Engineering Mathematics

UNIT : Partial Differential Equation

Today's Target

- Homogeneous Linear Partial Differential Equation with constant coefficient (Complementary function-CF)

- PYQs
- DPP

Course Details(Paid) : All Subjects

1	Recorded Video Lectures (100 % Syllabus Coverage)
2	Pdf Notes
3	Lecture wise DPP with Answers
4	DPP Solutions
5	Unit wise Live Doubt Classes
6	Unit wise Test
7	Unit wise set of PYQs with Answers
8	Course will be completed by 'Last week of Nov 2023'
	Tentative AKTU Semester Exam- Jan/Feb

Paid Courses are available in Gateway Classes Application
Link in Description

Unit-1

1. PDE
2. Order and degree of PDE
3. Types of PDE
4. Formation of PDE

Solution of PDE

1. Lagrange Method
2. Some standard form of non linear PDE
3. Charpit Method
4. Linear PDE with constant coefficient

(a) Homogeneous Linear PDE with constant coefficient

(b) Non-Homogeneous Linear PDE with constant coefficient

8. Equation reducible to Linear PDE with constant coefficient

Linear Partial Differential Equation with constant coefficient

Linear PDE with constant coefficient

Homogeneous Linear PDE with constant coefficient

(All partial derivatives are of same order)

Complete solution = CF + PI

Non-Homogeneous Linear PDE with constant coefficient

(All partial derivatives are not of same order)

Complete solution = CF + PI

Homogeneous Linear Partial Differential Equation with constant coefficient

General equation

$$a_0\frac{\partial^n z}{\partial x^n} + a_1\frac{\partial^n z}{\partial x^{n-1}\partial y} + \cdots\cdots\cdots\cdots + a_n\frac{\partial^n z}{\partial y^n} = F(x,y)$$

Put $\frac{\partial}{\partial x} = D$ and $\frac{\partial}{\partial y} = D'$

$$(a_0D^n + a_1D^{n-1}D' + \cdots\cdots\cdots\cdots a_nD'^n)z = F(x,y)$$

$$f(D,D')z = F(x,y)$$

CF

Complementary Function

Number of arbitrary function = order of PDF

PI

Particular integral

Does not contain any arbitrary function or constant

Steps to find CF

STEP-1: Convert the given PDE in the form

$$f(D,\ D')z = F(x,\ y) \quad \text{——} \quad (1)$$

Using $\frac{\partial}{\partial x} = D$, $\frac{\partial^2}{\partial x^2} = D^2$, $\frac{\partial}{\partial y} = D'$, $\frac{\partial^2}{\partial x \partial y} = DD'$ $\frac{\partial^2}{\partial y^2} = D'^2$ $etc.$

STEP-2: Put $D = m$ and $D' = 1$ in (1)

$$f(m,\ 1)z = F(x,\ y) \quad \text{——} \quad (2)$$

STEP-3: Form Aux. Equation

$$f(m,\ 1) = 0$$

STEP-4: Solve to find roots of Aux. Equation

CASE-1: When roots are different

S. No.	Roots of A E	C. F.
1.	m_1, m_2	$f_1(y + m_1x) + f_2(y + m_2x)$
2.	m_1, m_2, m_3	$f_1(y + m_1x) + f_2(y + m_2x) + f_3(y + m_3x)$
3.	$m_1, m_2, m_3 \ \ m_4$	$f_1(y + m_1x) + f_2(y + m_2x) + f_3(y + m_3x) + f_4\,(y + m_4x)$

Note: Where f_1, f_2, f_3 ..... are arbitrary function

Q.1 Solve: $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = 0$

$$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = 0$$

$$(D^2 - D'^2)z = 0$$

Put $D = m$, $D' = 1$

$$(m^2 - 1)z = 0$$

Aux. Eqn

$$m^2 - 1 = 0$$

$$m = 1, -1$$

$$CF = f_1(y + m_1 x) + f_2(y + m_2 x)$$

$$CF = f_1(y + x) + f_2(y - x)$$

$$PI = 0$$

complete solution

$$z = CF + PI$$

$$\boxed{z = f_1(y + x) + f_2(y - x)}$$

Q.2 :- Solve the following partial differential equation : $\frac{\partial^2 z}{\partial x^2} + 4\frac{\partial^2 z}{\partial x \partial y} - 5\frac{\partial^2 z}{\partial y^2} = 0$

$$\frac{\partial^2 z}{\partial x^2} + 4\frac{\partial^2 z}{\partial x \partial y} - 5\frac{\partial^2 z}{\partial y^2} = 0$$

$$(D^2 + 4DD' - 5D'^2)z = 0$$

Put $D = m$, $D' = 1$

$$(m^2 + 4m - 5)z = 0$$

Aux Eqn

$$m^2 + 4m - 5 = 0$$

$m = 1$

$m = -5$

$$CF = f_1(y + m_1 x) + f_2(y + m_2 x)$$

$$CF = f_1(y + x) + f_2(y - 5x)$$

$PI = 0$

Complete solution

$$z = CF + PI$$

$$\boxed{z = f_1(y + x) + f_2(y - 5x)}$$

Q.3 :- Solve the following partial differential equation $(D^3 - 6D^2D' + 11DD'^2 - 6D'^3)z = 0$

