ENGINEERING MATHEMATICS

UNIT-5 : *Statistical Techniques – III*

(Testing a Hypothesis and Statistical Quality Control)

Lec-08

Today's Target

- $Control\ Charts\ for\ variables\ (\bar{X} - chart\ and\ R - chart)$
- PYQ
- DPP

Statistical Quality control (SQC)

- A quality control system performs inspection, testing and analysis to ensure that the quality of the products produced is as per the quality standards.
- It is called SQC , When statistical techniques are used to control, improve and maintain quality or to solve quality problems.

Techniques of SQC

To control the quality characteristics of the product there are two main techniques.

(i) Process Quality Control

Control the quality of product during production process.

(ii) Product Quality control

Control the quality of product after production.

Control Chart

Control chart is a graphical representation of quality characteristics, which indicates whether the process is under control or not.

Types of control charts

There are many types of control charts. Most commonly used control charts are

(i) Control Charts for variables

They are used to measure quality characteristics

(1) Control chart for sample mean ($\bar{X}-chart$)

(2) Control chart for sample range ($R-chart$)

L – 8

(ii) Control Charts for attributes

They are used to maintain and achieve an acceptable quality level

(1) Control chart for fraction defective ($p-chart$)

(2) Control chart for number of defective ($np-chart$)

(3) Control chart for number of defects ($c-chart$)

L – 9

Control Limits

Control limits are the limits within which variations are acceptable

A Control chart consist of 3 Horizontal lines

(i) Control line (C L) : It is a solid line and represent desired control level of porcess.

(ii) Upper control line (UCL) : It is a dotted line and represents upper tolerance limit.

(iii) Lower control line (LCL) : It is a also a dotted line and repreent lower tolerance limit.



Control Charts for variables

Consider a random sample of size n is drawn during manufacturing process

Sample mean

$$\bar{x} = \frac{x_1 + x_2 + x_3 + \ldots\ldots + x_n}{n} = \frac{\sum x}{n}$$

Sample range

$$R = x_{max} - x_{min}$$

Let K consecutive samples are selected with mean $\bar{x}_1, \bar{x}_2, \bar{x}_3, \ldots \ldots \bar{x}_K$ and range $R_1, R_2, R_3 \ldots . R_K$

Mean of sample mean ($\bar{\bar{x}}$)

$$\bar{\bar{x}} = \frac{\bar{x}_1 + \bar{x}_2 + \bar{x}_3 + \ldots\ldots \bar{x}_K}{K} = \frac{\sum \bar{x}}{K}$$

Mean of sample Range

$$\bar{R} = \frac{R_1 + R_2 + R_3 + \ldots . + r_n}{K}$$

3σ Control limits for $\bar{X}$ – chart

$$CL = \begin{cases} \bar{\bar{x}}, & \text{When tolerance limits are not given} \\ \mu, & \text{When tolerance limits are given} \end{cases}$$

When σ is not given

(i) $ULC = \bar{\bar{x}} + A_2\bar{R}$

(ii) $LCL = \bar{\bar{x}} - A_2\,\bar{R}$

Where A_2 depend on sample size and found from table or given in the question

When σ is given

(i) $ULC = \bar{\bar{x}} + 3\,\sigma_x$

(ii) $LCL = \bar{\bar{x}} - 3\,\sigma_x$

$$\text{Where, } \sigma_x = \frac{\bar{\sigma}}{\sqrt{n}}$$

Control limits for $R-$ chart

(i) $CL = \bar{R}$

(ii) $LCL = \bar{R}\, D_3$

(iii) $UCL = \bar{R}\, D_4$

Where D_3, D_4 depend on sample size and found form table or given in the question

Process capability (6σ)

$$6\sigma = 6\frac{\bar{R}}{d_2}$$

Where d_2 depend on sample size and found from table or given in the questions.

Q.1 *The given table shows the values of sample mean $\bar{X}$ and the range (R) for 10 samples each of size 5 each.* AKTV-2022-23

Draw mean and range charts and also comment on the state of control of the process.

