# ENGINEERING MATHEMATICS

## UNIT-4 : *Statistical Techniques – II*

### *Lec-02*

**Today's Target**

- ***Random variables (PART-2)***
- PYQ
- DPP

## Types of Random variable

(1) Discrete random variable

(2) Continous random variable

## Discrete random variable

A random variable which can assume only a finite number of values is called a discrete random variable.

- Probability Mass Function
- Probability Distribution
- Expectation and Variance

# Expectation and Variance

Mean

*If X is a discrete random variable with probability distribution*

| $X$: | $x_1$ | $x_2$ | $x_3$ | …… | $x_n$ |
|---|---|---|---|---|---|
| $p(X)$ : | $p(x_1)$ | $p(x_2)$ | $p(x_3)$ | …… | $p(x_n)$ |

## Expectation

$$E(x) = x_1 p(x_1) + x_2 p(x_2) + x_3 p(x_3) + \ldots\ldots\ldots\ldots\ldots + x_n p(x_n)$$

$$\boxed{E(x) = \sum x\, p(x)}$$

$$E(x) = \bar{x} = \mu = \mu_1' = \sum x\, p(x)$$

*Expectation* or Mathematical *Expectation* or *Mean* or *First moment about origin*

## Important points:

$E(x) = \sum x\, p(x) = \mu_1'$ $(First\ moment\ about\ origin)$

$E(x^2) = \sum (x^2) p(x) = \mu_2'$ $(Second\ moment\ about\ origin)$

$E(x^3) = \sum (x^3) p(x) = \mu_3'$ $(Third\ moment\ about\ origin)$

$E(x^r) = \sum (x^r) p(x) = \mu_r'$ $(r^{th} moment\ about\ origin)$

## Properties of Expectation:

$(i)\ E(c) = c$ $\quad E(1) = 1$

$(ii)\ E(ax) = a\,E(x)$

$(iii)\ E(x \pm y) = E(x) \pm E(y)$

$(iv)\ E(xy) = E(x).E(y)$

# Variance

## $r^{th}$ moment about mean

$$\mu_r = \frac{\sum p(x-\bar{x})^r}{\sum pi}$$

But $\sum p_i = 1$

$$\mu_\gamma = \sum p(x-\bar{x})^\gamma$$

$$\mu_1 = \sum p(x-\bar{x})$$

$$\mu_1 = E(x-\bar{x})$$

$$\mu_1 = E(x) - E(\bar{x})$$

$$\mu_1 = \bar{x} - \bar{x}$$

$$\mu_1 = 0$$

$$\mu_2 = \sum p(x-\bar{x})^2$$

$$\mu_2 = \sum p(x-\mu)^2$$

$$\text{Variance} = \sum p(x-\mu)^2$$

$$\mu_2 = \sum p(x-\bar{x})^2$$

$$\mu_2 = E(x-\bar{x})^2$$

$$\mu_2 = E(x^2 + \bar{x}^2 - 2x\bar{x})$$

$$\mu_2 = E(x^2) + E(\bar{x}^2) - E(2x\bar{x})$$

$$\mu_2 = E(x^2) + \bar{x}^2 - 2\bar{x}E(x)$$

$$\mu_2 = E(x^2) + \bar{x}^2 - 2\bar{x}\,\bar{x}$$

$$\mu_2 = E(x^2) + \bar{x}^2 - 2\bar{x}^2$$

$$\mu_2 = E(x^2) - (\bar{x})^2$$

$$\mu_2 = E(x^2) - [E(x)]^2$$

$\mu_2$ : Variance OR Second moment about mean

## Impotant Formulae

### 1. *Expectation* (*Mean or First moment about origin*)

$$E(x) = x_1 p(x_1) + x_2 p(x_2) + x_3 p(x_3) + \ldots\ldots\ldots\ldots\ldots + x_n p(x_n)$$

$$E(x) = \sum x\, p(x)$$

### 2. *Variance* (*Second moment about mean*)

$$Variance = \mu_2 = \sum p\,(x-\mu)^2$$

$$Variance = \mu_2 = E(x^2) - [E(x)]^2$$

$$E(x^2) = x_1^2 P(x_1) + x_2^2 P(x_2) + \ldots + x_n^2 P(x_n)$$

### 3. *Standard Deviation*

$$S.D = \sqrt{Variance}$$

**Q.1 Find the Mean, Variance and Standard deviation of the number of heads when three coins are tossed.**

$S = \{HHH, HHT, HTH, THH, TTT, TTH, THT, HTT\}$

$X$ = Number of heads

$X(TTT) = 0$

$X(TTH, THT, THH) = 1$

$X(HHT, HTH, THH) = 2$

$X(HHH) = 3$

$X = 0, 1, 2, 3$

Probabilities

$P(X=0) = \frac{1}{8}$

$P(X=1) = \frac{3}{8}$

$P(X=2) = \frac{3}{8}$

$P(X=3) = \frac{1}{8}$

# Probability Distribution

| X | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| P(X) | $\frac{1}{8}$ | $\frac{3}{8}$ | $\frac{3}{8}$ | $\frac{1}{8}$ |

