3\int \underset{①}{x}\,\underset{②}{\cos x}\, dx \right]$$

$$PI = \frac{1}{D-2D'} \left[c\sin x + 3\{x(\sin x) - 1\times(-\cos x)\} \right]$$

$$PI = \frac{1}{D-2D'} \left[c\sin x + 3x\sin x + 3\cos x \right]$$

$$PI = \frac{1}{D-2D'} \left[\underline{(c+3x)}\sin x + 3\cos x \right]$$

$$PI = \frac{1}{D-2D'} \left[y\sin x + 3\cos x \right]$$

$$PI = \int \left[\{(c-2x)\sin x + 3\cos x \right] dx$$

$$PI = \int (c\sin x - 2x\sin x + 3\cos x)\, dx$$

$$PI = c\int \sin x\, dx - 2\int x\sin x\, dx +$$

$$3\int \cos x\, dx$$

GW GATEWAY CLASSES
GATEWAY CLASSES

$$PI = -c\times\cos x - 2\{x(-\cos x) - 1\times(-\sin x)\} + 3\sin x$$

$$= -c\times\cos x + 2x\cos x - 2\sin x + 3\sin x$$

$$= -\cos x(c - 2x) + \sin x$$

$$PI = -y\cos x + \sin x$$

complete solution

$$z = CF + PI$$

$$\boxed{z = f_1(y+2x) + f_2(y-3x) - y\cos x + \sin x}$$

GATEWAY CLASSES

Engineering Mathematics by Gulshan Sir

PDE : DPP-11

Topic : Homogeneous Linear Partial Differential Equation with constant coefficient

Particular Integral(PI) : Case-II and Case-III

Q .1 *Solve* $[D^2 + (a+b)DD' + ab\,D'^2]z = xy$ *(GBTU-2011 & 2013)*

$$z = f_1(y-ax) + f_2(y-bx) + \frac{1}{6}x^3y - \frac{(a+b)}{24}x^4$$

Q .2 *Solve :* $(D^3 + 2D^2D' - DD'^2 - 2D'^3)z = (y+2)e^x.$

$$z = f_1(y+x) + f_2(y-x) + f_3(y-2x) + ye^x$$

Q . *Solve :* $(D^2 + DD' - 6\ D'^2)z = y\sin x.$ *(MTU-2011 & 2012)*

$$z = f_1(y+2x) + f_2(y-3x) - y\sin x - \cos x$$

GATEWAY CLASSES

Engineering Mathematics

UNIT : Partial Differential Equation

Today's Target

- Non-Homogeneous Linear Partial Differential Equation with constant coefficient

Complementary Function (CF)

- PYQs
- DPP

GATEWAY CLASSES

Non - Homogeneous Linear Partial Differential Equation with constant coefficient
(All Partial Derivatives are not of same order)

Given PDE

$$\phi(D,\ D') = F(x,\ y) \text{ --------} ①$$

Methods to find CF

CASE-1:- When $\phi(D, D')$ resolve in to linear factors

$$(D - m_1 D' - a_1)(D - m_2 D' - a_2)(D - m_3 D' - a_3)z = F(x, y)$$

$$CF = e^{a_1 x} f_1(y + m_1 x) + e^{a_2 x} f_2(y + m_2 x) + e^{a_3 x} f_3(y + m_3 x)$$

NOTE : (i) CF corresponding to D = $f_1(y)$ (ii)CF corresponding to D′ = $f_1(x)$

GATEWAY CLASSES

Q.1 *Solve the linear partial differential equation* $(D + D' - 1)(D + 2D' - 2)z = 0.$

$$(D + D' - 1)(D + 2D' - 2)z = 0$$

$$CF = e^{x} f_1(y - x) + e^{2x} f_2(y - 2x)$$

$$PI = 0$$

Complete solution

$$z = CF + PI$$

$$\boxed{z = e^{x} f_1(y - x) + f_2(y - 2x)}$$

GATEWAY CLASSES

Q.2 Solve: $DD'(D + 2D' + 1)z = 0$.

$$DD'(D+2D'+1)z = 0$$

CF corresponding to $D = f_1(y)$

CF " " $D' = f_2(x)$

CF " $(D+2D'+1) = e^{-x} f_3(y-2x)$

$$CF = f_1(y) + f_2(x) + e^{-x} f_3(y-2x)$$

$$PI = 0$$

Complete solution

$$z = CF + PI$$

GATEWAY CLASSES

Q.3 Solve : $r - t + p - q = 0$. (UPTU-2013,2018)

$$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} + \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y} = 0$$

$$(D^2 - D'^2 + D - D')z = 0$$

$$[(D-D')(D+D') + (D-D')]z = 0$$

$$(D-D')(D+D'+1)z = 0$$

$$(D-D'+0)(D+D'+1)z = 0$$

$$CF = e^{0x} f_1(y+x) + e^{-x} f_2(y-x)$$

$$CF = f_1(y+x) + e^{-x} f_2(y-x)$$

$$PI = 0$$

complete solution

$$z = CF + PI$$

$$\boxed{z = f_1(y+x) + e^{-x} f_2(y-x)}$$

GATEWAY CLASSES

CASE-2:-

When $\phi(D,\ D')$ contains repeated factors

$$(D - m\,D' - a)^3\ z = F(x, y)$$

$$CF = e^{ax}\, f_1(y + mx) + xe^{ax}\, f_2(y + mx) + x^2 e^{ax}\, f_3(y + mx)$$

GATEWAY CLASSES

Q.4 *Solve :* $(D + 4D' + 5)^2 z = 0.$ *(AKTU-2018)*

$$(D+4D'+5)^2 z = 0$$

$$CF = e^{-5x} f_1(y-4x) + x e^{-5x} f_2(y-4x)$$

$$PI = 0$$

Complete solution

$$z = CF + PI$$

$$\boxed{z = e^{-5x} f_1(y-4x) + x f_2(y-4x)}$$

GATEWAY CLASSES

Q.5 Solve: $r + 2s + t + 2p + 2q + z = 0$.