Sample No.	1	2	3	4	5	6	7	8	9	10
$\bar{X}$	45	46	48	52	53	37	51	46	47	38
R	4	5	6	7	4	5	7	6	6	4

Here $n = 5$, $A_2 = 0.58$, $D_4 = 2.115$ *and* $D_3 = 0$.

$$\bar{\bar{X}} = \frac{\Sigma \bar{X}}{n} = \frac{45+46+48+52+53+37+51+46+47+38}{10} = \frac{463}{10}$$

$$\boxed{\bar{\bar{X}} = 46.3}$$

$$\bar{R} = \frac{\Sigma R}{n} = \frac{4+5+6+7+4+5+7+6+6+4}{10} = \frac{54}{10}$$

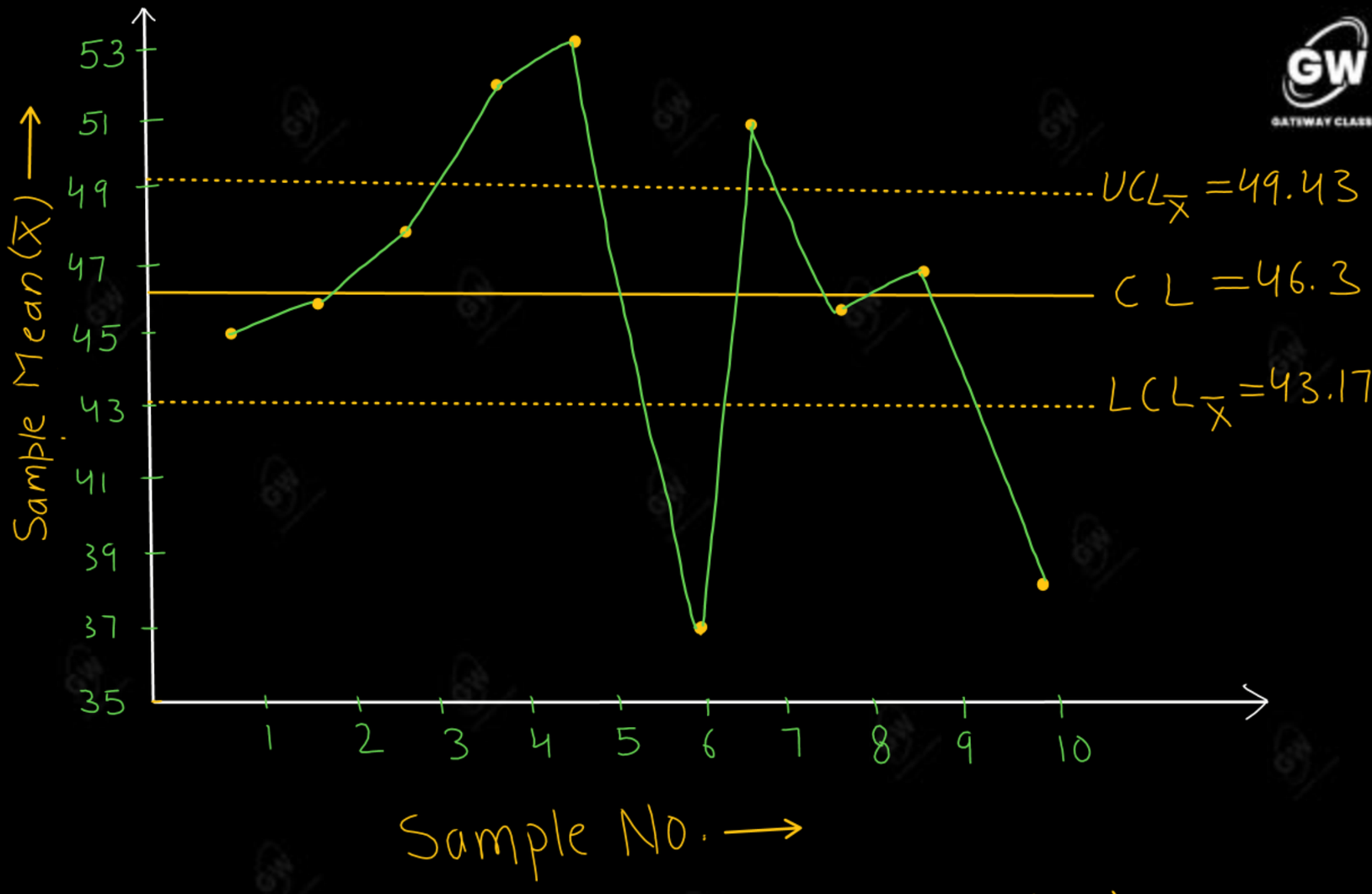
$$\boxed{\bar{R} = 5.4}$$

control limits for $\bar{X}$-Chart

$CL_{\bar{X}} = 46.3$

$$UCL_{\bar{X}} = \bar{\bar{X}} + A_2\bar{R}$$
$$= 46.3 + 0.58 \times 5.4$$
$$= 49.43$$

$$LCL_{\bar{X}} = \bar{\bar{X}} - A_2\bar{R}$$
$$= 46.3 - 0.58 \times 5.4$$
$$= 43.17$$