## Mean

$$E(X) = \bar{x} = 0 \times \frac{1}{8} + 1 \times \frac{3}{8} + 2 \times \frac{3}{8} + 3 \times \frac{1}{8}$$

$$\bar{x} = 0 + \frac{3}{8} + \frac{6}{8} + \frac{3}{8}$$

$$\bar{x} = \frac{12}{8}$$

$$\boxed{\bar{x} = \frac{3}{2}}$$

## Variance

$$E(x^2) = 0^2 \times \frac{1}{8} + 1^2 \times \frac{3}{8} + 2^2 \times \frac{3}{8} + 3^2 \times \frac{1}{8}$$

$$E(x^2) = 0 + \frac{3}{8} + \frac{12}{8} + \frac{9}{8}$$

$$E(x^2) = \frac{3+12+9}{8} = \frac{24}{8} = 3$$

$$\text{Variance} = E(x^2) - [E(x)]^2$$

$$= 3 - \left(\frac{3}{2}\right)^2 = 3 - \frac{9}{4}$$

$$\boxed{\text{Variance} = \frac{3}{4}}$$

$$\boxed{SD = \frac{\sqrt{3}}{2}}$$

**Q.2** Two cards are drawn simultaneously from a well shuffled pack of 52 cards. Find the Expectation and Variance of the number of kings.

Let $X$ = Number of kings

$X = 0, 1, 2$

$$P(X=0) = \frac{{}^{48}C_2}{{}^{52}C_2}$$

$$= \frac{48!}{2! \times 46!} \times \frac{2! \times 50!}{52!}$$

$$= \frac{48 \times 47 \times 46! \times 50!}{46! \times 52 \times 51 \times 50!}$$

$$P(X=0) = \frac{188}{221}$$

$$P(X=1) = \frac{{}^{4}C_1 \times {}^{48}C_1}{{}^{52}C_2}$$

$$P(X=1) = \frac{32}{221}$$

$$P(X=2) = \frac{{}^{4}C_2}{{}^{52}C_2}$$

$$P(X=2) = \frac{1}{221}$$

$${}^{n}C_r = \frac{n!}{r!\,(n-r)!}$$

# Probality Distribution

| X | 0 | 1 | 2 |
|---|---|---|---|
| P(X) | $\frac{188}{221}$ | $\frac{32}{221}$ | $\frac{1}{221}$ |

## Expectation

$$E(X) = 0 \times \frac{188}{221} + 1 \times \frac{32}{221} + \frac{2}{221}$$

$$\boxed{E(X) = \frac{2}{13}}$$

$$E(X^2) = 0^2 \times \frac{188}{221} + 1^2 \times \frac{32}{221} + 2^2 \times \frac{1}{221}$$

$$E(X^2) = \frac{36}{221}$$

## Variance

$$Var(X) = E(X^2) - [E(X)]^2$$

$$= \frac{36}{221} - \left(\frac{2}{13}\right)^2$$

$$\boxed{Var(X) = 0.1392}$$

## Standard deviation

$$S.D = \sqrt{0.1392}$$

$$\boxed{S.D = 0.37}$$

**Q.3** Let X denote the sum of the numbers obtained when two fair dice are rolled. Find the Expectation of X

$X$ = sum of numbers on two dice

| X | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| P(X) | $\frac{1}{36}$ | $\frac{2}{36}$ | $\frac{3}{36}$ | $\frac{4}{36}$ | $\frac{5}{36}$ | $\frac{6}{36}$ | $\frac{5}{36}$ | $\frac{4}{36}$ | $\frac{3}{36}$ | $\frac{2}{36}$ | $\frac{1}{36}$ |

Expectation

$$E(X) = \frac{2}{36} + \frac{6}{36} + \frac{12}{36} + \frac{20}{36} + \frac{30}{36} + \frac{42}{36} + \frac{40}{36} + \frac{36}{36} + \frac{30}{36} + \frac{22}{36} + \frac{12}{36}$$

$$E(X) = 7$$

**Q.4** A random variable X has the following distribution

| X: | -2 | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|
| $p(X)$: | 0.1 | $k$ | 0.2 | $2k$ | 0.3 | k |

**Determine (i) k (ii) Mean (iii) Variance**

(i) We know that

$$\sum p(x) = 1$$

$$0.1 + k + 0.2 + 2k + 0.3 + k = 1$$

$$4k + 0.6 = 1$$

$$4k = 0.4$$

$$\boxed{k = 0.1}$$

Mean

$$\bar{x} = -0.2 - k + 0 + 2k + 0.6 + 3k$$

$$\bar{x} = -0.2 - 0.1 + 0.2 + 0.6 + 0.3$$

$$\boxed{\bar{x} = 0.8}$$

Variance

$$E(x^2) = (-2)^2 \times 0.1 + 1^2 \times k + 0 + 1^2 \times 2k + 2^2 \times 0.3 + 3^2 \times k$$

$$E(X^2) = 0.4 + K + 2K + 1.2 + 9K$$

$$= 0.4 + 0.1 + 0.2 + 1.2 + 0.9$$

$$E(X^2) = 2.8$$

$$\text{Variance} = E(X^2) - [E(X)]^2$$

$$\text{Variance} = 2.8 - (0.8)^2$$

$$= 2.8 - 0.64$$

$$\boxed{\text{Var}(X) = 2.16}$$

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DPP- 02

Topic : $Random\ variables$ (PART-2)

$Q.1$ Let X denote the number of heads when a pair of coin is tossed. What is the expected value?

$Q.2$ $Two\ dice\ are\ thrown\ simultaneously.\ if\ X\ denote\ the\ number\ of\ sixes,\ find\ the\ expectation\ of\ X$

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# Thank You

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