$$r + 2s + t + 2p + 2q + z = 0$$

$$\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} + 2\frac{\partial z}{\partial x} + 2\frac{\partial z}{\partial y} + z = 0$$

$$\left[(D^2 + 2DD' + D'^2) + 2D + 2D' + 1\right] z = 0$$

$$\left[(D + D')^2 + 2(D + D') + 1\right] z = 0$$

$$(D + D' + 1)^2 z = 0$$

$$CF = e^{-x} f_1(y - x) + x e^{-x} f_2(y - x)$$

$$PI = 0$$

complete solution

$$z = CF + PI$$

$$z = e^{-x} f_1(y - x) + x e^{-x} f_2(y - x)$$

GATEWAY CLASSES

CASE-3:-

When Function $\phi(D,\ D')$ con not be factorized into linear factor

Take a trial solution

$$z = Ae^{hx+ky}$$

Where A, h, k *are constants.*

GATEWAY CLASSES

Q. 6 **Solve:** $(D^2 + D'^2 - p^2)z = 0.$

$$(D^2 + D'^2 - p^2)z \quad \text{——} (1)$$

consider a trial solution

$$z = Ae^{hx+ky}$$

From eqn (1)

$$(D^2+D'^2-p^2)Ae^{hx+ky} = 0$$

$$A\left[D^2 e^{hx+ky} + D'^2 e^{hx+ky} - p^2 e^{hx+ky}\right] = 0$$

$$A\left[h^2 e^{hx+ky} + k^2 e^{hx+ky} - p^2 e^{hx+ky}\right] = 0$$

$$Ae^{hx+ky}(h^2+k^2-p^2) = 0$$

But $Ae^{hx+ky} \neq 0$

$$\therefore h^2+k^2-p^2 = 0$$

$$CF = \sum Ae^{hx+ky}$$

$$z = CF + PI$$

Where $h^2+k^2-p^2 = 0$

$$PI = 0$$

GATEWAY CLASSES

Engineering Mathematics by Gulshan Sir

PDE : DPP-12

Topic : Non-Homogeneous Linear Partial Differential Equation with constant coefficient

Complementary Function (CF)

Q .1 *Solve* $2s + t - 3q = 0$

Q .2 Solve $DD'(2D + D' + 11)^2 z = 0$

GATEWAY CLASSES

Engineering Mathematics

UNIT : Partial Differential Equation

Today's Target

- Non-Homogeneous Linear Partial Differential Equation with constant coefficient

Particular Integral (PI) : Case-1 and Case-2

- PYQs
- DPP

GATEWAY CLASSES

Course Details(Paid) : All Subjects

1	Recorded Video Lectures (100 % Syllabus Coverage)
2	Pdf Notes
3	Lecture wise DPP with Answers
4	DPP Solutions
5	Unit wise set of PYQs

GATEWAY CLASSES

Paid Courses are available in Gateway Classes Application
Link in Description

Non - Homogeneous Linear Partial Differential Equation with constant coefficient
(All Partial Derivatives are not of same order)

Given PDE

$$\phi(D,\ D') = F(x,\ y)$$

$\phi(D, D')$ → CF ; $F(x, y)$ → PI

Methods to find *PI*

CASE-1:- When $F(x,\ y) = e^{ax+by}$

$$PI = \frac{1}{\phi(D, D')} e^{ax+by}$$

Put $D = a$

$D' = b$

$$PI = \frac{1}{\phi(a, b)} e^{ax+by}$$

When $\phi(a, b) \neq 0$

GATEWAY CLASSES

Q.1 Solve $(D^2 - 4DD' + 4D'^2 - D + 2D')z = e^{3x+4y}$

$$(D^2 - 4DD' + 4D'^2 - D + 2D')z = e^{3x+4y}$$

$$\left[(D-2D')^2 - (D-2D')\right]z = e^{3x+4y}$$

$$(D-2D')(D-2D'-1)z = e^{3x+4y}$$

$$(D-2D'-0)(D-2D'-1)z = e^{3x+4y}$$

$$CF = e^{0x} f_1(y+2x) + e^{-x} f_2(y+2x)$$

$$\boxed{CF = f_1(y+2x) + e^{+x} f_2(y+2x)}$$

$$PI = \frac{1}{(D^2 - 4DD' + 4D'^2 - D + 2D')} e^{3x+4y}$$

Put $D = 3$ and $D' = 4$

$$PI = \frac{1}{9 - 48 + 64 - 3 + 8} e^{3x+4y}$$

$$PI = \frac{1}{30} e^{3x+4y}$$

Complete solution

$$z = CF + PI$$

$$\boxed{z = f_1(y+2x) + e^{x} f_2(y+2x) + \frac{1}{30} e^{3x+4y}}$$

GATEWAY CLASSES

Q.2 Solve: $D(D-2D'-3)z = e^{x+2y}$.