Hence, process is not under control

control limits for R-chart

$$CL_R = \bar{R}$$

$$\boxed{CL_R = 5.4}$$

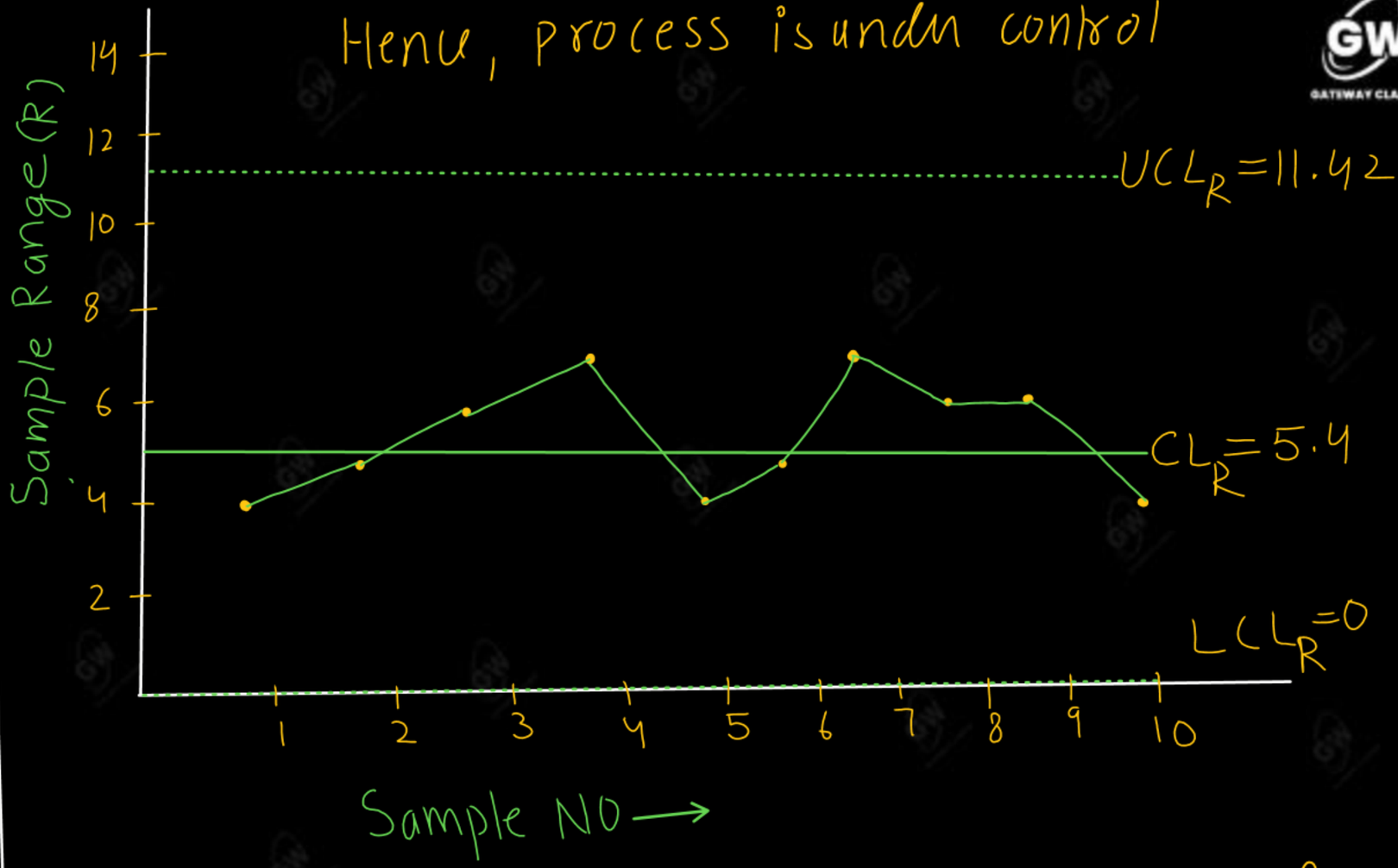
$$UCL_R = D_4\bar{R} = 2.115 \times 5.4$$

