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DPP - 14 Maths - IV Unit - 1

Topic - Particular Integral (PI): Case 3 and Case 4

Q-1- Solve the linear partial differential equation
$(D^2 - D'^2 - 3D + 3D')z = xy + e^{x+2y}$

Sol- $(D^2 - D'^2 - 3D + 3D')z = xy + e^{x+2y}$

$[(D+D')(D-D') - 3(D-D')]z = xy + e^{x+2y}$

$(D-D')[D+D'-3]z = xy + e^{x+2y}$

$\boxed{C.F. = f_1(y+x) + e^{3x} f_2(y-x)}$

$PI = \frac{1}{D^2 - D'^2 - 3D + 3D'} xy + e^{x+2y}$

$PI = \frac{1}{D^2 - D'^2 - 3D + 3D'} xy + \frac{1}{D^2 - D'^2 - 3D + 3D'} e^{x+2y}$

$PI = P_1 + P_2$

$P_1 = \frac{1}{(D-D')(D+D'-3)} xy$

$P_1 = \frac{1}{D\left(1 - \frac{D'}{D}\right) - 3\left(1 - \left(\frac{D}{3} + \frac{D'}{3}\right)\right)} xy$

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$$P_1 = \frac{-1}{3D}\left(1-\frac{D'}{D}\right)^{-1}\left[1-\left(\frac{D}{3}+\frac{D'}{3}\right)\right]^{-1}$$

$$P_1 = \frac{-1}{3D}\left(1+\frac{D'}{D}+\frac{D'^2}{D^2}+\cdots\right)\left(1+\frac{D}{3}+\frac{D'}{3}+\left(\frac{D}{3}+\frac{D'}{3}\right)^2+\cdots\right)(xy)$$

$$P_1 = \frac{-1}{3D}\left(1+\frac{D'}{D}+\frac{D'^2}{D^2}+\cdots\right)\left(1+\frac{D}{3}+\frac{D'}{3}+\frac{D^2}{9}+\frac{D'^2}{9}+\frac{2DD'}{9}+\cdots\right)(xy)$$

$$P_1 = \frac{-1}{3D}\left(1+\frac{D}{3}+\frac{D'}{3}+\frac{2DD'}{9}+\frac{\not{9}D'}{3\not{9}}+\frac{D'^2}{3D}+\frac{2DD'^2}{9D}+\frac{D'^2}{D}\right)(xy)$$

$$P_1 = \frac{-1}{3D}\left(1+\frac{D}{3}+\frac{D'}{3}+\frac{2DD'}{9}+\frac{D'}{D}+\frac{D'}{3}\right)(xy)$$

$$PI = \frac{-1}{3D}\left[xy+\frac{D}{3}(xy)+\frac{D'}{3}(xy)+\frac{2DD'}{9}(xy)+\frac{D'}{D}(xy)+\frac{D'}{3}(xy)\right]$$

$$P_1 = \frac{-1}{3D}\left[xy+\frac{y}{3}+\frac{x}{3}+\frac{2}{9}+\frac{x^2}{2}+\frac{x}{3}\right]$$

$$P_1 = \frac{-1}{3D}\left[xy+\frac{y}{3}+\frac{2x}{3}+\frac{x^2}{2}+\frac{2}{9}\right]$$

$$\boxed{P_1 = \frac{-1}{3}\left[\frac{x^2y}{2}+\frac{xy}{3}+\frac{x^2}{3}+\frac{x^3}{6}+\frac{2x}{9}\right]}$$

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$$P_2 = \frac{1}{D^2 - D'^2 - 3D + 3D'} e^{x+2y}$$

Put $D = 1$, $D' = 2$

$$P_2 = \frac{1}{1 - 4 - 3 + 6} e^{x+2y}$$

{ Case of failure }

Now multiply with x and differentiate w.r.t D

$$P_2 = \frac{x}{2D - 3} e^{x+2y}$$

$$\boxed{P_2 = -x e^{x+2y}}$$

$$PI = \frac{-1}{3}\left[\frac{x^2y}{2} + \frac{xy}{3} + \frac{x^2}{3} + \frac{x^3}{6} + \frac{2x}{9}\right] - x e^{x+2y}$$