$$D(D-2D'-3)z = e^{x+2y}$$

CF corresponding to $D = f_1(y)$

CF " $(D-2D'-3) = e^{3x} f_2(y+2x)$

$$\boxed{CF = f_1(y) + e^{3x} f_2(y+2x)}$$

$$PI = \frac{1}{D(D-2D'-3)} e^{x+2y}$$

Put $D = 1$

$D' = 2$

$$PI = \frac{1}{1(1-2\times2-3)} e^{x+2y}$$

$$PI = -\frac{1}{6} e^{x+2y}$$

complete solution

$$z = CF + PI$$

$$\boxed{z = f_1(y) + f_2(y+2x) - \frac{1}{6} e^{x+2y}}$$

GATEWAY CLASSES

Non - Homogeneous Linear Partial Differential Equation with constant coefficient

(All Partial Derivatives are not of same order)

Given PDE

$$\phi(D,\ D') = F(x,\ y)$$

where

$$\phi(D, D') \neq 0$$

Methods to find *PI*

CASE-2:- When $F(x,\ y) = sin(ax + by)/Cos(ax + by)$

$$PI = \frac{1}{\phi(D, D')} \sin(ax+by)/\cos(ax+by)$$

Put $D^2 = -a^2$

$DD' = -ab$

$D'^2 = -b^2$

GATEWAY CLASSES

Q.3 Solve: $(D - D' - 1)(D - D' - 2)z = \sin(2x + 3y)$ **(AKTU-2021-22)**

$$(D-D'-1)(D-D'-2)z = \sin(2x+3y)$$

$$CF = e^{x} f_1(y+x) + e^{2x} f_2(y+x)$$

$$PI = \frac{1}{(D-D'-1)(D-D'-2)} \sin(2x+3y)$$

$$PI = \frac{1}{D^2 - DD' - 2D - D'D + D'^2 + 2D' - D + D' + 2} \sin(2x+3y)$$

$$PI = \frac{1}{D^2 - 2DD' + D'^2 - 3D + 3D' + 2} \sin(2x+3y)$$

Put $D^2 = -4$

$DD' = -6$

$D'^2 = -9$

$$PI = \frac{1}{-4 + 12 - 9 - 3D + 3D' + 2} \sin(2x+3y)$$

$$PI = \frac{1}{-3D + 3D' + 1} \sin(2x+3y)$$

$$PI = \frac{1}{-[3D - 3D' - 1]} \sin(2x+3y)$$

GATEWAY CLASSES

$$PI = \frac{-1 \times [(3D-3D')+1]}{[(3D-3D')-1][(3D-3D')+1]} e^{2x+3y}$$

$$PI = \frac{-[(3D-3D')+1]}{(3D-3D')^2 - 1^2} e^{2x+3y}$$

$$PI = \frac{-[3D-3D'+1]}{9D^2+9D'^2-18DD'-1} e^{2x+3y}$$

Put $D^2 = -4$

$D^2 = -9$

$DD' = -6$

$$PI = \frac{-(3D-3D'+1)}{-36-81+108-1} \sin(2x+3y)$$

$$PI \quad \frac{+(3D-3D'+1)}{+10} \sin(2x+3y)$$

$$PI = \frac{1}{10}\left[3D\sin(2x+3y) - 3D'\sin(2x+3y) + \sin(2x+3y)\right]$$

$$PI = \frac{1}{10}\left[3\times 2\cos(2x+3y) - 9\cos(2x+3y) + \sin(2x+3y)\right]$$

GATEWAY CLASSES

$$PI = \frac{1}{10}\left[\sin(2x+3y) - 3\cos(2x+3y)\right]$$

complete solution

$$\boxed{Z = CF + PI}$$

GATEWAY CLASSES

Q.4 *Solve the following partial differential equations:*

$$(D^2 - DD' - 2D'^2 + 2D + 2D')z = \sin(2x + y)$$ *(AKTU-2016)*

$(D^2 - DD' - 2D'^2 + 2D + 2D')z = \sin(2x+y)$

$(D^2 + DD' - DD' - DD' - 2D'^2 + 2D + 2D')z = \sin(2x+y)$

$(D^2 + DD' - 2DD' - 2D'^2 + 2D + 2D')z = \sin(2x+y)$

$[D(D+D') - 2D'(D+D') + 2(D+D')]z = \sin(2x+y)$

$(D+D')(D-2D'+2)z = \sin(2x+y)$

$CF = e^{0x} f_1(y-x) + e^{-2x} f_2(y+2x)$

$CF = f_1(y-x) + f_2(y+2x)$

GATEWAY CLASSES

$$PI = \frac{1}{D^2 - DD' - 2D'^2 + 2D + 2D'} \sin(2x+y)$$

Put $D^2 = -4$

$D'^2 = -1$

$DD' = -2$

$$PI = \frac{1}{-4 + 2 + 2 + 2D + 2D'} \sin(2x+y)$$

$$PI = \frac{1}{2(D+D')} \sin(2x+y)$$

$$PI = \frac{D-D'}{2(D+D')(D-D')} \sin(2x+y)$$

$$PI = \frac{D-D'}{2(D^2-D'^2)} \sin(2x+y)$$

Put $D^2 = -4$

$D'^2 = -1$

$$PI = \frac{(D-D')}{2(-4+1)} \sin(2x+y)$$

$$PI = -\frac{1}{6}\left[D\sin(2x+y) - D'\sin(2x+y)\right]$$

$$PI = -\frac{1}{6}\left[2\cos(2x+y) - \cos(2x+y)\right]$$

GATEWAY CLASSES

$$PI = -\frac{1}{6}\cos(2n+y)$$

complete solution

$$Z = CF + PI$$

GATEWAY CLASSES

PDE : DPP-13

➢ Topic : Non-Homogeneous Linear Partial Differential Equation with constant coefficient

Particular Integral (PI) : Case-1 and Case-2

Q .1 *Solve:* $(D^3 - 3DD' + D' + 4)z = e^{2x+y}$.