$$\boxed{UCL_R = 11.42}$$

$$LCL_R = D_3\bar{R} = 0$$

$$\boxed{LCL_R = 0}$$

Hence, process is under control



conclusion: Process is not under statistical control

Q.2 *The following are the mean lengths and ranges of lengths of a finished product from 10 samples each of size 5. The specification limits for length are* 200 ± 5 *cm. Construct* $\bar{X}-$ *chart and* $R-$ *chart and examine whether then process is undercontrol and state your recommendations.*

Sample No.	1	2	3	4	5	6	7	8	9	10
Mean $(\bar{X})$	201	198	202	200	203	204	199	196	199	201
Range (R)	5	0	7	3	3	7	2	8	5	6

Assume for $n = 5$, $A_2 = 0.58$, $D_4 = 2.11$ *and* $D_3 = 0$.

Given

$\mu = \bar{\bar{X}} = 200$

$$\bar{R} = \frac{\Sigma R}{n} = \frac{5+0+7+3+3+7+2+8+5+6}{10} = \frac{46}{10}$$

$$\bar{R} = 4.6$$

Control limits for $\bar{X}$-chart

$$CL_{\bar{X}} = 200$$

$$UCL_{\bar{X}} = \bar{\bar{X}} + A_2\bar{R}$$

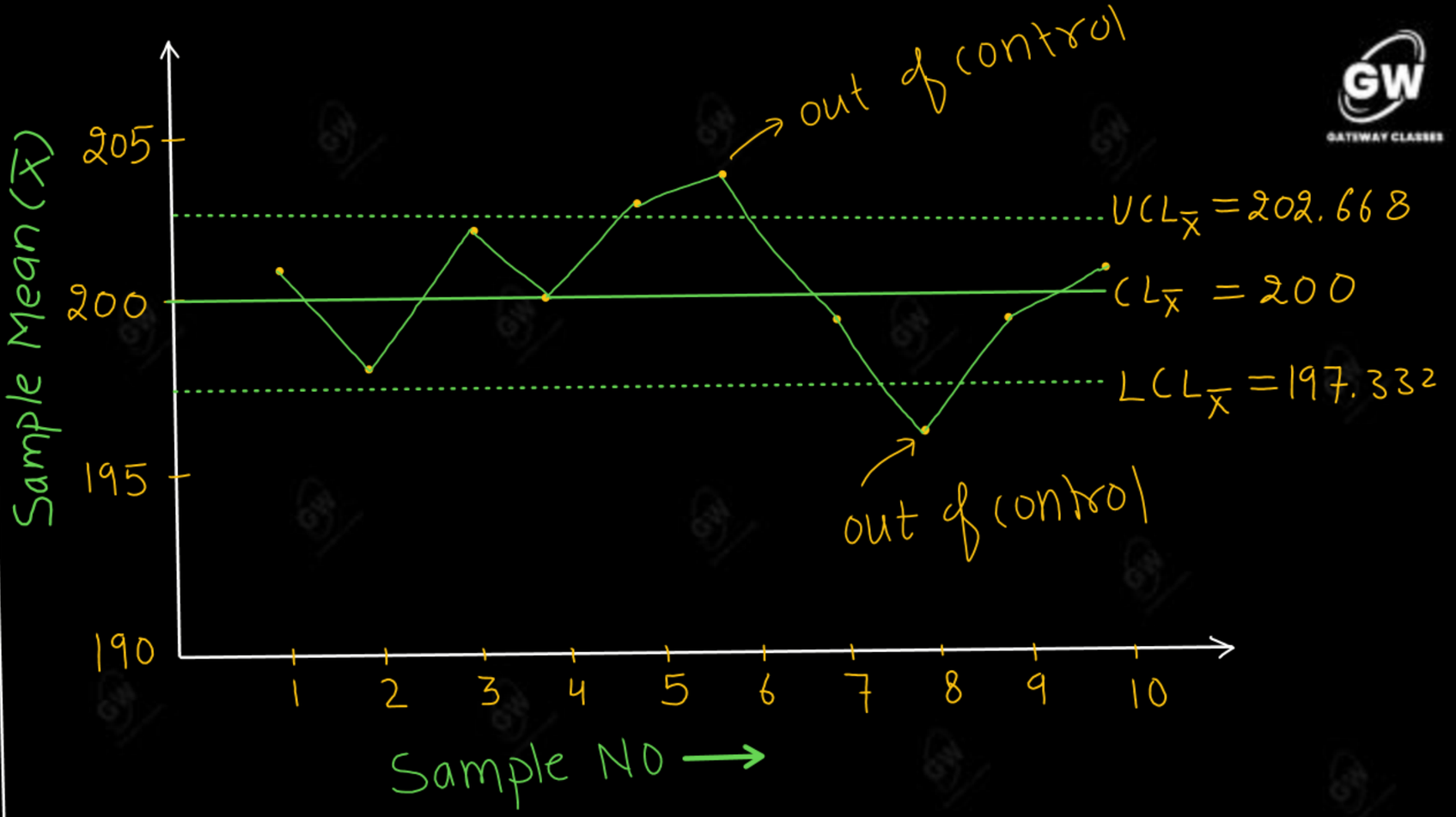
$$= 200 + 0.58 \times 4.6$$

$$= 202.668$$

$$LCL_{\bar{X}} = \bar{\bar{X}} - A_2\bar{R}$$

$$= 200 - 0.58 \times 4.6$$

$$= 197.332$$