$(D^3 - 6D^2D' + 11DD'^2 - 6D'^3)z = 0$

Put $D = m$ and $D' = 1$

$(m^3 - 6m^2 + 11m - 6)z = 0$

Aux Eq^n

$m^3 - 6m^2 + 11m - 6 = 0$

$m = 3, 1, 2$

$CF = f_1(y + m_1x) + f_2(y + m_2x) + f_3(y + m_3x)$

$CF = f_1(y + 3x) + f_2(y + x) + f_3(y + 2x)$

$PI = 0$

Complete solution

$z = CF + PI$

$$z = f_1(y + 3x) + f_2(y + x) + f_3(y + 2x)$$

Q.4 :- Solve : $\frac{\partial^4 z}{\partial x^4} - \frac{\partial^4 z}{\partial y^4} = 0.$

$$\frac{\partial^4 z}{\partial x^4} - \frac{\partial^4 z}{\partial y^4} = 0$$

$$(D^4 - D'^4)z = 0$$

Put $D = m$, $D' = 1$

$$(m^4 - 1)z = 0$$

Aux. Eqn

$$m^4 - 1 = 0$$

$$(m^2 - 1)(m^2 + 1) = 0$$

$$m = \pm 1$$

$$m = \pm i$$

$$CF = f_1(y + m_1 x) + f_2(y + m_2 x) + f_3(y + m_3 x) + f_4(y + m_4 x)$$

$$CF = f_1(y + x) + f_2(y - x) + f_3(y + ix) + f_4(y - ix)$$

$$PI = 0$$

complete solution

$$\boxed{z = CF + PI}$$

Q.5 :- Solve the linear partial differential equation $\frac{\partial^4 z}{\partial x^4} + \frac{\partial^4 z}{\partial y^4} = 0$

$$\frac{\partial^4 z}{\partial x^4} + \frac{\partial^4 z}{\partial y^4} = 0$$

$$(D^4 + D'^4)z = 0$$

Put $D = m$, $D' = 1$

$$(m^4 + 1)z = 0$$

Aux. Eqn

$$m^4 + 1 = 0$$

$$m^4 + 1 = 0$$

$$m^4 + 2m^2 + 1 = 2m^2$$

$$(m^2)^2 + 2m^2 + 1 = 2m^2$$

$$(m^2 + 1)^2 = 2m^2$$

$$(m^2 + 1)^2 - 2m^2 = 0$$

$$(m^2 + 1)^2 - (\sqrt{2}\, m)^2 = 0$$

$$(m^2 + \sqrt{2}m + 1)(m^2 - \sqrt{2}m + 1) = 0$$

$$m^2 + \sqrt{2}m + 1 = 0$$

$$m = \frac{-\sqrt{2} \pm \sqrt{2-4}}{2}$$

$$m = \frac{-\sqrt{2}}{2} \pm \frac{i\sqrt{2}}{2}$$

$$m = -\frac{1}{\sqrt{2}} \pm i\frac{1}{\sqrt{2}}$$

$$m = z_1, \bar{z}_1$$

$$m^2 - \sqrt{2}m + 1 = 0$$

$$m = \frac{\sqrt{2} \pm \sqrt{2-4}}{2}$$

$$m = \frac{\sqrt{2}}{2} \pm \frac{i\sqrt{2}}{2}$$

$$m = \frac{1}{\sqrt{2}} \pm i\frac{1}{\sqrt{2}}$$

$$m = z_2, \bar{z}_2$$

$$CF = f_1(y+m_1x) + f_2(y+m_2x) + f_3(y+m_3x) + f_4(y+m_4x)$$

$$CF = f_1(y+z_1x) + f_2(y+\overline{z}_1x) + f_3(y+z_2x) + f_4(y+\overline{z}_2x)$$

$$PI = 0$$

complete solution

$$\boxed{Z = CF + PI}$$

CASE-2: When roots are repeated

S. No.	Roots of A E	C. F.
1.	m, m	$f_1(y+mx)+xf_2(y+mx)$
2.	m, m, m	$f_1(y+mx)+xf_2(y+mx)+x^2f_3(y+mx)$

Note: Where f_1, f_2, f_3 ..... are arbitrary function

Q.6 :- Solve $25r - 40s + 16t = 0$

$$25r - 40s + 16t = 0$$

$$25\frac{\partial^2 z}{\partial x^2} - 40\frac{\partial^2 z}{\partial x \partial y} + 16\frac{\partial^2 z}{\partial y^2} = 0$$