Q .2 *Solve* $(D^2 - D'^2 - 3D + 3D')z = e^{x-2y}$

Q .3 *Find the particular integral of* $2s + t - 3q = 5\cos(3x - 2y)$.

Q .4 *Solve the following partial differential equations:* $(D^2 - DD' + D' - 1)z = \cos(x + 2y)$

GATEWAY CLASSES

Engineering Mathematics

UNIT : Partial Differential Equation

Today's Target

- Non-Homogeneous Linear Partial Differential Equation with constant coefficient

Particular Integral (PI) : Case-3 and Case-4

- PYQs
- DPP

GATEWAY CLASSES

Course Details(Paid) : All Subjects

1	Recorded Video Lectures (100 % Syllabus Coverage)
2	Pdf Notes
3	Lecture wise DPP with Answers
4	DPP Solutions
5	Unit wise set of PYQs

GATEWAY CLASSES

Paid Courses are available in Gateway Classes Application
Link in Description

Non - Homogeneous Linear Partial Differential Equation with constant coefficient
(All Partial Derivatives are not of same order)

Given PDE

$$\phi(D,\ D') = F(x,\ y)$$

CF PI

Methods to find *PI*

CASE-3:- When $F(x,\ y) = x^m y^n$

$$PI = \frac{1}{\phi(D, D')}\, x^m y^n$$

$$PI = \left[1 + \phi(D, D')\right]^{-1} x^m y^n$$

Note :-

(i) If a separate constant term is present , take it common

(ii) If $n = m$ (Taking either D or D' as common factor)

(iii) If $n > m$ (Taking D' as a common factor)

(iv) If $m > n$ (Taking D as a common factor)

GATEWAY CLASSES

After taking common, Apply the formulae given below

(1) $(1+x)^{-1} = 1 - x + x^2 - x^3 + x^4 + \ldots\ldots$

(2) $(1-x)^{-1} = 1 + x + x^2 + x^3 + x^4 + \ldots\ldots$

(3) $(1+x)^{-2} = 1 - 2x + 3x^2 - 4x^3 + \cdots$

(4) $(1-x)^{-2} = 1 + 2x + 3x^2 + 4x^3 + \cdots$

GATEWAY CLASSES

Q.1 Solve : $s + p - q = z + xy$.

$$s + p - q = z + xy$$

$$\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y} - z = xy$$

$$(DD' + D - D' - 1)z = xy$$

$$(DD' - D' + D - 1)z = xy$$

$$[D'(D-1) + 1(D-1)]z = xy$$

$$\underline{(D-1)(D'+1)z = xy}$$

CF corresponding to $D - 1 = e^x f_1(y)$

CF " " " $\underline{(D'+1)} = e^{-y} f_2(x)$

$$\boxed{CF = e^x f_1(y) + e^{-y} f_2(x)}$$

$$PI = \frac{1}{(D-1)(D'+1)} xy$$

$$PI = \frac{-1}{(1-D)(1+D')} xy$$

$$PI = -1\left[(1-D)^{-1}(1+D')^{-1}\right] xy$$

GATEWAY CLASSES

$$PI = -\left[(1 + D + D^2 + \cdots)(1 - D' + D'^2 - \cdots)\right] xy$$

$$PI = -\left[1 - D' + D - DD' + \cdots\right] xy$$

$$PI = -\left[xy - D'(xy) + D(xy) - DD'(xy)\right]$$

$$PI = -(xy - x + y - 1)$$

$$PI = -xy + x - y + 1$$

complete solution

$$Z = CF + PI$$

$$\boxed{Z = e^{x} f_1(y) + e^{-y} f_2(x) - xy + x - y + 1}$$

GATEWAY CLASSES

Q.2 Solve: $D(D+D'-1)(D+3D'-2)z = x^2-4xy+2y^2$.

$$D(D+D'-1)(D+3D'-2)z = x^2-4xy+2y^2$$

$$CF = f_1(y)+e^{x}f_2(y-x)+e^{2x}f_3(y-3x)$$

$$PI = \frac{1}{D(D+D'-1)(D+3D'-2)}(x^2-4xy+2y^2)$$

$$PI = \frac{1}{D(-1)\left[1-(D+D')\right](-2)\left[1-\left(\frac{D}{2}+\frac{3}{2}D'\right)\right]}(x^2-4xy+2y^2)$$

$$PI = \frac{1}{2D}\left[1-(D+D')\right]^{-1}\left[1-\left(\frac{D}{2}+\frac{3}{2}D'\right)\right]^{-1}(x^2-4xy+2y^2)$$