All points do not lie within control limits, Hence process is not under control

Control limits for R-chart

$$CL = \bar{R}$$

$$CL = 4.6$$

$$UCL_R = D_4 \bar{R}$$

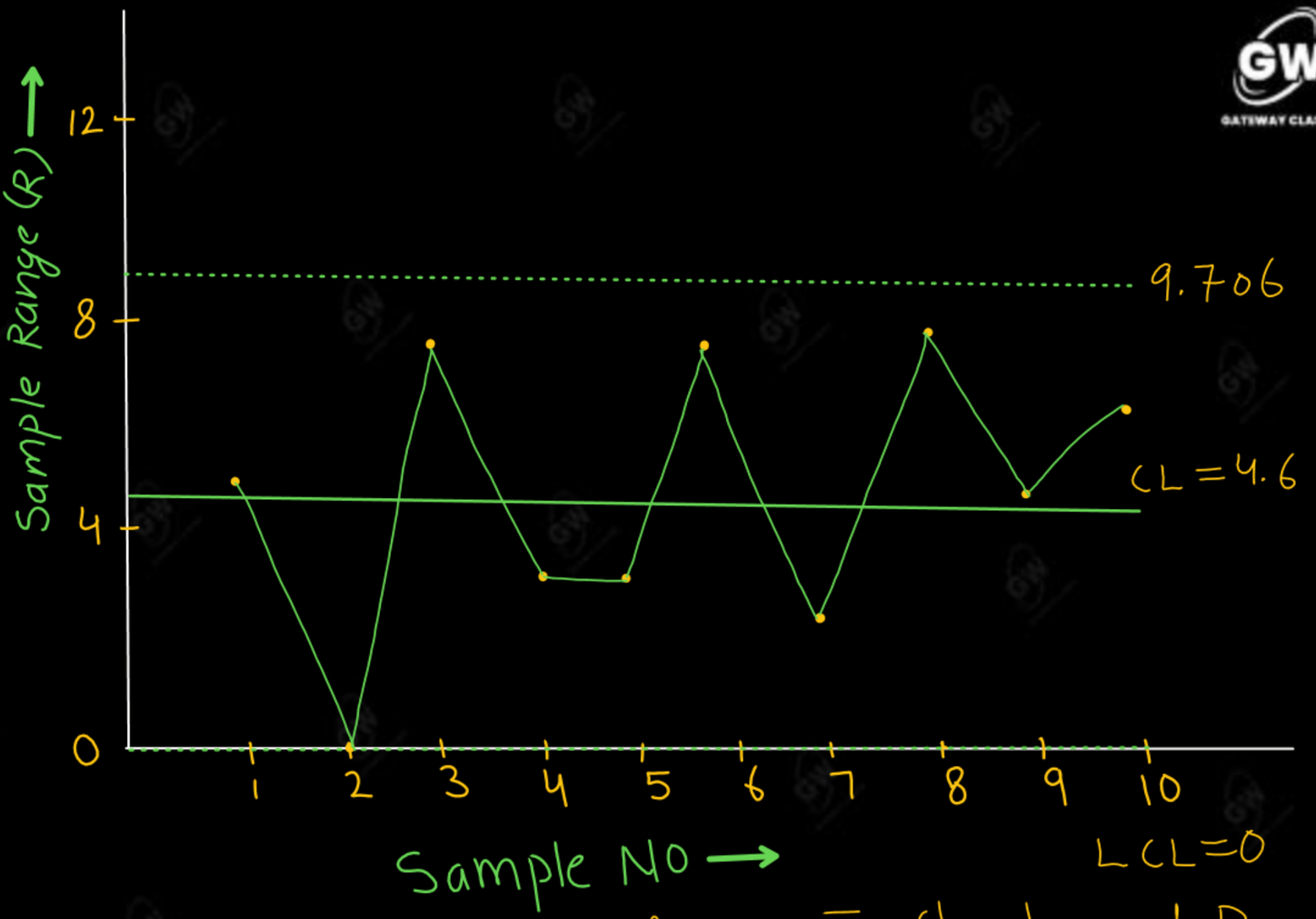
$$= 2.11 \times 4.6$$

$$= 9.706$$

$$LCL_R = D_3 \bar{R}$$

$$= 0 \times 4.6$$

$$= 0$$



Conclusion: It is clear from $\bar{x}$-chart and R-chart that process is not under statistical control

Q.3 *A drilling machine bores holes with a mean diameter of 0.5230 cm and a standard deviation of 0.0032 cm. Calculate the 2 – sigma and 3 – sigma upper and lower control limits for means of sample of 4.* *(GBTU – 2010)*

Given

$\bar{\bar{x}} = 0.5230$

$\sigma = 0.0032$

$n = 4$

2 – Sigma limit

$$UCL = \bar{\bar{x}} + 2\frac{\sigma}{\sqrt{n}}$$

$$= 0.5230 + \frac{2 \times 0.0032}{\sqrt{4}}$$

$$UCL = 0.5262 \text{ cm}$$

$$LCL = \bar{\bar{x}} - 2\frac{\sigma}{\sqrt{n}}$$

$$= 0.5230 - \frac{2 \times 0.0032}{\sqrt{4}}$$

$$LCL = 0.5198 \text{ cm}$$

3 – Sigma Limit

$$UCL = \bar{\bar{x}} + 3\frac{\sigma}{\sqrt{n}}$$

$$UCL = 0.5278 \text{ cm}$$

$$LCL = \bar{\bar{x}} - 3\frac{\sigma}{\sqrt{n}}$$

$$LCL = 0.5182$$