$$(25D^2 - 40DD' + 16D'^2)z = 0$$

Put $D = m$, $D' = 1$

$$(25m^2 - 40m + 16)z = 0$$

Aux Eqn

$$25m^2 - 40m + 16 = 0$$

$$(5m - 4)^2 = 0$$

$$(5m - 4)(5m - 4) = 0$$

$$m = \frac{4}{5}, \frac{4}{5}$$

$$CF = f_1(y + mx) + x f_2(y + mx)$$

$$CF = f_1\left(y + \frac{4}{5}x\right) + x f_2\left(y + \frac{4}{5}x\right)$$

complete solution

$$\boxed{Z = CF + PI}$$

Q.7 :- Solve: $(D^3 - 6D^2D' + 12DD'^2 - 8D'^3)z = 0$

$(D^3 - 6D^2D' + 12DD'^2 - 8D'^3)z = 0$

Put $D = m$, $D' = 1$

$(m^3 - 6m^2 + 12m - 8)z = 0$

Aux. Eqn

$m^3 - 6m^2 + 12m - 8 = 0$

$m = 2, 2, 2$

$CF = f_1(y + mx) + x f_2(y + mx) + x^2 f_3(y + mx)$

$CF = f_1(y + 2x) + x f_2(y + 2x) + x^2 f_3(y + 2x)$

$PI = 0$

complete solution

$$z = CF + PI$$

Q.8 :- Solve: $(D+2D')(D-3D')^2 z = 0$

$$(D+2D')(D-3D')^2 z = 0$$

Put $D = m$, $D' = 1$

$$(m+2)(m-3)^2 z = 0$$

Aux Eqn

$$(m+2)(m-3)^2 = 0$$

$$m = -2$$

$$m = 3, 3$$

$$CF = f_1(y+m_1x) + f_2(y+mx) + x f_3(y+mx)$$

$$CF = f_1(y-2x) + f_2(y+3x) + x f_3(y+3x)$$

$$PI = 0$$

Complete solution

$$Z = CF + PI$$

CASE-3: When D , D' , both D and D' are common factor

(i) C F corresponding to $D'\ z = f_1(x)$

(ii) C F corresponding to $D'^{\,2}\ z = f_1(x) + yf_2(x)$

(iii) C F corresponding to $D'^{\,3}\ z = f_1(x) + yf_2(x) + y^2 f_3(x)$

(ii) C F corresponding to $Dz = f_1(y)$

(iii) C F corresponding to $D^2\ z = f_1(y) + xf_2(y)$

(iv) C F corresponding to $D^3\ z = f_1(y) + xf_2(y) + x^2 f_3(y)$

Q.9 :- Solve: $\frac{\partial^3 z}{\partial x^3} - \frac{\partial^3 z}{\partial x^2 \partial y} = 0$

$$\frac{\partial^3 z}{\partial y^3} - \frac{\partial^3 z}{\partial x^2 \partial y} = 0$$

$$(D'^3 - D^2 D')z = 0$$

$$D'(D'^2 - D^2)z = 0$$

Aux. Eqn

$$D'(D'^2 - D^2) = 0$$

$$D' = 0$$

$$CF = f_1(x)$$

$$D'^2 - D^2 = 0$$

Put $D = m$

$$D' = 1$$

$$1 - m^2 = 0$$

$$m = \pm 1$$

$$CF = f_2(y+m_1x) + f_3(y+m_2x)$$

$$CF = f_2(y+x) + f_3(y-x)$$

Final

$$CF = f_1(x) + f_2(y+x) + f_3(y-x)$$

$PI = 0$

Complete solution

$$z = CF + PI$$

$$\boxed{z = f_1(x) + f_2(y+x) + f_3(y-x)}$$

Q.10 :- Solve: $(D^3D'^2 + D^2D'^3)z = 0$

$$(D^3D'^2 + D^2D'^3)z = 0$$

$$D^2D'^2(D + D')z = 0$$

Aux Eqn

$$D^2 = 0$$

$$CF = f_1(y) + xf_2(y)$$

$$D'^2 = 0$$

$$CF = f_3(x) + yf_4(x)$$

$$D + D' = 0$$

Put $D = m$

$D' = 1$

$$m + 1 = 0$$

$$m = -1$$

$$CF = f_5(y + mx)$$

$$CF = f_5(y - x)$$

$$CF = f_1(y) + xf_2(y) + f_3(x) + yf_4(x) + f_5(y - x)$$

Complete solution

$$\boxed{z = CF + PI}$$

Engineering Mathematics by Gulshan Sir

PDE : DPP-8

Topic : Homogeneous Linear Partial Differential Equation with constant coefficient (Complementary function)

Q .1 Solve : $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} - 6\frac{\partial^2 z}{\partial y^2} = 0$ Ans $z = f_1(y+3x) + f_2(y-2x)$

Q .2 Solve $\frac{\partial^3 z}{\partial x^3} - 3\frac{\partial^3 z}{\partial x^2 \partial y} + 2\frac{\partial^3 z}{\partial x \partial y^2} = 0$ Ans $z = f_1(y) + f_2(y+x) + f_3(y+2x)$

Q .3 Solve: $4r - 12s + 9t = 0$ Ans $z = f_1(y+\frac{3}{2}x) + x f_2(y+\frac{3}{2}x)$

Q .4 Solve $(D^3D' - 4D^2D'^2 + 4DD'^3)z = 0$ Ans

$z = f_1(y) + f_2(x) + f_3(y+2x) + x f_4(y+2x)$

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Thank You