GATEWAY CLASSES

$$PI = \frac{1}{2D}\left[1+(D+D')+(D+D')^2+\cdots\right]\left[1+\left(\frac{D}{2}+\frac{3}{2}D'\right)+\left(\frac{D}{2}+\frac{3}{2}D'\right)^2+\cdots\right]x^2-4xy+2y^2$$

$$PI = \frac{1}{2D}\left[1+D+D'+D^2+D'^2+2DD'\right]\left[1+\frac{D}{2}+\frac{3}{2}D'+\frac{D^2}{4}+\frac{9}{4}D'^2+\frac{3}{2}DD'\right]\left(x^2-4xy+2y^2\right)$$

$$PI = \frac{1}{2D}\left[1+\frac{D}{2}+\frac{3}{2}D'+\frac{D^2}{4}+\frac{9}{4}D'^2+\frac{3}{2}DD'+D+\frac{D^2}{2}+\frac{3}{2}DD'+D'+\frac{DD'}{2}+\frac{3}{2}D'^2+\right.$$
$$\left.D^2+D'^2+2DD'\right]\left(x^2-4xy+2y^2\right)$$

$$PI = \frac{1}{2D}\left[1+\frac{3}{2}D+\frac{5D'}{2}+\frac{7D^2}{4}+\frac{19D'^2}{4}+\frac{11}{2}DD'\right]\left(x^2-4xy+2y^2\right)$$

$$PI = \frac{1}{2D}\left[x^2-4xy+2y^2+\frac{3}{2}D\left(x^2-4xy+2y^2\right)+\frac{5}{2}D'\left(x^2-4xy+2y^2\right)+\frac{7}{4}D^2\left(x^2-4xy+2y^2\right)\right.$$
$$\left.+\frac{19}{4}D'^2\left(x^2-4xy+2y^2\right)+\frac{11}{2}DD'\left(x^2-4xy+2y^2\right)\right.$$

GATEWAY CLASSES

$$PI = \frac{1}{2D}\left[x^2 - 4xy + 2y^2 + \frac{3}{2}(2x-4y) + \frac{5}{2}(-4x+4y) + \frac{7}{4}(2) + \frac{19}{4}(4) + \frac{11}{2}(-4)\right]$$

$$PI = \frac{1}{2D}\left[x^2 - 4xy + 2y^2 + 3x - 6y - 10x + 10y + \frac{7}{2} + 19 - 22\right]$$

$$PI = \frac{1}{2D}\left[x^2 - 4xy + 2y^2 - 7x + 4y + \frac{1}{2}\right]$$

$$PI = \frac{1}{2}\left[\frac{x^3}{3} - 2x^2y + 2xy^2 - \frac{7x^2}{2} + 4xy + \frac{1}{2}x\right]$$

complete solution

$$z = CF + PI$$

GATEWAY CLASSES

Non - Homogeneous Linear Partial Differential Equation with constant coefficient
(All Partial Derivatives are not of same order)

Given PDE

$$\phi(D,\ D') = F(x,\ y)$$

Methods to find *PI*

CASE-4:- When $F(x,\ y) = e^{ax+by}$ V

$$PI = \frac{1}{\phi(D, D')} e^{(ax+by)} V(x, y)$$

$$PI = e^{ax+by} \times \frac{1}{\phi(D+a, D'+b)} V(x, y)$$

GATEWAY CLASSES

Q.3 Solve : $(D - 3D' - 2)^2 z = 2e^{2x} \tan(y + 3x)$.

$$(D-3D'-2)^2 z = 2e^{2x}\tan(y+3x)$$

$$CF = e^{2x} f_1(y+3x) + x e^{2x} f_2(y+3x)$$

$$PI = \frac{1}{(D-3D'-2)^2} 2e^{2x}\tan(y+3x)$$

$$PI = 2 \times \frac{1}{(D-3D'-2)^2} e^{2x+0y}\tan(y+3x)$$

$$PI = 2e^{2x+0y} \times \frac{1}{(D+2-3D'-2)^2}\tan(y+3x)$$

$$PI = 2e^{2x} \times \frac{1}{(D-3D')^2}\tan(y+3x)$$

Put $D \longrightarrow 3,\ D' \longrightarrow 1$

case of failure

$$PI = 2e^{2x} \times x \times \frac{1}{2(D-3D')}\tan(y+3x)$$

Again case of failure

$$PI = 2x^2 e^{2x} \times \frac{1}{2}\tan(y+3x)$$

GATEWAY CLASSES

$$PI = x^2 e^{2x} \tan(y+3x)$$

complete solution

$$z = CF + PI$$

GATEWAY CLASSES

Q.4 *Solve the linear partial differential equation :* $(D^2 - DD' + D' - 1)z = \cos(x + 2y) + e^y$.

$$(D^2 - DD' + D' - 1)z = \cos(x+2y) + e^y$$

$$[(D^2 - 1) + D' - DD']z = \cos(x+2y) + e^y$$

$$[(D-1)(D+1) - D'(D-1)]z = \cos(x+2y) + e^y$$

$$(D-1)(D+1-D')z = \cos(x+2y) + e^y$$

$$CF = e^x f_1(y) + e^{-x} f_2(y+x)$$

$$PI = \frac{1}{(D^2 - DD' + D' - 1)} \{\cos(x+2y) + e^y\}$$

$$PI = \frac{1}{(D^2 - DD' + D' - 1)} \cos(x+2y) + \frac{1}{(D^2 - DD' + D' - 1)} e^y$$