Topic : *Control Charts for variables ($\bar{X}$ – chart and R – chart)*

Q. 1 The following data shows the value of sample mean $\bar{X}$ and arange R for 10 samples of size 5 each. Calculate the values for central line and control limits for $\bar{X}$ – chart and R chart and determine whether the process is under control.

Sample no. :	1	2	3	4	5	6	7	8	9	10
Mean $\bar{X}$:	11.2	11.8	10.8	11.6	11	9.6	10.4	9.6	10.6	10
Range R :	7	4	8	5	7	4	8	4	7	9

Assume for $n = 5$, $A_2 = 0.577$, $D_3 = 0$ *and* $D_4 = 2.115$.

Q. 2 A company manufactures screws to a nominal diameter 0.500 ± 0.030 *cm. Five samples were taken randomly from the manufactured lots and 3 measurements were taken on each sample at different lengths. Following are the readings:*

Sample no.	*Measurement per sample* x*(in cm)*		
	1	2	3
1	0.488	0.489	0.505
2	0.494	0.495	0.499
3	0.498	0.515	0.487
4	0.492	0.509	0.514
5	0.490	0.508	0.499

Calculate the control limits of $\bar{X}$ and R charts. Draw $\bar{X}$ and R charts and examine whether the process s in statistical control ?

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