DPP-6 Maths-4 Unit-5

Topic- X^2- test

Q-1- Test for goodness of fit a poisson distribution at 5% level of significance to the following frequency distribution

x :	0	1	2	3	4	5	6	7	8
f :	52	151	130	102	45	12	5	1	2

Null Hypothesis (H_0)

H_0: Poisson Distribution is a good fit to the data.

x	f	fx
0	52	0
1	151	151
2	130	260
3	102	306
4	45	180
5	12	60
6	5	30
7	1	7
8	2	16
	$\Sigma f = 500$	$\Sigma fx = 1010$

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f} = \frac{1010}{500}$$

$$\boxed{\lambda = 2.02}$$

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By Poisson Distribution

$$P(x) = \frac{e^{-\lambda} \times \lambda^x}{x!}$$

Expected frequency

$$E_i = \frac{N \times e^{-\lambda} \times \lambda^x}{x!}$$

$$E_i = \frac{500 \times e^{-2.02} \times (2.02)^x}{x!}$$

$$\boxed{E_i = \frac{66.32 \times (2.02)^x}{x!}}$$

$E_1 = 66.32$

$E_2 = 66.32 \times 2.02$
$= 133.96$

$E_3 = \frac{66.32 \times 2.02 \times 2.02}{2}$
$= 135.30$

$E_4 = \frac{66.32 \times 2.02 \times 2.02 \times 2.02}{6}$

$E_4 = 91.10$

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$$E_5 = \frac{66.32 \times 2.02 \times 2.02 \times 2.02 \times 2.02}{24}$$

$E_5 = 46$

$E_6 = 18.58$

$E_7 = 6.25$

$E_8 = 1.80$

$E_9 = 0.45$

O_i	E_i	$O_i - E_i$	$(O_i - E_i)^2$	$\frac{(O_i - E_i)^2}{E_i}$
52	66.32	-14.32	205.06	3.091
151	133.96	17.04	290.36	2.167
130	135.30	-5.3	28.09	0.207
102	91.10	10.9	118.81	1.304
45	46	-1	1	0.021
12	18.58	-6.58	43.29	2.329
5	6.25	-0.5	0.25	0.029
1	1.80			
2	0.45			
8	8.5			

$\sum \frac{(O_i - E_i)^2}{E_i} = 9.148$

$$\boxed{X^2_{cal} = 9.148}$$

Degree of freedom = $n - K = 9 - 1 - 1 - 2$
$= 5$

Level of Significance (α) = 5% = 0.05

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From χ^2 table -

$$\boxed{\chi^2_{tab} = 11.07}$$

Conclusion - $\because \chi^2_{tab} > \chi^2_{cal}$

$\therefore H_0$ is accepted.

Hence, Poisson Distribution is good fit to given Data.

Q-2- Test for goodness of fit of a poisson distribution at 5% level of significance to the following frequency distribution

x	0	1	2	3	4	5
f	275	138	75	7	4	1

x	f	fx
0	275	0
1	138	138
2	75	150
3	7	21
4	4	16
5	1	5
	$\Sigma f = 500$	$\Sigma fx = 330$

$$\text{Mean} = \lambda = \frac{330}{500} = 0.66$$

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H_0: Poisson Distribution is a good fit to the data.

By Poisson Distribution

$$P(x) = \frac{e^{-\lambda}\lambda^x}{x!}$$

Expected Frequency

$$E_i = N \times \frac{e^{-\lambda}\lambda^x}{x!}$$

$$E_i = \frac{500 \times e^{-0.66} \times (0.66)^x}{x!}$$

$$\boxed{E_i = \frac{258.425 \times (0.66)^x}{x!}}$$

$E_1 = 258.425$

$E_2 = 170.560$

$E_3 = 56.284$

$E_4 = 12.382$

$E_5 = 2.04$

$E_6 = 0.269$

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O_i	E_i	$O_i - E_i$	$(O_i - E_i)^2$	$\frac{(O_i - E_i)^2}{E_i}$
275	258.425	16.575	274.73	1.06
138	170.560	-32.56	1060.15	6.21
75	56.284	16.03	256.96	3.62
7	12.382	70.97		
4	2.04			
1	0.269			

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$$\sum \frac{(O_i - E_i)^2}{E_i} = 10.89$$

$$\boxed{X^2_{cal} = 10.89}$$

$$d.o.f = n - K$$
$$= n - 1 - 1 - 3$$
$$= 6 - 5$$
$$= 1$$

Level of Significance $(\alpha) = 5\% = 0.05$

$$\boxed{X^2_{tab} = 3.841}$$

Conclusion - $\because X^2_{cal} > X^2_{tab}$

$\therefore H_0$ is rejected.

Hence, Poisson Distribution is not good fit to given Data.

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Q-3- Records taken of the number of male and female births in 800 families having four children are as follows

No. of male births	0	1	2	3	4
No. of female births	4	3	2	1	0
No. of families	32	178	290	236	64

Test whether data are consistent with the hypothesis that the Binomial law holds and chance of male birth is equal to female birth, $p = q = \frac{1}{2}$.

Ans- H_0: Data are binomially distributed and probability of male and female both are equal.

$$p = q = \frac{1}{2}$$

$N = 800$

$n = 4$

$r = 0, 1, 2, 3, 4$

Expected Frequency

$$E_i = N\, {}^nC_r\, p^r q^{n-r}$$

$$\boxed{E_i = 800 \times {}^4C_r\, p^r q^{4-r}}$$

$$E_1 = 800 \times {}^4C_0 \left(\frac{1}{2}\right)^0 \times \left(\frac{1}{2}\right)^4 = 50$$

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$$E_2 = 800 \times {}^4C_1 \times \left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right)^3$$

$$= 200$$

$$E_3 = 800 \times {}^4C_2 \times \left(\frac{1}{2}\right)^2 \times \left(\frac{1}{2}\right)^2 = 300$$

$$E_4 = 800 \times {}^4C_3 \times \left(\frac{1}{2}\right)^3 \times \left(\frac{1}{2}\right)^1 = 200$$

$$E_5 = 800 \times {}^4C_4 \times \left(\frac{1}{2}\right)^4 \times \left(\frac{1}{2}\right)^0 = 50$$

O_i	E_i	$O_i - E_i$	$(O_i - E_i)^2$	$\frac{(O_i - E_i)^2}{E_i}$
32	50	-18	324	6.48
178	200	-22	484	2.42
290	300	-10	100	0.33
236	200	-36	1296	6.48
64	50	-14	196	3.92

$$\sum \frac{(O_i - E_i)^2}{E_i} = 19.63$$

$$\boxed{X^2_{cal} = 19.63}$$

d.o.f = 5 - 1 = 4

Level of Significance (α) = 5% = 0.05%

$$\boxed{X^2_{tab} = 9.49}$$

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Conclusion :- $\therefore X^2_{cal} > X^2_{tab}$

$\therefore$ H_0 is rejected.

Hence, Data are not Binomially distributed and probability of birth of boy and girl are not equal.

$\Rightarrow p \neq q$

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