$$PI = P_1 + P_2 \quad \text{——} (1)$$

$$P_1 = \frac{1}{(D^2 - DD' + D' - 1)} \cos(x+2y)$$

GATEWAY CLASSES

Put $D^2 = -1$

$DD' = -2$

$$P_1 = \frac{1}{-1+2+D'-1}\cos(x+2y)$$

$$P_1 = \frac{1}{D'}\cos(x+2y)$$

$$\boxed{P_1 = -\frac{\sin(x+2y)}{2}}$$

$$P_2 = \frac{1}{(D-1)(D-D'+1)}e^y$$

$$P_2 = \frac{1}{(D-D'+1)}\left[\frac{1}{D-1}e^{0x+y}\right]$$

$$P_2 = \frac{1}{(D-D'+1)}\left(-e^{0x+y}\right)$$

$$P_2 = -e^{0x+y}\left[\frac{1}{D-(D'+1)+1}\right]$$

$$P_2 = -e^y\left(\frac{1}{D-D'-1+1}\right)$$

$$P_2 = -e^y \times \frac{1}{D-D'}e^{0x+0y}$$

GATEWAY CLASSES

$$P_2 = -e^y \times \frac{1}{0-0} e^{0x+0y}$$

case of failure

$$P_2 = -xe^y \times \frac{1}{1} e^{0x+0y}$$

$$P_2 = -xe^y$$

Put P_1 and P_2 in (1)

$$P_I = -\frac{\sin(x+2y)}{2} - xe^y$$

complete solution

$$z = CF + PI$$

GATEWAY CLASSES

PDE : DPP-14

➢ Topic : Non-Homogeneous Linear Partial Differential Equation with constant coefficient

Particular Integral (PI) : Case-3 and Case-4

Q .1 *Solve the linear partial differential equation :* $(D^2 - D'^2 - 3D + 3D')z = xy + e^{x+2y}$. *(GBTU-2011)*

Q .2 *Solve* $(D + D' - 1)^2 z = xy$ *(GBTU-2011)*

Q .3 *Solve :* $(D - 3D' - 2)^3 z = 6e^{2x} \sin(3x + y)$.

Q .4 *Solve :* $r - 4s + 4t + p - 2q = e^{x+y}$.

GATEWAY CLASSES

Engineering Mathematics

UNIT : Partial Differential Equation

Today's Target

- Equation Reducible to Linear PDE with constant coefficient
- PYQs
- DPP

GATEWAY CLASSES

Course Details(Paid) : All Subjects

1	Recorded Video Lectures (100 % Syllabus Coverage)
2	Pdf Notes
3	Lecture wise DPP with Answers
4	DPP Solutions
5	Unit wise set of PYQs

GATEWAY CLASSES

Paid Courses are available in Gateway Classes Application
Link in Description

Equation Reducible to Linear PDE with constant coefficient

General Form

$$a_0 x^2 \frac{\partial^2 z}{\partial x^2} + a_1 xy \frac{\partial^2 z}{\partial x \partial y} + a_2 y^2 \frac{\partial^2 z}{\partial y^2} + b_1 \frac{x \partial z}{\partial x} + b_2 y \frac{\partial z}{\partial y} = F(x, y)$$

Standard substitution to convert above equation in Linear PDE with constant coefficient

(1) Put $x = e^u \Rightarrow u = \log x$

$y = e^v \Rightarrow v = \log y$

(2) Put $x \frac{\partial z}{\partial x} = DZ$,

$$x^2 \frac{\partial^2 z}{\partial x^2} = D(D-1)\, Z,$$

where, $D \equiv \frac{\partial}{\partial u}$

$$x^3 \frac{\partial^2 z}{\partial x^3} = D(D-1)\,(D-2) Z$$

GATEWAY CLASSES

(3) : $y\frac{\partial z}{\partial y} = D'Z$

$$y^2\frac{\partial^2 z}{\partial y^2} = D'(D'-1)\,Z,$$ where $D' \equiv \frac{\partial}{\partial v}$

$$y^3\frac{\partial^3 z}{\partial y^3} = D'(D'-1)\,(D'-2)Z$$

(4) : $xy\frac{\partial^2 z}{\partial x\,\partial y} = DD'Z$

(5): After above substitution given PDE is converted in to Linear PDE with constant coefficient

GATEWAY CLASSES

Linear Partial Differential Equation with constant coefficient



Homogeneous Linear PDE with constant coefficient

(All partial derivatives are of same order)

Complete solution = CF + PI

Non-Homogeneous Linear PDE with constant coefficient

(All partial derivatives are not of same order)

Complete solution = CF + PI

GATEWAY CLASSES

Q.1 Solve the differential equation : $x^2\frac{\partial^2 z}{\partial x^2} - y^2\frac{\partial^2 z}{\partial y^2} = xy$. **AKTU-2022-23**

$$x^2\frac{\partial^2 z}{\partial x^2} - y^2\frac{\partial^2 z}{\partial y^2} = xy$$