Complete solution $\Rightarrow$ $z = CF + PI$

$$\boxed{z = f_1(y+x) + e^{3x} f_2(y-x) - \frac{1}{3}\left[\frac{x^2y}{2} + \frac{xy}{3} + \frac{x^2}{3} + \frac{x^3}{6} + \frac{2}{9}\right] - x e^{x+2y}}$$

Ans

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Q-2 : Solve : $(D+D'-1)^2 z = xy$

$(D+D'-1)^2 z = xy$

$CF = e^x f_1(y-x) + x e^x f_2(y-x)$

$PI = \frac{1}{(D+D'-1)^2} xy$

$PI = \frac{1}{[1-(D+D')]^2} xy$

$PI = [1-(D+D')]^{-2} xy$

Using expansion :

$(1-x)^{-2} = 1+2x+3x^2+4x^3+ \ldots\ldots$

$PI = [1+2(D+D')+3(D+D')^2+ \ldots\ldots] xy$

$PI = [1+2D+2D'+3D^2+3D'^2+6DD'+ \ldots] xy$

$PI = [xy+2D(xy)+2D'(xy)+6DD'(xy)]$

$\boxed{PI = xy+2y+2x+6}$

Complete solution $\Rightarrow$ $Z = CF+PI$

$\boxed{Z = e^x f_1(y-x) + x e^x f_2(y-x) + xy+2y+2x+6}$

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Que-3 : Solve : $(D-3D'-2)^3 z = 6e^{2x}\sin(3x+y)$

$$(D-3D'-2)^3 z = 6e^{2x}\sin(3x+y)$$

$$\boxed{C.F. = e^{2x}f_1(y+3x) + xe^{2x}f_2(y+3x) + x^2e^{2x}f_3(y+3x)}$$

$$P.I. = \frac{1}{(D-3D'-2)^3}\,6e^{2x}\sin(3x+y)$$

Replace D by D+2

$$P.I. = 6e^{2x}\frac{1}{(D+2-3D'-2)^3}\sin(3x+y)$$

$$P.I. = 6e^{2x}\frac{1}{(D-3D')^3}\sin(3x+y)$$

Put $D \to 3$, $D' \to 1$

{Case of failure}

$$PI = 6e^{2x} \times x \times \frac{1}{3(D-3D')^2}\sin(3x+y)$$

Again Put $D \to 3$, $D' \to 1$

Again case of failure

$$PI = 6e^{2x} \times x^2 \times \frac{1}{6(D-3D')}\sin(3x+y)$$

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Put $D \to 3$, $D' \to 1$

{ Again case of failure }

$$PI = \frac{1}{6} e^{2x} \times x^3 \times \sin(3x+y)$$

$$\boxed{PI = x^3 e^{2x} \sin(3x+y)}$$

Complete solution $\Rightarrow$ $z = CF + PI$

$$\boxed{z = e^{2x} f_1(y+3x) + x e^{2x} f_2(y+3x) + x^2 e^{2x} f_3(y+2x) + x^3 e^{2x} \sin(3x+y)}$$

Q-4 : Solve $r - 4s + 4t + p - 2q = e^{x+y}$

$$r - 4s + 4t + p - 2q = e^{x+y}$$

$$(D^2 - 4DD' + 4D'^2 + D - 2D')z = e^{x+y}$$

$$[(D-2D')^2 + (D-2D')]z = e^{x+y}$$

$$(D-2D')(D-2D'+1)z = e^{x+y}$$

$$\boxed{C.F. = f_1(y+2x) + e^{-x} f_2(y+2x)}$$

$$PI = \frac{1}{D^2 - 4DD' + 4D'^2 + D - 2D'} e^{x+y}$$

Put $D \to 1$, $D' \to 1$

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{ Case of failure }

$$PI = x \times \frac{1}{2D - 4D' + 1} e^{x+y}$$

$D \to 1, \; D' \to 1$

$$PI = x \times \frac{1}{2 - 4 + 1} e^{x+y}$$

$$\boxed{PI = -x e^{x+y}}$$

Complete solution $\Rightarrow$ $z = CF + PI$

$$\boxed{z = f_1(y+2x) + e^{-x}(y+2x) - x e^{x+y}}$$

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