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DPP-3 Maths-4 Unit-4

Topic- Random Variable

Q-1- Find the constant k so function f(x) defined as follows be a density function

$$\begin{cases} 1/k, & a \le x \le b \\ 0, & \text{elsewhere} \end{cases}$$

Sol- Since f(x) is a pdf i) $f(x) \ge 0 \quad \forall x$

ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

From ii)

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_{-\infty}^{a} f(x)\,dx + \int_{a}^{b} f(x)\,dx + \int_{b}^{\infty} f(x)\,dx = 1$$

$$\int_{a}^{b} \frac{1}{k}\,dx = 1$$

$$\left[\frac{x}{k}\right]_a^b = 1$$

$$\frac{1}{k}(b-a) = 1$$

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$$\boxed{k = b - a}$$

Ans

Q.2- A continuous random variable X has,

$$f(x) = \begin{cases} \frac{1}{2}(x+1) & , \quad -1 \le x \le 1 \\ 0 & , \quad \text{elsewhere} \end{cases}$$

represent the density. Find mean, variance and S.D.

Sol- Mean :- $E(x) = \int_{-\infty}^{\infty} x f(x)\, dx$

$$E(x) = \int_{-\infty}^{-1} x f(x) dx + \int_{-1}^{1} x f(x) dx + \int_{1}^{\infty} x f(x)\, dx$$

$$= \int_{-1}^{1} \frac{x}{2}(x+1)\, dx$$

$$= \frac{1}{2}\int_{-1}^{1} (x^2 + x)\, dx$$

$$= \frac{1}{2}\left[\frac{x^3}{3} + \frac{x^2}{2}\right]_{-1}^{1}$$

$$= \frac{1}{2}\left[\frac{1}{3} + \frac{1}{2} + \frac{1}{3} - \frac{1}{2}\right]$$

$$\boxed{E(x) = \frac{1}{3}}$$

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ii) $E(x^2) = \int_{-\infty}^{\infty} x^2 f(x)\,dx$

$$E(x^2) = \int_{-\infty}^{-1} x^2 f(x)\,dx + \int_{-1}^{1} x^2 f(x)\,dx + \int_{1}^{\infty} x^2 f(x)\,dx$$

$$E(x^2) = \frac{1}{2}\int_{-1}^{1} (x^3 + x^2)\,dx$$

$$= \frac{1}{2}\left[\frac{x^4}{4} + \frac{x^3}{3}\right]_{-1}^{1}$$

$$= \frac{1}{2}\left[\frac{1}{4} + \frac{1}{3} - \frac{1}{4} + \frac{1}{3}\right]$$

$$= \frac{1}{3}$$

$$\text{Variance} = E(x^2) - [E(x)]^2$$

$$= \frac{1}{3} - \frac{1}{9}$$

$$= \frac{3-1}{9}$$

$$\boxed{\text{Variance} = \frac{2}{9}}$$

Standard deviation = $\sqrt{\text{Variance}}$

$$\boxed{\text{S.D.} = \frac{\sqrt{2}}{3}}$$

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