Put $x = e^u \Rightarrow u = \log x$

$y = e^v \Rightarrow v = \log y$

$$x^2\frac{\partial^2 z}{\partial x^2} = D(D-1)z$$

$$y^2\frac{\partial^2 z}{\partial y^2} = D'(D'-1)z$$

$$D(D-1)z - D'(D'-1)z = e^u \times e^v$$

$$[D(D-1) - D'(D'-1)]z = e^{u+v}$$

$$(D^2 - D - D'^2 + D')z = e^{u+v}$$

$$(D^2 - D'^2 + D' - D)z = e^{u+v}$$

$$[(D-D')(D+D') - (D-D')]z = e^{u+v}$$

$$(D-D')(D+D'-1)z = e^{u+v}$$

GATEWAY CLASSES

$$(D-D')(D+D'-1)z = e^{u+v}$$

$$CF = e^{0u} f_1(v+u) + e^{u} f_2(v-u)$$

$$CF = f_1(v+u) + e^{u} f_2(v-u)$$

$$CF = f_1(\log y + \log x) + x f_2(\log y - \log x)$$

$$CF = f_1\{\log xy\} + x f_2\left(\log \tfrac{y}{x}\right)$$

$$\boxed{CF = g_1(xy) + x g_2\left(\tfrac{y}{x}\right)}$$

$$PI = \frac{1}{(D-D')(D+D'-1)} e^{u+v}$$

$$PI = \frac{1}{(D-D')}\left[\frac{1}{(D+D'-1)} e^{u+v}\right]$$

$$PI = \frac{1}{D-D'}\left[\frac{1}{1+1-1} e^{u+v}\right]$$

$$PI = \frac{1}{D-D'} e^{u+v}$$

case failure

GW GATEWAY CLASSES

$$PI = \frac{1}{D-D'} e^{u+v}$$

$$PI = u \times \frac{1}{1-0} e^{u+v}$$

$$PI = u e^{u+v}$$

$$PI = u \times e^{u} \times e^{v}$$

$$\boxed{PI = nylogn}$$

complete solution

$$z = CF + PI$$

$$\boxed{z = g_1(nx) + n\, g_2(y/n) + nylogn}$$

GATEWAY CLASSES

Q.2 Solve the linear partial differential equation : $x^2\frac{\partial^2 z}{\partial x^2} - 4xy\frac{\partial^2 z}{\partial x \partial y} + 4y^2\frac{\partial^2 z}{\partial y^2} + 6y\frac{\partial z}{\partial y} = x^3y^4$.

AKTU-2021-22

$$x^2\frac{\partial^2 z}{\partial x^2} - 4xy\frac{\partial^2 z}{\partial x \partial y} + 4y^2\frac{\partial^2 z}{\partial y^2} + 6y\frac{\partial z}{\partial y} = x^3y^4$$

Put $x = e^u \Rightarrow u = \log x$

$y = e^v \Rightarrow v = \log y$

$$x^2\frac{\partial^2 z}{\partial x^2} = D(D-1)z$$

$$xy\frac{\partial^2 z}{\partial x \partial y} = DD'z$$

$$y^2\frac{\partial^2 z}{\partial y^2} = D'(D'-1)z$$

$$y\frac{\partial z}{\partial y} = D'z$$

$$D(D-1)z - 4DD'z + 4D'(D'-1)z + 6D'z = (e^u)^3(e^v)^4$$

$$[D^2 - D - 4DD' + 4D'^2 - 4D' + 6D']z = e^{3u} \times e^{4v}$$

$$[D^2 - D - 4DD' + 4D'^2 + 2D']z = e^{3u+4v}$$

$$[(D^2 - 4DD' + 4D'^2) - D + 2D']z = e^{3u+4v}$$

GATEWAY CLASSES

$$[(D-2D')^2-(D-2D')]z = e^{3u+4v}$$

$$(D-2D')(D-2D'-1)z = e^{3u+4v}$$

$$CF = e^{0u} f_1(v+2u) + e^{u} f_2(v+2u)$$

$$CF = f_1(\log y + 2\log x) + x f_2(\log y + 2\log x)$$

$$CF = f_1(\log y + \log x^2) + x f_2(\log y + \log x^2)$$

$$CF = f_1(\log yx^2) + x f_2(\log yx^2)$$

$$\boxed{CF = g_1(yx^2) + x g_2(yx^2)}$$

$$PI = \frac{1}{(D-2D')(D-2D'-1)} e^{3u+4v}$$

Put $D = 3$ and $D' = 4$

$$PI = \frac{1}{(3-8)(3-8-1)} e^{3u+4v}$$

$$PI = \frac{1}{30} e^{3u} \times e^{4v}$$

GATEWAY CLASSES

$$PI = \frac{1}{30}(e^u)^3 \times (e^v)^4$$

$$\boxed{PI = \frac{1}{30} x^3 y^4}$$

complete solution

$$z = CF + PI$$

$$\boxed{z = g_1(yx^2) + x\, g_2(yx^2) + \frac{1}{30} x^3 y^4}$$

GATEWAY CLASSES

Q.3 Solve : $x^2r - y^2t + px - qy = \log x$

AKTU-2017-18

$$x^2\frac{\partial^2 z}{\partial x^2} - y^2\frac{\partial^2 z}{\partial y^2} + x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y} = \log x$$

Put $x = e^u \Rightarrow u = \log x$

$y = e^v \Rightarrow v = \log y$

$$x^2\frac{\partial^2 z}{\partial x^2} = D(D-1)z$$

$$y^2\frac{\partial^2 z}{\partial y^2} = D'(D'-1)z$$

$$x\frac{\partial z}{\partial x} = Dz$$

$$y\frac{\partial z}{\partial y} = D'z$$

$$D(D-1)z - D'(D'-1)z + Dz - D'z = u$$

$$[D^2 - D - D'^2 + D' + D - D']z = u$$

$$(D^2 - D'^2)z = u$$

Aux. Eqn

$$D^2 - D'^2 = 0$$

Put $D = m_1$, $D' = 1$

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$$m^2 - 1 = 0$$

$$m = \pm 1$$

$$CF = f_1(y+x) + f_2(y-x)$$

$$CF = f_1(\log y + \log x) + f_2(\log y - \log x)$$

$$CF = f_1(\log xy) + f_2\left(\log \frac{y}{x}\right)$$

$$\boxed{CF = g_1(xy) + g_2\left(\frac{y}{x}\right)}$$

$$PI = \frac{1}{(D^2 - D'^2)} u$$

$$PI = \frac{1}{(D^2 - D'^2)} (u + 0v)$$

Put $D = 1$, $D' = 0$ and $u + 0v = t$

$$PI = \frac{1}{1 - 0} \int\int t\, dt + dt$$

$$PI = \int \frac{t^2}{2} dt$$

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$$PI = \frac{t^3}{6} = \frac{u^3}{6}$$

$$PI = \frac{(\log n)^3}{6}$$

Complete solution

$$z = CF + PI$$

$$\boxed{z = g_1(xy) + g_2(y/x) + \frac{1}{6}(\log n)^3}$$

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PDE : DPP-15

➢ Topic : Non-Homogeneous Linear Partial Differential Equation with constant coefficient

Particular Integral (PI) : Case-3 and Case-4

Q .1 *Solve the linear partial differential equation :* $\left(x^2D^2 + 2xyDD' + y^2D'^2\right)z = x^m y^n$. *AKTU-2020-21*

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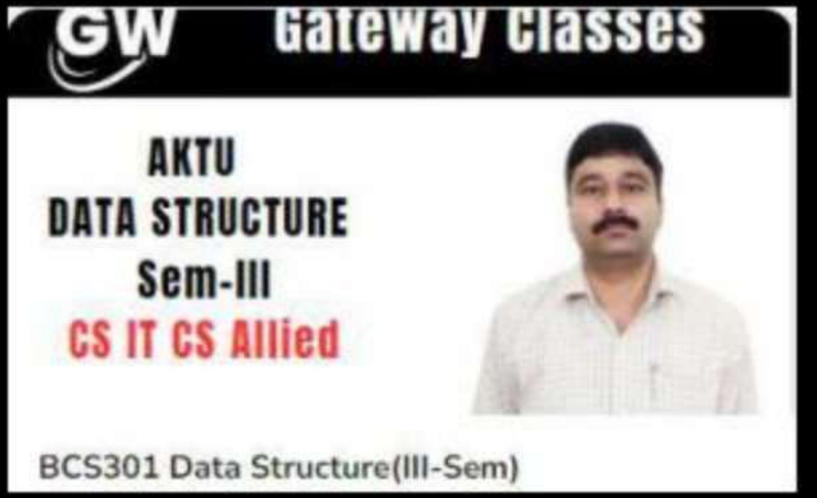
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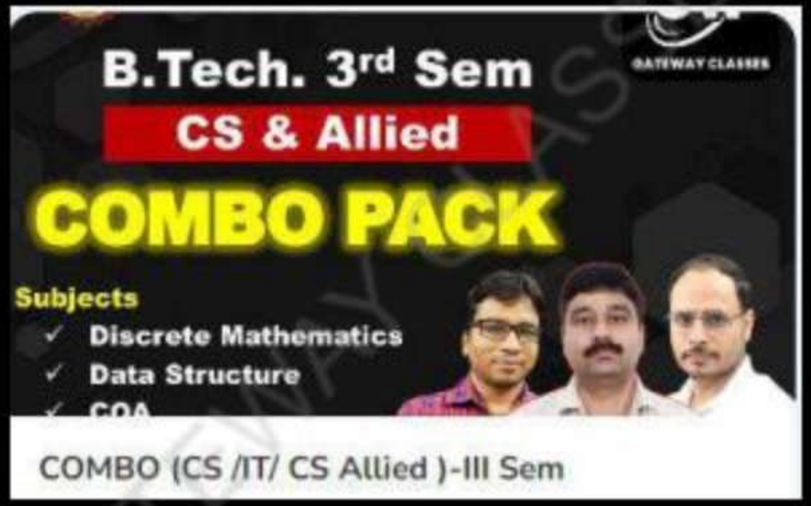


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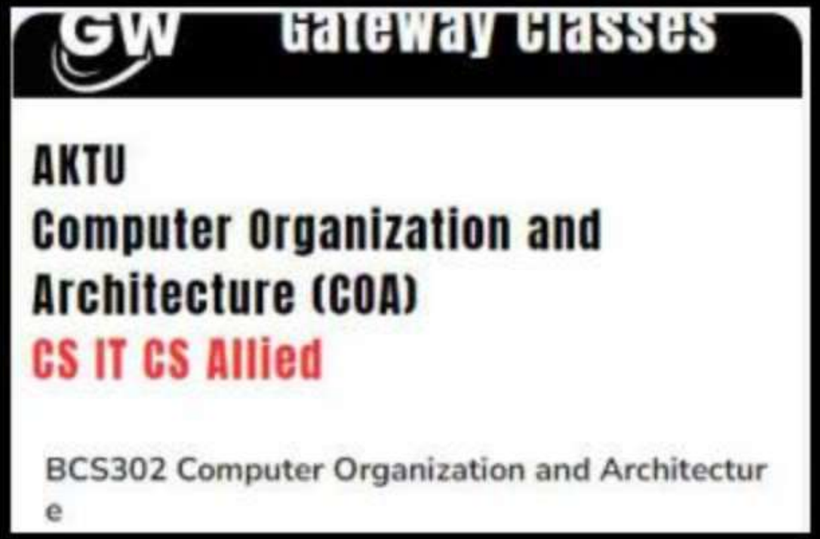
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